Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced Level

Tuesday 26 June 2007 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. A bead of mass 0.5 kg is threaded on a smooth straight wire. The only forces acting on the bead are a constant force $(4\mathbf{i} + 7\mathbf{j} + 2\mathbf{k})$ N and the normal reaction of the wire. The bead starts from rest at the point A with position vector $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ m and moves to the point B with position vector $(4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$ m.

Find the speed of the bead when it reaches B.

(4)

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2. At time t seconds, the position vector of a particle P is $\mathbf{r}$ metres, where $\mathbf{r}$ satisfies the differential equation

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} + 3\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \mathbf{0}.$$

When $t = 0$, the velocity of P is $(8\mathbf{i} - 12\mathbf{j})$ m s^{-1}.

Find the velocity of P when $t = \frac{2}{3}\ln 2$.

(7)

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3. A uniform rod AB, of mass m and length $2a$, is free to rotate about a fixed smooth axis which passes through A and is perpendicular to the rod. The rod has angular speed ω when it strikes a particle P of mass m and adheres to it. Immediately before the rod strikes P, P is at rest and at a distance x from A. Immediately after the rod strikes P, the angular speed of the rod is $\frac{3}{4}\omega$.

Find x in terms of a.

(5)

4.

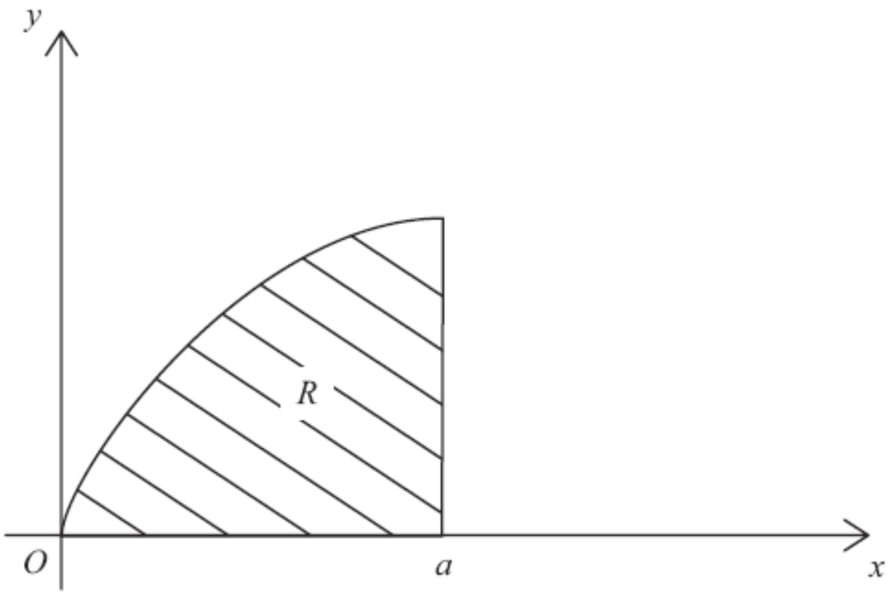

Figure 1

A region R is bounded by the curve $y^2 = 4ax$ ($y > 0$), the x-axis and the line $x = a$ ($a > 0$), as shown in Figure 1. A uniform solid S of mass M is formed by rotating R about the x-axis through 360°. Using integration, prove that the moment of inertia of S about the x-axis is $\frac{4}{3}Ma^2$.

(You may assume without proof that the moment of inertia of a uniform disc, of mass m and radius r, about an axis through its centre perpendicular to its plane is $\frac{1}{2}mr^2$.)

(7)

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5. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a rigid body, where

$\mathbf{F}_1 = (3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$ N and

$\mathbf{F}_2 = (5\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N.

The force $\mathbf{F}_1$ acts at the point with position vector $(\mathbf{i} - 2\mathbf{j})$ m , and the force $\mathbf{F}_2$ acts at the point with position vector $(3\mathbf{i} - \mathbf{k})$ m. The two forces are equivalent to a single force $\mathbf{F}$ acting at the point with position vector $(\mathbf{i} - \mathbf{k})$ m, together with a couple $\mathbf{G}$.

(*a*) Find $\mathbf{F}$.

(1)

(*b*) Find the magnitude of $\mathbf{G}$.

(8)

6.

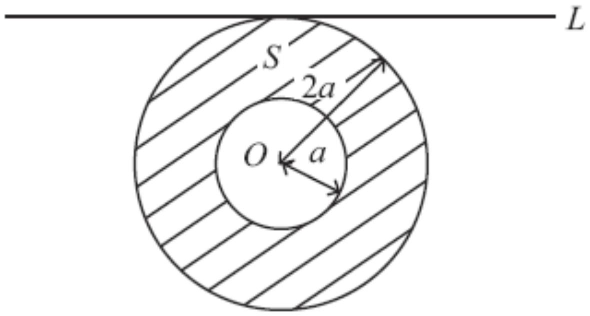

Figure 2

A lamina S is formed from a uniform disc, centre O and radius $2a$, by removing the disc of centre O and radius a, as shown in Figure 2. The mass of S is M.

(*a*) Show that the moment of inertia of S about an axis through O and perpendicular to its plane is $\frac{5}{2}Ma^2$.

(3)

The lamina is free to rotate about a fixed smooth horizontal axis L. The axis L lies in the plane of S and is a tangent to its outer circumference, as shown in Figure 2.

(*b*) Show that the moment of inertia of S about L is $\frac{21}{4}Ma^2$.

(4)

S is displaced through a small angle from its position of stable equilibrium and, at time $t = 0$, it is released from rest. Using the equation of motion of S, with a suitable approximation,

(*c*) find the time when S first passes through its position of stable equilibrium.

(6)

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7. A motor boat of mass M is moving in a straight line, with its engine switched off, across a stretch of still water. The boat is moving with speed U when, at time $t = 0$, it develops a leak. The water comes in at a constant rate so that at time t, the mass of water in the boat is λt. At time t the speed of the boat is v and it experiences a total resistance to motion of magnitude $2\lambda v$.

(*a*) Show that $(M + \lambda t)\dfrac{\mathrm{d}v}{\mathrm{d}t} + 3\lambda v = 0$.

(6)

(*b*) Show that the time taken for the speed of the boat to reduce to $\frac{1}{2}U$ is $\dfrac{M}{\lambda}(2^{\frac{1}{3}} - 1)$.

(6)

The boat sinks when the mass of water inside the boat is M.

(*c*) Show that the boat does not sink before the speed of the boat is $\frac{1}{2}U$.

(2)

8.

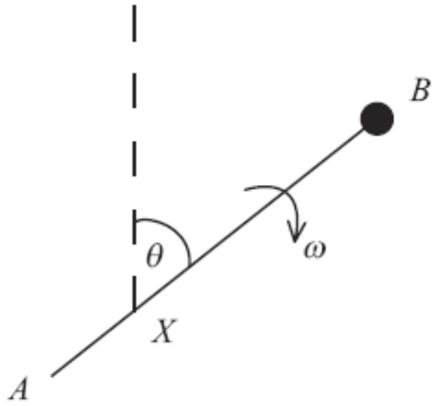

Figure 3

A uniform rod AB has mass $3m$ and length $2a$. It is free to rotate in a vertical plane about a smooth fixed horizontal axis through the point X on the rod, where $AX = \frac{1}{2}a$. A particle of mass m is attached to the rod at B. At time $t = 0$, the rod is vertical, with B above A, and is given an initial angular speed $\sqrt{\frac{g}{a}}$. When the rod makes an angle θ with the upward vertical, the angular speed of the rod is ω, as shown in Figure 3.

(*a*) By using the principle of the conservation of energy, show that

$$\omega^2 = \frac{g}{2a}(5 - 3\cos\theta).$$

(8)

(*b*) Find the angular acceleration of the rod when it makes an angle θ with the upward vertical.

(3)

When $\theta = \phi$, the resultant force of the axis on the rod is in a direction perpendicular to the rod.

(*c*) Find $\cos\phi$.

(5)

TOTAL FOR PAPER: 75 MARKS

END

June 2007
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
1.	$\mathbf{d} = \mathbf{AB} = 3\mathbf{i} + \mathbf{j} - 5\mathbf{k}$ $\frac{1}{2}.0.5v^2 = \mathbf{F}.\mathbf{d} = (4\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}).(3\mathbf{i} + \mathbf{j} - 5\mathbf{k})$ $v = 6 \text{ m s}^{-1}$	B1 M1 A1 A1 (4)
2.	$\frac{d\mathbf{v}}{dt} + 3\mathbf{v} = \mathbf{0}$ $\text{IF} = e^{3t} \Rightarrow \frac{d(\mathbf{v}e^{3t})}{dt} = \mathbf{0}$ $\Rightarrow \mathbf{v}e^{3t} = \mathbf{A}$ $t = 0, \mathbf{v} = 8\mathbf{i} - 12\mathbf{j} \Rightarrow \mathbf{v} = (8\mathbf{i} - 12\mathbf{j})e^{-3t}$ $t = \frac{2}{3}\ln 2 \Rightarrow \mathbf{v} = (8\mathbf{i} - 12\mathbf{j})e^{-2\ln 2} = (2\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$	B1 M1 A1 M1 A1 DM1 A1 (7)
3.	$\frac{4}{3}ma^2\omega = (\frac{4}{3}ma^2 + mx^2)\frac{3}{4}\omega$ $\Rightarrow x = \frac{2}{3}a$	M1A1A1 DM1A1 (5)
4.	$V = \pi\int_0^a 4ax \, dx$ $= 2\pi a^3$ $\delta m = \frac{M}{2\pi a^3}.\pi 4ax\delta x \qquad (= \frac{2M}{a^2}x\delta x)$ $\delta I = \frac{1}{2}\frac{2M}{a^2}x\delta x.y^2 = \frac{4M}{a}x^2\delta x$ $I = \frac{4M}{a}\int_0^a x^2 \, dx = \frac{4}{3}Ma^2$	M1 A1 M1 M1A1 DM1A1 (7)

Question Number	Scheme	Marks
5. (a)	$\mathbf{F} = \Sigma\, \mathbf{F}_i = (8\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$ N	B1 (1)
(b)	$\Sigma\, \mathbf{r}_i \times \mathbf{F}_i = (\mathbf{i} - 2\mathbf{j})\text{x}(3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}) + (3\mathbf{i} - \mathbf{k})\text{x}(5\mathbf{i} - \mathbf{j} + 2\mathbf{k})$	M1
	$= (12\mathbf{i} + 6\mathbf{j} + 10\mathbf{k}) + (-\mathbf{i} - 11\mathbf{j} - 3\mathbf{k}) \quad (= (11\mathbf{i} - 5\mathbf{j} + 7\mathbf{k})$	A2 , 1 , 0
	$\mathbf{G} + (\mathbf{i} - \mathbf{k})\text{x}(8\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) = (11\mathbf{i} - 5\mathbf{j} + 7\mathbf{k})$	M1
	$(3\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}),\ \mathbf{G} = (8\mathbf{i} - \mathbf{j} + 4\mathbf{k})$	B1, A1
	$\mathbf{G} = \sqrt{8^2 + (-1)^2 + 4^2} = 9$ Nm.	M1A1 (8)
		(9)
6. (a)	$I_O = \frac{M}{3\pi a^2}\left(\frac{\pi}{2}(2a)^2(2a)^2 - \frac{\pi}{2}(a)^2(a)^2\right)$	M1 A1
	$= \frac{5Ma^2}{2}$ *	A1 (3)
(b)	$I_{diameter} = \frac{1}{2}\frac{5Ma^2}{2}$ (perp. axes)	M1 A1
	$I_L = \frac{5Ma^2}{4} + M(2a)^2$ (parallel axes)	M1
	$= \frac{21Ma^2}{4}$	A1 (4)
(c)	M(L), $-Mg2a\sin\theta = \frac{21Ma^2}{4}\ddot{\theta}$	M1 A1
	$\sin\theta \approx \theta \Rightarrow \ddot{\theta} = -\frac{8g}{21a}\theta$, so SHM	DM1 A1
	Time$= \frac{1}{4}2\pi\sqrt{\frac{21a}{8g}}$	DM1
	$= \frac{\pi}{2}\sqrt{\frac{21a}{8g}}$	A1 (6)
		(13)

7.(a)	$(m+\delta m)(v+\delta v)-mv=-2\lambda v\delta t$	M1 A1
	$m\frac{\mathrm{d}v}{\mathrm{d}t}+v\frac{\mathrm{d}m}{\mathrm{d}t}=-2\lambda v$	
	$\frac{\mathrm{d}m}{\mathrm{d}t}=\lambda$; $m=M+\lambda t$	B1 ; B1
	$(M+\lambda t)\frac{\mathrm{d}v}{\mathrm{d}t}+3\lambda v=0$ *	D M1 A1
		(6)
(b)	$-\int\frac{\mathrm{d}v}{3\lambda v}=\int\frac{\mathrm{d}t}{(M+\lambda t)}$	M1
	$-\frac{1}{3\lambda}\left[\ln v\right]_{u}^{\frac{1}{2}u}=\frac{1}{\lambda}\left[\ln(M+\lambda t)\right]_0^T$	DM1 A1
	$\frac{1}{3}\ln 2=\ln\frac{(M+\lambda T)}{M}$	DM1
	$T=\frac{M}{\lambda}(2^{\frac{1}{3}}-1)$ *	DM1 A1
		(6)
(c)	Sinks at $T_S=\frac{M}{\lambda}$	
	Reaches speed $\frac{1}{2}U$ at $T=\frac{M}{\lambda}(2^{\frac{1}{3}}-1)$	
	Since $(2^{\frac{1}{3}}-1)<1$, $T<T_S$	M1
	i.e. Reaches speed $\frac{1}{2}U$ before it sinks	A1c.s.o.
		(2)
		(14)

8.(a)	MI of rod + particle $= \frac{1}{12}3m(2a)^2 + 3m(\frac{1}{2}a)^2, + m(\frac{3}{2}a)^2$ $= 4ma^2$ $\frac{1}{2}4ma^2(\omega^2 - \frac{g}{a}) = 3mg\frac{a}{2}(1-\cos\theta) + mg\frac{3a}{2}(1-\cos\theta)$ $\omega^2 = \frac{g}{2a}(5 - 3\cos\theta)$	M1, M1 A1 M1 A **ft** A1 DM1 A1 (8)
(b)	$4ma^2\ddot{\theta} = 3mg\frac{1}{2}a\sin\theta + mg\frac{3}{2}a\sin\theta$ $\Rightarrow \quad \ddot{\theta} = \frac{3g\sin\theta}{4a}$	M1 A1 **ft** A1 (3)
(c)	$F + 4mg\cos\theta = 3m\frac{1}{2}a\omega^2 + m\frac{3}{2}a\omega^2$ $= 3ma\frac{g}{2a}(5 - 3\cos\theta)$ $\Rightarrow F = \frac{mg}{2}(15 - 17\cos\theta)$ When $\theta = \phi$, $F = 0$ $\Rightarrow \cos\phi = \frac{15}{17}$	M1 A1 DM1 DM1 A1 (5) **16**