Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced Level

Monday 18 June 2007 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 6 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. A small ball is moving on a horizontal plane when it strikes a smooth vertical wall. The coefficient of restitution between the ball and the wall is e. Immediately before the impact the direction of motion of the ball makes an angle of 60° with the wall. Immediately after the impact the direction of motion of the ball makes an angle of 30° with the wall.

(*a*) Find the fraction of the kinetic energy of the ball which is lost in the impact.

(6)

(*b*) Find the value of e.

(4)

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2. A lorry of mass M moves along a straight horizontal road against a constant resistance of magnitude R. The engine of the lorry works at a constant rate RU, where U is a constant. At time t, the lorry is moving with speed v.

(*a*) Show that $Mv\frac{\mathrm{d}v}{\mathrm{d}t} = R(U - v)$.

(3)

At time $t = 0$, the lorry has speed $\frac{1}{4}U$ and the time taken by the lorry to attain a speed of $\frac{1}{3}U$ is $\frac{kMU}{R}$, where k is a constant.

(*b*) Find the exact value of k.

(7)

3.

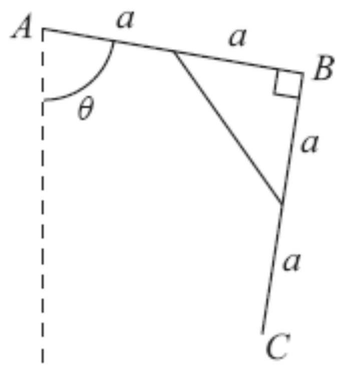

Figure 1

A framework consists of two uniform rods *AB* and *BC*, each of mass *m* and length $2a$, joined at *B*. The mid-points of the rods are joined by a light rod of length $a\sqrt{2}$, so that angle *ABC* is a right angle. The framework is free to rotate in a vertical plane about a fixed smooth horizontal axis. This axis passes through the point *A* and is perpendicular to the plane of the framework. The angle between the rod *AB* and the downward vertical is denoted by θ, as shown in Fig. 1.

(*a*) Show that the potential energy of the framework is

$$-mga(3\cos\theta + \sin\theta) + \text{constant.}$$

(4)

(*b*) Find the value of θ when the framework is in equilibrium, with *B* below the level of *A*.

(4)

(*c*) Determine the stability of this position of equilibrium.

(4)

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4. At 12 noon, ship *A* is 20 km from ship *B*, on a bearing of 300°. Ship *A* is moving at a constant speed of 15 km h^{-1} on a bearing of 070°. Ship *B* moves in a straight line with constant speed V km h^{-1} and intercepts *A*.

(*a*) Find, giving your answer to 3 significant figures, the minimum possible for V.

(3)

It is now given that $V = 13$.

(*b*) Explain why there are two possible times at which ship *B* can intercept ship *A*.

(2)

(*c*) Find, giving your answer to the nearest minute, the earlier time at which ship *B* can intercept ship *A*.

(8)

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5. A smooth uniform sphere A has mass $2m$ kg and another smooth uniform sphere B, with the same radius as A, has mass m kg. The spheres are moving on a smooth horizontal plane when they collide. At the instant of collision the line joining the centres of the spheres is parallel to $\mathbf{j}$. Immediately **after** the collision, the velocity of A is $(3\mathbf{i} - \mathbf{j})$ m s^{-1} and the velocity of B is $(2\mathbf{i} + \mathbf{j})$ m s^{-1}. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(*a*) Find the velocities of the two spheres immediately before the collision.

(7)

(*b*) Find the magnitude of the impulse in the collision.

(2)

(*c*) Find, to the nearest degree, the angle through which the direction of motion of A is deflected by the collision.

(4)

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6. A small ball is attached to one end of a spring. The ball is modelled as a particle of mass 0.1 kg and the spring is modelled as a light elastic spring AB, of natural length 0.5 m and modulus of elasticity 2.45 N. The particle is attached to the end B of the spring. Initially, at time $t = 0$, the end A is held at rest and the particle hangs at rest in equilibrium below A at the point E. The end A then begins to move along the line of the spring in such a way that, at time t seconds, $t \leq 1$, the downward displacement of A from its initial position is $2 \sin 2t$ metres. At time t seconds, the extension of the spring is x metres and the displacement of the particle below E is y metres.

(*a*) Show, by referring to a simple diagram, that $y + 0.2 = x + 2 \sin 2t$.

(3)

(*b*) Hence show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2} + 49y = 98 \sin 2t$.

(5)

Given that $y = \dfrac{98}{45} \sin 2t$ is a particular integral of this differential equation,

(*c*) find y in terms of t.

(5)

(*d*) Find the time at which the particle first comes to instantaneous rest.

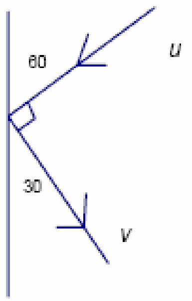
(4)

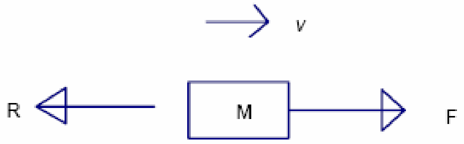
TOTAL FOR PAPER: 75 MARKS

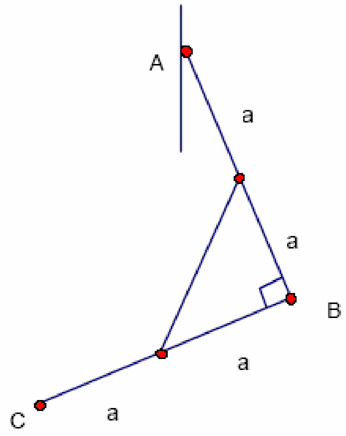
END

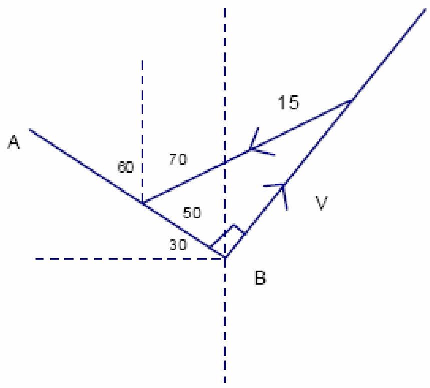
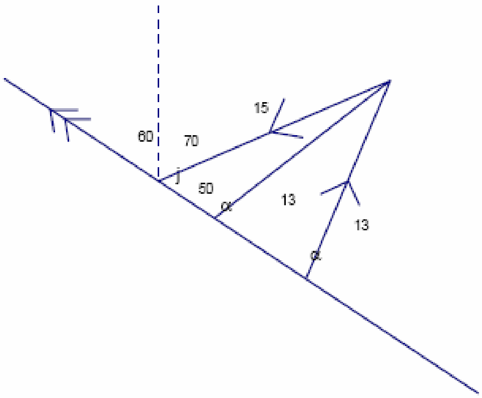
June 2007
6680 Mechanics M4
Mark Scheme

General:
For M marks, correct number of terms, dimensionally correct, all terms that need resolving are resolved.
Omission of g from a resolution is an accuracy error, not a method error.
Omission of mass from a resolution is a method error.
Omission of a length from a moments equation is a method error.
Where there is only one method mark for a question or part of a question, this is for a *complete* method.
Omission of units is not (usually) counted as an error.

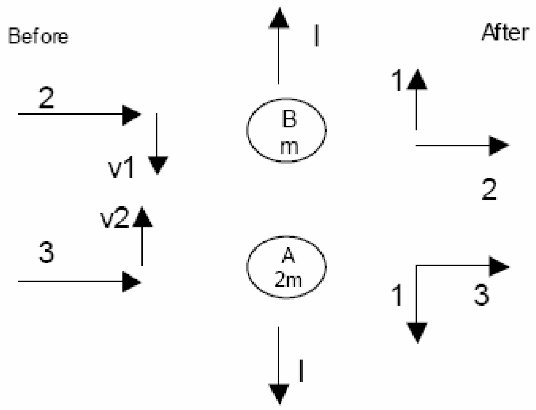
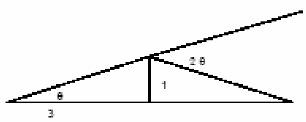
Question Number	Scheme	Marks
1(a)	 $u\cos 60^0 = v\cos 30^0$ $u = v\sqrt{3}$ KE lost $= \frac{1}{2}m(u^2 - v^2)$ Fraction of KE lost $= 1 - \left(\frac{v}{u}\right)^2$ $= 1 - \frac{1}{3} = \frac{2}{3}$ or at least 3sf ending in 7 or $\frac{3}{4}(1-e^2)$	M1A1 A1 M1 DM1 A1 (6)
(b)	$e = \frac{v\sin 30°}{u\sin 60°}$ $= \frac{v}{u}\cdot\frac{1}{\sqrt{3}}$ $= \frac{1}{3}$	M1A1 DM1 A1 (4)
a)	M1 Resolve parallel to the wall *Alt: reasonable attempt at equation connecting two variables* A1 Correct as above or equivalent *equation correct* A1 u in terms of v or v.v. - not necessarily simplified. *or ration of the two variables correct* M1 expression for KE lost DM1 expression in one variable for fraction of KE lost – could be u/v as above A1 cao	*The first three marks can be awarded in (b) if not seen in (a)*
b)	M1 Use NIL perpendicular to the wall and form equation in e A1 Correct unsimplified expression as above or $eu\sin 60^o = v\sin 30^o$ or equivalent DM1 Substitute values for trig functions or use relationship from (a) and rearrange to e = ….. A1 cao accept decimals to at least 3sf	*The first two marks can be awarded in (a)*

2(a)		
	$F = \frac{Ru}{v}$	B1
	R(→), $\frac{Ru}{v} - R = M\frac{dv}{dt}$	M1
	$R(u-v) = Mv\frac{dv}{dt}$ *	A1
		(3)
(b)	$\int_0^T dt = \frac{M}{R}\int_{\frac{1}{4}U}^{\frac{1}{3}U}\frac{v\,dv}{u-v}$	M1A1
	$\Rightarrow T = \frac{M}{R}\int_{\frac{1}{4}U}^{\frac{1}{3}U} -1 + \frac{u}{u-v}dv$	DM1
	$= \frac{M}{R}\left[-v - u\ln(u-v)\right]_{\frac{1}{4}U}^{\frac{1}{3}U}$	A1
	$= \frac{M}{R}\left[-\frac{u}{3} - u\ln\left(\frac{2u}{3}\right) + \frac{u}{4} + u\ln\left(\frac{3u}{4}\right)\right]$ $\left(C = -\frac{Mu}{R}\left(\ln\frac{3u}{4} + \frac{1}{4}\right)\right)$	M1
	$= \frac{Mu}{R}\left(-\frac{1}{12} + \ln\frac{9}{8}\right)$	M1
	Hence $k = \ln\frac{9}{8} - \frac{1}{12}$	A1
		(7)
a)	B1 Correct expression involving the driving force. M1 Use of F = ma to form a differential equation. Condone sign errors. a must be expressed as a derivative, but could be any valid form. A1 Rearrange to **given form**.	
b)	M1 Separate the variables A1 Separation correct (limits not necessarily seen at this stage) DM1 Attempt a complete integration process A1 Integration correct M1 Correct use of both limits – substitute and subtract. Condone wrong order. M1 Simplify to find k from an expression involving a logarithm A1 Answer as given, or exact equivalent. Need to see k = lnA + B	

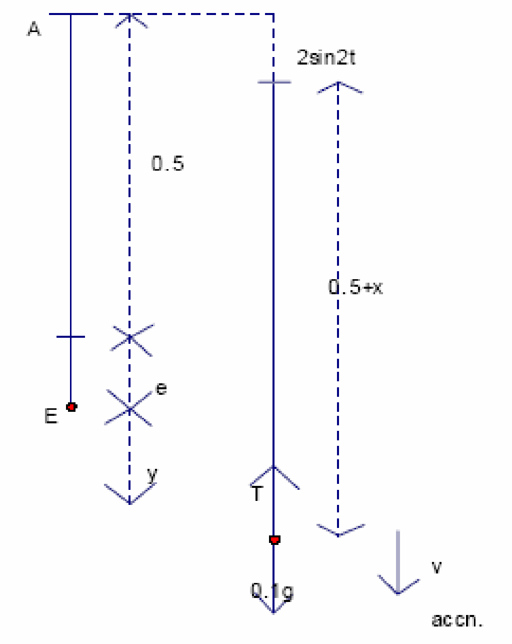
Question Number	Scheme	Marks
3. (a)	 $V = -mga\cos\theta - mg(2a\cos\theta + a\sin\theta)$ $= -mga(3\cos\theta + \sin\theta) \quad (+const)$ *	M1A1A1 A1 (4)
(b)	$\dfrac{dV}{d\theta} = -mga(-3\sin\theta + \cos\theta)$	M1A1
	$= 0 \quad \Rightarrow \quad \tan\theta = \dfrac{1}{3}$	M1
	$\Rightarrow$ θ = 0.32(1)^c or 18.4° accept awrt	A1 (4)
(c)	$\dfrac{d^2V}{d\theta^2} = -mga(-3\cos\theta - \sin\theta)$ $= mga(3\cos\theta + \sin\theta)$	M1A1
	Hence, when θ = 0.32^c, $\dfrac{d^2V}{d\theta^2} > 0$	M1
	i.e. stable	A1 (4)
a)	M1 Expression for the potential energy of the two rods. Condone trig errors. Condone sign errors. BC term in two parts A1 correct expression for AB A1 correct expression for BC A1 Answer **<u>as given</u>** .	
b)	M1 Attempt to differentiate V. Condone errors in signs and in constants. A1 Derivative correct M1 Set derivative = 0 and rearrange to a single trig function in θ A1 Solve for θ or M1A1 find the position of the center of mass M1A1 form and solve trig equation for θ	
c)	M1 Differentiate to obtain the second derivative A1 Derivative correct M1 Determine the sign of the second derivative A1 Correct conclusion. cso Or: M1 Find the value of $\dfrac{dV}{d\theta}$ on both sides of the minimum point A1 signs correct M1 Use the results to determine the nature of the turning point A1 Correct conclusion, cso.	*These 4 marks are dependent on the use of derivatives*

4 (a)	 Fix A $v_{\min} = 15\sin 50°$ $= 11.5$ km h^{-1} (3 s.f.) or: triangle without the right angle identified and $\frac{15}{\sin\theta} = \frac{v_B}{\sin 50}$ $\Rightarrow v_B = \frac{15\sin 50}{\sin\theta}$ minimum value $\Rightarrow \theta = 90$ for M1 As above for A1A1	M1A1 A1 (3)
(b)	 Ambiguous Sine Rule: 2 possible solutions for α	B1B1 (2)
(c)	$\frac{\sin\alpha}{15} = \frac{\sin 50}{13}$	M1A1
	<u>α = 62.1°</u> (or 118°) (smaller value gives larger relative velocity)	A1
	$\Rightarrow$ either $v = 13\cos 62.1 + 15\cos 50 = 15.72 kmh^{-1}$	M1A1
	Or $v^2 = 15^2 + 13^2 - 390\cos 67.9 = 247.27$ $v = 15.7 kmh^{-1}$	M1 A1
	Time = $\frac{20}{their\,15.72.....}$ = 1.272…… hrs	M1 A1
	Earliest time is 13.16hrs or 13.17 hrs accept 1.16 (pm) or 1.17 (pm)	A1 (8)

a)	M1 Velocity of B relative to A is in the direction of the line joining AB. Minimum V requires a right angled triangle. Convincing attempt to find the correct side. A1 15 x sin(their 50^0) A1 Q specifies 3sf, so 11.5 only	
b)	B1B1 Convincing argument B1B0 Argument with some merit	
c)	M1 Use of Sine Rule A1 Correct expression A1 (2 possible values,) pick the correct value. M1 Use trig. to form an equation in v A1 correct equation M1 $time = \frac{dis\tan ce}{speed}$ A1ft correct expression with their v (not necessarily evaluated) A1 correct time in hours & minutes Or: M1 Use of cosine rule A1 $13^2 = 15^2 + v^2 - 2\times 15\times v\times \cos 50$ A1 (Award after the next two marks) 15.72 or awrt 15.72 M1 Attempt to solve the equation for v A1 $\frac{30\cos 50 \pm \sqrt{(30\cos 50)^2 - 4\times 56}}{2}$ (15.72 or 3.562) Finish as above	

5. (a)		
	CLM: $2v_2 - v_1 = 1 - 2 = -1$	M1A1
	NIL: $1+1 = \frac{1}{2}(v_1 + v_2)$	M1A1
	$\therefore v_2 = 1, v_1 = 3$ *Dependent on both M's above*	DM1
	Horizontal components unchanged (i.e. 2 & 3) *Independent of all other marks*	A1
	$\mathbf{v}_A = 3\mathbf{i} + \mathbf{j}$; $\mathbf{v}_B = 2\mathbf{i} - 3\mathbf{j}$	A1
		(7)
(b)	For B: I = m(1-(-3)) = 4m	M1A1
	(Or For A: -I = 2m(-1 – 1) $\therefore$ I = 4m)	(2)
(c)	$\begin{pmatrix}3\\1\end{pmatrix}.\begin{pmatrix}3\\-1\end{pmatrix} = \sqrt{3^2+1^2}.\sqrt{3^2+(-1)^2}\cos\theta$	M1A1
	$\Rightarrow 8 = 10\cos\theta$	M1
	$\theta = 37°$	A1
	Alternative:	(4)
	 M1	
	where $\tan\theta = \frac{1}{3}$ A1	
	required angle is 2θ M1A1	

a)	M1 Conservation of momentum along the line of centres. Condone sign errors A1 equation correct M1 Impact law along the line of centres. *e* must be used correctly, but condone sign errors. A1 equation correct. The signs need to be consistent between the two equations M1 Solve the simultaneous equations for their v_1 and v_2. A1 **i** components correct – independent mark A1 $\mathbf{v}_A$ & $\mathbf{v}_B$ correct	
b)	M1 Impulse = change in momentum for one sphere. Condone order of subtraction. A1 Magnitude correct.	
c)	M1 Any complete method to find the trig ratio of a relevant angle. A1 $\cos\theta = \frac{4}{5}$, $\tan\frac{\theta}{2} = \frac{1}{3}$, ... Or M1 find angle of approach to the line of centres and angle after collision. A1 values correct. (both 71.56 …..) M1 solve for θ A1 37^0 (Q specifies nearest degree) Special case: candidates who act as if the line of centres is in the direction of **i**: CLM u+2v = 8 NIL v-u = 2 u=4/3, v=10/3 4/3i + j ; 10/3i – j Impulse 2m-4/3m = 2/3m $\frac{10+1}{\sqrt{10}\sqrt{\frac{109}{9}}} = \cos\theta$ θ = 1.70^0 Work is equivalent, so treat as a MR: M1A0M1A0M1A1A1 M1A1 M1A1M1A1	

6 (a)		At E, $\frac{2.45e}{0.5} = 0.1g$ $\Rightarrow e = 0.2$ $\Rightarrow$ $0.5(orl) + 0.2 + y = 2\sin 2t + 0.5(orl) + x$ $\Rightarrow 0.2 + y = 2\sin 2t + x$ *	M1 A1 B1 (3)
(b)	$0.1g - T = 0.1\ddot{y}$ R(↓) $0.1g - \frac{2.45x}{0.5} = 0.1\ddot{y}$ $0.98 - 4.9(0.2 + y - 2\sin 2t) = 0.1\ddot{y}$ ($-4.9y + 9.8\sin 2t = 0.1\ddot{y}$) $\Rightarrow \frac{d^2y}{dt^2} + 49y = 98\sin 2t$ *		M*1 M1 DM*1A1 A1 cso (5)
(c)	CF is $y = A\cos 7t + B\sin 7t$ Hence GS is $y = A\cos 7t + B\sin 7t + \frac{98}{45}\sin 2t$ t = 0, y = o: 0 = A so, $y = B\sin 7t + \frac{98}{45}\sin 2t$ $\dot{y} = 7B\cos 7t + \frac{196}{45}\cos 2t$ t = 0, $\dot{y} = 0 : 0 = 7B + \frac{196}{45}$ $\Rightarrow B = -\frac{28}{45}$ $\Rightarrow y = \frac{14}{45}(7\sin 2t - 2\sin 7t)$		M1 A1 B1 M1 A1 (5)
(d)	$\dot{y} = \frac{14}{45}(14\cos 2t - 14\cos 7t)$ $\dot{y} = 0 \Rightarrow \cos 2t = \cos 7t$ $\Rightarrow 7t = 2k\pi \pm 2t$ $k=1 \Rightarrow 9t = 2\pi$ (or $5t=2\pi$) $t = \frac{2\pi}{9}$, accept 0.698s, 0.70s.		B1 M1 M1 A1 (4)

a)	M1 Hooke's law to find extension at equilibrium A1 cao B1 Q specifies reference to a diagram. Correct reasoning leading to **given answer**.	
b)	M1 Use of F=ma. Weight, tension and acceleration. Condone sign errors. M1 Substitute for tension in terms of x M1 Use given result to substitute for x in terms of y A1 Correct unsimplified equation A1 Rearrange to **given form** cso.	
c)	M1 Correct form for CF A1 GS for y correct B1 Deduce coefficient of cos $\theta = 0$ M1 Differentiate their y and substitue t=0, $\dot{y} = 0$ A1 y in terms of t. Any exact equivalent.	
d)	B1 $\dot{y}$ correct M1 set $\dot{y} = 0$ M1 solve for general solution for t: $7t = 2k\pi \pm 2t$ or: $\sin\frac{9t}{2} \times \sin\frac{5t}{2} = 0 \Rightarrow \sin\frac{9t}{2} = 0 or \sin\frac{5t}{2} = 0$ A1 Select smallest value	