

Paper Reference(s)

**6679/01**

# **Edexcel GCE**

## **Mechanics M3**

### **Advanced/Advanced Subsidiary**

**Monday 10 June 2013 – Morning**

**Time: 1 hour 30 minutes**

**Materials required for examination**

Mathematical Formulae (Pink)

**Items included with question papers**

Nil

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.**

#### **Instructions to Candidates**

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In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper.

Answer ALL the questions.

You must write your answer for each question in the space following the question.

Whenever a numerical value of  $g$  is required, take  $g = 9.8 \text{ m s}^{-2}$ .

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

#### **Information for Candidates**

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A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

The marks for the parts of questions are shown in round brackets, e.g. **(2)**.

There are 7 questions in this question paper. The total mark for this paper is 75.

There are 28 pages in this question paper. Any blank pages are indicated.

#### **Advice to Candidates**

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You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

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1.

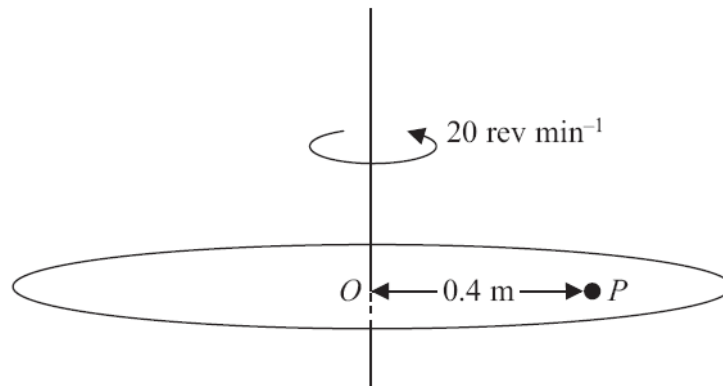


Figure 1

A rough disc is rotating in a horizontal plane with constant angular speed 20 revolutions per minute about a fixed vertical axis through its centre  $O$ . A particle  $P$  rests on the disc at a distance  $0.4 \text{ m}$  from  $O$ , as shown in Figure 1. The coefficient of friction between  $P$  and the disc is  $\mu$ . The particle  $P$  is on the point of slipping.

Find the value of  $\mu$ .

(6)

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2. A particle  $P$  of mass  $0.5 \text{ kg}$  is moving along the positive  $x$ -axis in the positive  $x$ -direction. The only force on  $P$  is a force of magnitude  $\left(2t + \frac{1}{2}\right) \text{ N}$  acting in the direction of  $x$  increasing, where  $t$  seconds is the time after  $P$  leaves the origin  $O$ . When  $t = 0$ ,  $P$  is at rest at  $O$ .

(a) Find an expression, in terms of  $t$ , for the velocity of  $P$  at time  $t$  seconds.

(3)

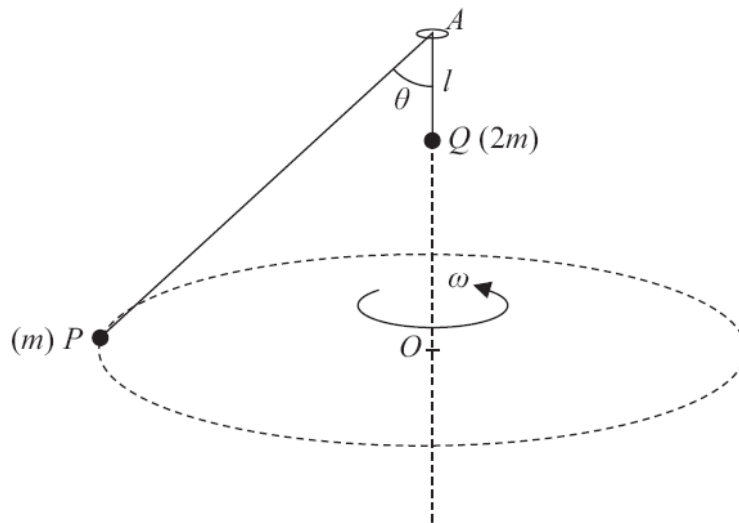
The particle passes through the point  $A$  with speed  $6 \text{ m s}^{-1}$ .

(b) Find the distance  $OA$ .

(6)

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3.



**Figure 2**

Two particles  $P$  and  $Q$ , of mass  $m$  and  $2m$  respectively, are attached to the ends of a light inextensible string of length  $6l$ . The string passes through a small smooth fixed ring at the point  $A$ . The particle  $Q$  is hanging freely at a distance  $l$  vertically below  $A$ . The particle  $P$  is moving in a horizontal circle with constant angular speed  $\omega$ . The centre  $O$  of the circle is vertically below  $A$ . The particle  $Q$  does not move and  $AP$  makes a constant angle  $\theta$  with the downward vertical, as shown in Figure 2.

Show that

(i)  $\theta = 60^\circ$ ,

(ii)  $\omega = \sqrt{\left(\frac{2g}{5l}\right)}$ .

**(8)**

4. A particle  $P$  of mass 2 kg is attached to one end of a light elastic string of natural length 1.2 m. The other end of the string is attached to a fixed point  $O$  on a rough horizontal plane. The coefficient of friction between  $P$  and the plane is  $\frac{2}{5}$ . The particle is held at rest at a point  $B$  on the plane, where  $OB = 1.5$  m. When  $P$  is at  $B$ , the tension in the string is 20 N. The particle is released from rest.

(a) Find the speed of  $P$  when  $OP = 1.2$  m.

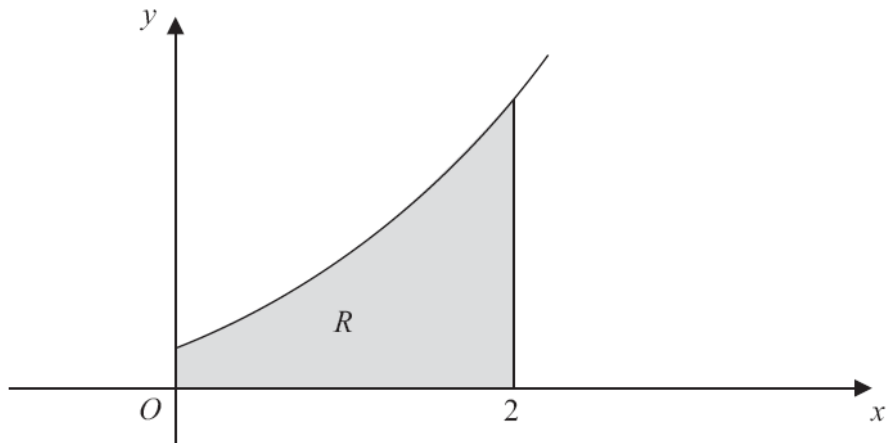
**(7)**

The particle comes to rest at the point  $C$ .

(b) Find the distance  $BC$ .

**(2)**

5.

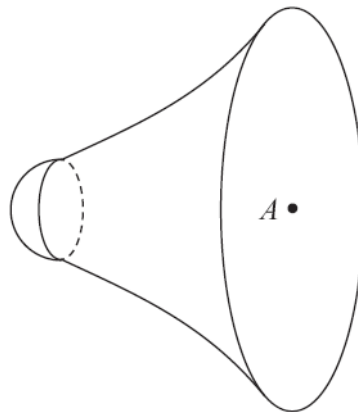


**Figure 3**

The shaded region  $R$  is bounded by the curve with equation  $y = (x + 1)^2$ , the  $x$ -axis, the  $y$ -axis and the line with equation  $x = 2$ , as shown in Figure 3. The region  $R$  is rotated through  $2\pi$  radians about the  $x$ -axis to form a uniform solid  $S$ .

(a) Use algebraic integration to find the  $x$  coordinate of the centre of mass of  $S$ .

**(8)**



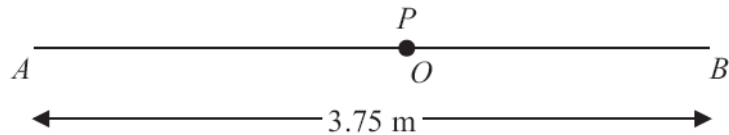
**Figure 4**

A uniform solid hemisphere is fixed to  $S$  to form a solid  $T$ . The hemisphere has the same radius as the smaller plane face of  $S$  and its plane face coincides with the smaller plane face of  $S$ , as shown in Figure 4. The mass per unit volume of the hemisphere is 10 times the mass per unit volume of  $S$ . The centre of the circular plane face of  $T$  is  $A$ . All lengths are measured in centimetres.

(b) Find the distance of the centre of mass of  $T$  from  $A$ .

**(5)**

6.



**Figure 5**

The points  $A$  and  $B$  are 3.75 m apart on a smooth horizontal floor. A particle  $P$  has mass 0.8 kg. One end of a light elastic spring, of natural length 1.5 m and modulus of elasticity 24 N, is attached to  $P$  and the other end is attached to  $A$ . The ends of another light elastic spring, of natural length 0.75 m and modulus of elasticity 18 N, are attached to  $P$  and  $B$ . The particle  $P$  rests in equilibrium at the point  $O$ , where  $AOB$  is a straight line, as shown in Figure 5.

(a) Show that  $AO = 2.4$  m.

**(4)**

The point  $C$  lies on the straight line  $AOB$  between  $O$  and  $B$ . The particle  $P$  is held at  $C$  and released from rest.

(b) Show that  $P$  moves with simple harmonic motion.

**(5)**

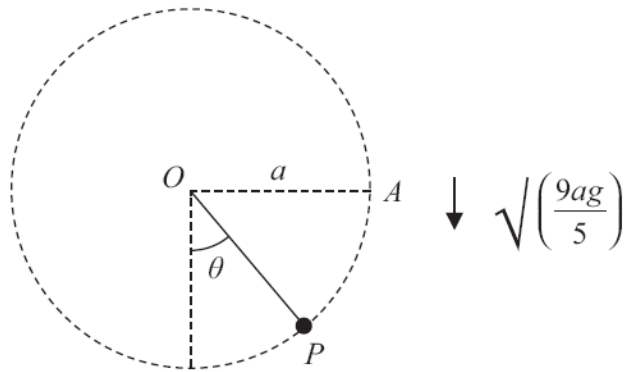
The maximum speed of  $P$  is  $\sqrt{2}$  m s<sup>-1</sup>.

(c) Find the time taken by  $P$  to travel 0.3 m from  $C$ .

**(5)**

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7.



**Figure 6**

A particle  $P$  of mass  $5m$  is attached to one end of a light inextensible string of length  $a$ . The other end of the string is attached to a fixed point  $O$ . The particle is held at the point  $A$ , where  $OA = a$  and  $OA$  is horizontal, as shown in Figure 6. The particle is projected vertically downwards with speed  $\sqrt{\left(\frac{9ag}{5}\right)}$ . When the string makes an angle  $\theta$  with the downward vertical through  $O$  and the string is still taut, the tension in the string is  $T$ .

(a) Show that  $T = 3mg(5\cos\theta + 3)$ . (6)

At the instant when the particle reaches the point  $B$  the string becomes slack.

(b) Find the speed of  $P$  at  $B$ . (3)

At time  $t = 0$ ,  $P$  is at  $B$ .

At time  $t$ , before the string becomes taut once more, the coordinates of  $P$  are  $(x, y)$  referred to horizontal and vertical axes with origin  $O$ . The  $x$ -axis is directed along  $OA$  produced and the  $y$ -axis is vertically upward.

(c) Find

(i)  $x$  in terms of  $t$ ,  $a$  and  $g$ ,

(ii)  $y$  in terms of  $t$ ,  $a$  and  $g$ . (7)

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**TOTAL FOR PAPER: 75 MARKS**

**END**

Question Number	Scheme	Marks
1.	$R(\uparrow) \quad R = mg$ $F = \mu mg$ $20 \text{ revs per min} = \frac{20}{60} \times 2\pi \text{ rad s}^{-1}$ $\left( = \frac{2}{3} \pi \text{ rad s}^{-1} \right)$ $R(\rightarrow) \quad \mu mg = m \times 0.4 \times \left( \frac{2}{3} \pi \right)^2$ $\mu = \frac{0.4 \times 4\pi^2}{9g}$ $\mu = 0.18 \text{ or } 0.179$	<p>B1</p> <p>M1A1</p> <p>M1A1ft</p> <p>A1</p> <p style="text-align: right;"><b>[6]</b></p>

#### Notes for Question 1

B1 for resolving vertically and using  $F = \mu R$  to obtain  $F = \mu mg$ . This may not be seen explicitly, but give B1 when seen used in an equation.

M1 for attempting to change revs per minute to  $\text{rad s}^{-1}$ , must see  $(2)\pi$ . (Can use 60 or  $60^2$ )

A1 for  $\frac{20}{60} \times 2\pi$  ( $\text{rad s}^{-1}$ ) oe

M1 for NL2 horizontally along the radius - acceleration in either form for this mark,  $F$  or  $\mu mg$  or  $\mu m$  all allowed.  $r$  to be 0.4 now or later. This is not dependent on the previous M mark.

A1ft for  $\mu mg = m \times 0.4 \times \left( \frac{2}{3} \pi \right)^2$  follow through on their  $\omega$

A1cso for  $\mu = 0.18$  or  $0.179$ , **must be 2 or 3 sf.**

**NB:** Use of  $\leq$ : is allowed, provided used correctly, until the final statement, which must be  $\mu = \dots$

Question Number	Scheme	Marks
<p><b>2</b></p> <p>(a)</p> <p>(b)</p>	$\left(2t + \frac{1}{2}\right) = 0.5 \frac{dv}{dt}$ $\int (4t + 1) dt = \int dv$ $2t^2 + t = v + c$ $t = 0 \quad v = 0 \quad c = 0$ $v = 2t^2 + t \text{ m s}^{-1}$ $\frac{dx}{dt} = 2t^2 + t$ $x = \frac{2}{3}t^3 + \frac{1}{2}t^2 + k$ $t = 0 \quad x = 0 \quad k = 0$ $x = \frac{2}{3}t^3 + \frac{1}{2}t^2$ $v = 6 \quad 6 = 2t^2 + t \quad 2t^2 + t - 6 = 0$ $(2t - 3)(t + 2) = 0 \quad t = \frac{3}{2}$ $x = \frac{2}{3} \times \left(\frac{3}{2}\right)^3 + \frac{1}{2} \left(\frac{3}{2}\right)^2$ $x = \frac{27}{8} \text{ (oe 3.4, 3.375, 3.38) m}$	<p>M1</p> <p>M1dep <i>c</i> not needed</p> <p>A1 inc the value for <i>c</i> (3)</p> <p>M1</p> <p>A1</p> <p>M1A1</p> <p>M1dep</p> <p>A1 cso (6)</p> <p style="text-align: right;"><b>[9]</b></p>



## Notes for Question 2

(a)

M1 for NL2 with acceleration in the form  $\frac{dv}{dt}$ , seen explicitly or implied by the integration

mass can be 0.5 or  $m$

M1dep for integrating with respect to  $t$  - constant not needed

A1cso for **showing** that  $c = 0$  and giving the final result  $v = 2t^2 + t$  Must see  $t = 0, v = 0$  as a minimum

*By definite integration:*

M1 as above

M1dep for integrating, ignore limits

A1 for substituting the limits 0 and  $v$  and 0 and  $t$  and obtaining  $v = 2t^2 + t$

(b)

M1 for integrating their  $v$  with respect to  $t$  constant not needed

A1 for **showing** that  $k = 0$  If no constant shown this mark is lost.

M1 for setting  $v = 6$  using their answer from (a) **and** attempting to solve the resulting quadratic equation, any valid method. If solved by calculator, **both** solutions must be shown.

A1 for  $t = \frac{3}{2}$  negative solution need not be shown with an algebraic solution

M1dep for using **their** (positive) value for  $t$  to obtain  $x = \dots$  If two positive values were obtained, then allow M1 for substituting either value. Dependent on the first M1 of (b) but not the second.

A1cso for  $x = \frac{27}{8}$  (oe eg 3.375, 3.38) (All marks for (b) must have been awarded)

*By definite integration:*

M1 for integrating their  $v$  with respect to  $t$  limits not needed

A1 for correct integration with lower limits 0.

M1 for setting  $v = 6$  using their answer from (a) **and** attempting to solve the resulting quadratic equation, any valid method. If solved by calculator, **both** solutions must be shown.

A1 for  $t = \frac{3}{2}$  negative solution need not be shown with an algebraic solution

M1dep for substituting **their** limits into **their** integrated  $v$  (sub should be shown). Dependent on the first M1 of (b) but not the second

A1cso for  $x = \frac{27}{8}$  (oe eg 3.375, 3.38)

Question Number	Scheme	Marks
<p><b>3</b></p> <p><b>(i)</b></p>	<p>For <math>Q</math>    <math>T = 2mg</math></p> <p>For <math>P</math>    <math>T \cos \theta = mg</math></p> <p style="text-align: center;"><math>\cos \theta = \frac{1}{2} \quad \theta = 60^\circ \quad *</math></p>	<p>B1</p> <p>M1</p> <p>A1cso</p>
<p><b>(ii)</b></p>	<p>For <math>P \rightarrow T \sin \theta = mr\omega^2</math></p> <p style="text-align: center;"><math>2mg \sin \theta = m \times 5l \sin \theta \times \omega^2</math></p> <p style="text-align: center;"><math>\omega^2 = \frac{2g}{5l} \quad \omega = \sqrt{\frac{2g}{5l}} \quad *</math></p>	<p>M1A1</p> <p>M1depA1</p> <p>A1cso</p> <p style="text-align: right;"><b>[8]</b></p>

### Notes for Question 3

In this question, award marks as though the question is not divided into two parts - ie give marks for equations wherever seen.

(i)

B1 for using  $Q$  (no need to state  $Q$  being used) to state that  $T = 2mg$  or  $T_Q = 2mg$  with  $T_P = T_Q$  seen or implied later.

M1 for attempting to resolve vertically for  $P$   $T$  must be resolved but sin/cos interchange or omission of  $g$  are accuracy errors.

$mg + 2mg = T + T \cos \theta$  gets M0

A1cso for combining the two equations to obtain  $\theta = 60^\circ$  \*

**NB:** This is a "show" question, so if no expression is seen for  $T$  and just  $2mg \cos \theta = mg$  shown, award 0/3 as this equation could have been produced from the required result, so insufficient working.

(ii)

M1 for attempting NL2 for  $P$  along the radius. The mass used must be  $m$  if the particle is not stated to be  $P$ ; a mass of  $2m$  would imply use of  $Q$ .  $T$  must be resolved. Acceleration can be in either form.

A1 for  $T \sin \theta = m r \omega^2$  or  $T \frac{\sqrt{3}}{2} = m r \omega^2$

M1 dep for eliminating  $T$  between the two equations for  $P$  and substituting for  $r$  in terms of  $l$  and  $\theta$  dependent on the second but not the first M mark.

A1 for  $2mg \sin \theta = m \times 5l \sin \theta \times \omega^2$  or  $\frac{T \sin \theta}{T \cos \theta} = \tan \theta = 5l \sin \theta \left( \frac{\omega^2}{g} \right)$   $\theta$  or  $60^\circ$

A1cso for re-arranging to obtain  $\omega = \sqrt{\frac{2g}{5l}}$  \* ensure the square root is correctly placed

*Alternatives:* Some candidates "cancel" the  $\sin \theta$  without ever showing it.

M1A1 for  $T = m \times 5l \omega^2$

M1A1 for  $2mg = 5ml \omega^2$

A1cso as above

**Vector Triangle method: Triangle must be seen**

$T = 2mg$  B1

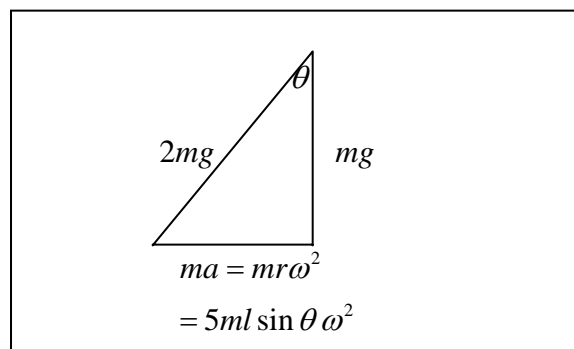
$\cos \theta = \frac{mg}{2mg}$  M1

$\theta = 60^\circ$  A1

Correct triangle M1A1

$\sin \theta = \frac{5ml \sin \theta \omega^2}{2mg}$  M1A1

$\omega = \dots$  A1cso (as above)



Question Number	Scheme	Marks
<p><b>4</b></p> <p><b>(a)</b></p>	$T = \frac{\lambda x}{l}$ $20 = \frac{\lambda \times 0.3}{1.2}$ $\lambda = 80 \text{ N}$ $\text{Initial EPE} = \frac{\lambda x^2}{2l} = \frac{80 \times 0.3^2}{2.4} (= 3 \text{ J})$ $\frac{80 \times 0.3^2}{2.4} - 0.4 \times 2g \times 0.3 = \frac{1}{2} \times 2v^2$ $v^2 = 0.648$ $v = 0.80 \text{ or } 0.805 \text{ m s}^{-1}$ <p><b>(b)</b> Comes to rest <math>0.4 \times 2g \times y = 3</math></p> $y = \frac{3}{0.4 \times 2 \times 9.8} = 0.38 \text{ or } 0.383 \text{ m}$ <p><i>Alternatives:</i> Energy from string going slack to rest:</p> $\frac{1}{2} \times 2 \times 0.648 = 0.4 \times 2g \times x$ $x = 0.8265\dots$ $y = 0.3 + 0.08265\dots = 0.38 \text{ or } 0.383$ <p>NL2 to obtain the accel when string is slack <math>\left(-\frac{2g}{5}\right)</math> and <math>v^2 = u^2 + 2as</math></p> $0 = 0.648 + 2 \times \left(-\frac{2g}{5}\right) s$ $BC = \frac{0.648 \times 5}{4g} + 0.3 = 0.38 \text{ or } 0.383$	<p>M1A1</p> <p>A1</p> <p>B1</p> <p>M1A1ft</p> <p>A1 (7)</p> <p>M1</p> <p>A1 (2)</p> <p><b>[9]</b></p> <p>M1 Complete method A1</p> <p>M1A1</p>

### Notes for Question 4

(a)

M1 for attempting Hooke's Law, formula must be correct, either explicitly or by correct substitution.

A1 for  $20 = \frac{\lambda \times 0.3}{1.2}$

A1 for obtaining  $\lambda = 80$

B1 for the initial EPE  $\frac{\lambda \times 0.3^2}{2.4}$  (= 3 J) their value for  $\lambda$  allowed. May only be seen in the equation.

M1 for a work-energy equation with one EPE term, one KE term and work done against friction (Award if second EPE/KE terms included **provided** these become 0). The EPE must be dimensionally correct, but need not be fully correct (eg denominator 1.2 instead of 2.4)

A1ft for a completely correct equation follow through their EPE

A1 cao for  $v = 0.80$  or  $0.805$  must be 2 or 3 sf

**NB:** This is damped harmonic motion (due to friction) so all SHM attempts lose the last 4 marks.

(b)

M1 for any **complete** method leading to a value for either  $BC$ . If the distance travelled after the string becomes slack is found the work must be completed by adding 0.3 Their EPE found in (a) used in energy methods.

**MS** method is energy from  $B$  to  $C$  ie work done against friction = loss of EPE.

**OR** Energy from point where the string becomes slack to  $C$  ie work done against friction = loss of KE and completed for the required distance

**OR** NL2 to obtain the acceleration  $\left(-\frac{2g}{5}\right)$  while the string is slack **and**  $v^2 = u^2 + 2as$  to find the distance and completed for the required distance

A1cso for  $BC = 0.38$  or  $0.383$  (m) **must be 2 or 3 sf**

Question Number	Scheme	Marks
5(a)	$V = \int_0^2 \pi y^2 dx = \pi \int_0^2 (x+1)^4 dx$ $= \pi \left[ \frac{1}{5} (x+1)^5 \right]_0^2$ $= \frac{1}{5} \pi [3^5 - 1] \quad \left( = \frac{242\pi}{5} \right)$	M1 A1 M1
	$\int_0^2 \pi y^2 x dx = \pi \int_0^2 x(x+1)^4 dx$ $= \pi \left[ \frac{x(x+1)^5}{5} \right]_0^2 - \pi \int_0^2 \frac{(x+1)^5}{5} dx, = \frac{2 \times 3^5 \pi}{5} - \pi \left[ \frac{(x+1)^6}{30} \right]_0^2$ $\left[ \frac{2 \times 3^5}{5} - \frac{3^6}{30} + \frac{1}{30} \right] \pi \quad (= 72.933... \pi)$	M1 A1 M1
	<b>ALT:</b> by expanding $= \pi \int_0^2 (x^5 + 4x^4 + 6x^3 + 4x^2 + x) dx$ $= \pi \left[ \frac{x^6}{6} + \frac{4}{5} x^5 + \frac{6}{4} x^4 + \frac{4}{3} x^3 + \frac{1}{2} x^2 \right]_0^2$ $= \pi \left[ \frac{2^6}{6} + \frac{4}{5} \times 2^5 + \frac{6}{4} \times 2^4 + \frac{4}{3} \times 2^3 + \frac{1}{2} \times 2^2 \right]$	M1A1 M1
	<b>OR by subst:</b> $\pi \int_1^3 (u-1) u^4 du, = \pi \left[ \frac{u^6}{6} - \frac{u^5}{5} \right]_1^3, = \pi \left[ \frac{3^6}{6} - \frac{3^5}{5} - \left( \frac{1}{6} - \frac{1}{5} \right) \right]$	M1A1M1
(b)	$\bar{x} = \frac{\pi \left[ \frac{2 \times 3^5}{5} - \frac{3^6 - 1}{30} \right]}{\frac{242\pi}{5}} \text{ OR } \frac{\pi \left[ \frac{2^6}{6} + \frac{4 \times 2^5}{5} + \frac{6 \times 2^4}{4} + \frac{4 \times 2^3}{3} + \frac{2^2}{2} \right]}{\frac{242\pi}{5}}, = 1.5068$ <p style="text-align: center;">hemisphere                      S                      T</p> <p>Mass ratio                      <math>10 \times \frac{2\pi}{3} \times 1</math>                      <math>\frac{242\pi}{5}</math>                      <math>\left( \frac{20}{3} + \frac{242}{5} \right) \pi = \frac{826}{15} \pi</math></p> <p>Dist from A                      <math>2 + \frac{3 \times 1}{8}</math>                      0.493                      <math>\bar{x}</math></p> $\frac{20}{3} \times \frac{19}{8} + \frac{242}{5} \times 0.493 = \left( \frac{20}{3} + \frac{242}{5} \right) \bar{x}$ <p><math>\bar{x} = 0.7208... \text{ cm}</math> (awrt 0.72)</p>	M1, A1 (8) B1ft on S B1ft on S M1A1ft A1 (5) <b>[13]</b>

### Notes for Question 5

**NB:** Some candidates will omit  $\pi$  throughout (as they know it cancels). In such cases award all marks if earned. If  $\pi$  is omitted from one integration only but then appears in the result of that integration at the last stage or is then omitted from the second integration, all marks can be gained. But if omitted from one integration, including the last stage, and included with the other mark strictly according to the MS.

(a)

M1 for using  $V = \int_0^2 \pi y^2 dx = \pi \int_0^2 (x+1)^4 dx$  - limits not needed and attempting the integration by inspection or expansion (algebra **must** be seen)

A1 for correct integration - limits not needed

M1 for substituting the correct limits into **their** integrated function - no need to simplify

M1 for attempting to integrate  $\int_0^2 \pi y^2 x dx = \pi \int_0^2 x(x+1)^4 dx$  - limits not needed - by parts. This mark can be awarded once the integral has been expressed as the difference of an appropriate integrated function and an integral

A1 for correct, complete integration  $\pi \left[ \frac{x(x+1)^5}{5} \right]_0^2 - \pi \left[ \frac{(x+1)^6}{30} \right]_0^2$  or  $\frac{2 \times 3^5 \pi}{5} - \pi \left[ \frac{(x+1)^6}{30} \right]_0^2$  Limits not needed

M1 for substituting the correct limits into **their** integrated function - no need to simplify

*Alternative methods for  $\int_0^2 \pi y^2 x dx = \pi \int_0^2 x(x+1)^4 dx$*

M1 for expanding and integrating or making a suitable substitution and attempting the integration - limits not needed

A1 for correct integration - limits not needed

M1 for substituting the correct limits into **their** integrated function - no need to simplify

M1 for using  $\bar{x} = \frac{\int \pi y^2 x dx}{\int \pi y^2 dx}$  Their integrals need not be correct.

A1cao for  $\bar{x} = 1.5068...$  Accept 1.5, 1.51 or better or  $\frac{547}{363}$

(b)

B1ft for correct mass ratio, follow through their volume for  $S$  need  $\pi$  now

B1ft for correct distances, follow through their distance for  $S$ , but remember it must be 2 - answer from (a) if working from A. Distances from the common face are  $-\frac{3}{8}$ , ans from (a),  $\bar{x}$  Distances from other end are  $\frac{5}{8}$ , 1+ ans from (a),  $\bar{x}$

M1 for a dimensionally correct moments equation

A1ft for a fully correct moments equation, follow through their distances and mass ratio

A1cao for 0.7208... Accept 0.72 or better (Exact is  $\frac{1191}{1652}$ )

Question Number	Scheme	Marks
6(a)	$\frac{24e}{1.5} = \frac{18(1.5 - e)}{0.75}$ $16e = 36 - 24e$ $e = 0.9$ $AO = 2.4 \text{ m}^*$	M1A1  A1  A1ft (4)
6(b)	$\frac{18(0.6 - x)}{0.75} - \frac{24(0.9 + x)}{1.5} = m\ddot{x} \text{ or } 0.8\ddot{x}$ $14.4 - 24x - 14.4 - 16x = m\ddot{x} \text{ or } 0.8\ddot{x}$ $\ddot{x} = -\frac{40x}{0.8 \text{ or } m} (= -50x) \therefore \text{SHM}$	M1A1A1   M1depA1 (5)
6(c)	$\ddot{x} = -50x \Rightarrow \omega = \sqrt{50} \text{ or } 5\sqrt{2}$ $\text{max. speed} = \sqrt{2} \Rightarrow a \times 5\sqrt{2} = \sqrt{2}$ $a = \frac{1}{5}$ $-0.1 = 0.2 \cos(5\sqrt{2})t$ $t = \frac{1}{5\sqrt{2}} \cos^{-1}\left(-\frac{1}{2}\right)$ $t = \frac{1}{5\sqrt{2}} \times \frac{2\pi}{3} = \frac{\pi\sqrt{2}}{15} \text{ or } 0.296 \text{ s } (0.2961\dots) \text{ Accept } 0.30, \text{ or better}$	B1  M1  A1  M1   A1 (5)
		<b>[14]</b>



### Notes for Question 6

(a)

M1 for using Hooke's Law for each string, equating the two tensions and solving to find the extension in either string. The extensions should add to 1.5. The formula for Hooke's law must be correct, either shown explicitly in its general form or implicitly by the substitution.

A1 for a correct equation

A1 for  $e = 0.9$

A1cso for 2.4 (m) \*

*Alternative:* Find the ratio of the two extensions and divide 1.5 m in that ratio.

M1 complete method      A1 correct ratio      A1 extension in  $AO$

A1 2.4 (m)

(b)

M1 for an equation of motion for  $P$ . There must be a difference of two tensions. The acceleration can be  $a$  or  $\ddot{x}$  here and  $x$  should be measured from the equilibrium position ( $O$ ) unless a suitable substitution is made later. Mass can be  $m$  or 0.8

A1,A1 for  $\frac{18(0.6-x)}{0.75} - \frac{24(0.9+x)}{1.5} = m\ddot{x}$  or  $0.8\ddot{x}$  or  $a$  instead of  $\ddot{x}$  Give A1A1 if the equation is completely correct and A1 if only one error. Note that if the difference of the tensions is the wrong way round, this is *one* error

M1dep for simplifying to  $\ddot{x} = f(x)$  Must be  $\ddot{x}$  now.

A1 for  $\ddot{x} = -\frac{40x}{0.8 \text{ or } m}$  ( $= -50x$ ) **and the conclusion** (ie  $\therefore$  SHM)

(c)

B1 for  $\omega = \sqrt{50}$  or  $5\sqrt{2}$  need not be shown explicitly

M1 for using max speed  $= a\omega = \sqrt{2}$  with **their**  $\omega$

A1 for  $a = \frac{1}{5}$

M1 for using  $x = a \cos \omega t$  with **their**  $\omega$  and  $a$  and  $x = \pm(0.3 - a)$  **or**  $x = a \sin \omega t$  provided the work is completed by adding a quarter of their period is added to the time to complete the method.

A1cao for  $t = \frac{\pi\sqrt{2}}{15}$  or 0.296s (0.2961...) Accept 0.30 or better

Question Number	Scheme	Marks
7	$T - 5mg \cos \theta = \frac{5mv^2}{a}$	M1A1
(a)	$\frac{1}{2} \times 5mv^2 - \frac{1}{2} \times 5m \times \frac{9ag}{5} = 5mga \cos \theta$ $5mv^2 = 10mga \cos \theta + 9mga$	M1A1
	$T = 5mg \cos \theta + 10mg \cos \theta + 9mg$	M1dep
	$T = 3mg(5 \cos \theta + 3) \quad *$	A1 (6)
(b)	$T = 0 \quad \cos \theta = -\frac{3}{5}$	B1
	$v^2 = \frac{9ag}{5} - \frac{6ag}{5} = \frac{3ag}{5}$	M1
	$v = \sqrt{\frac{3ag}{5}}$	A1 (3)
(c)	<p>horiz comp of vel at B = <math>\sqrt{\frac{3ag}{5}} \times \frac{3}{5}</math></p>	M1
	<p>vert comp = <math>\sqrt{\frac{3ag}{5}} \times \frac{4}{5}</math></p>	M1
(i)	$x = -\frac{4a}{5} + \frac{3}{5} \sqrt{\frac{3ag}{5}} t$	M1depA1
	$y - \frac{3a}{5} = \frac{4}{5} \sqrt{\frac{3ag}{5}} t - \frac{1}{2} gt^2$	M1depA1ft
(ii)	$y = \frac{4}{5} \sqrt{\frac{3ag}{5}} t - \frac{1}{2} gt^2 + \frac{3a}{5}$	A1 (7)
		<b>[16]</b>

### Notes for Question 7

**(a)**

M1 for attempting NL2 along the radius when the string makes an angle  $\theta$  with the downward vertical. The acceleration can be in either form, the weight must be resolved and  $T$  must be included (not resolved). Sin/cos interchange or omission of  $g$  are accuracy errors as is omission of 5 in one or both terms. Radius can be  $a$  or  $r$ .

A1 for a correct equation  $T - 5mg \cos \theta = \frac{5mv^2}{a}$  Acceleration must be in the  $\frac{v^2}{r}$  form now.

M1 for a conservation of energy equation from the horizontal to the same point. There must be a difference of 2 KE terms and a loss of PE term (which may be indicated by a difference of 2 PE terms). The initial KE can be  $\frac{1}{2} \times \text{mass} \times \left( \sqrt{\frac{9ag}{3}} \right)^2$  or  $\frac{1}{2} \times \text{mass} \times u^2$  for this mark. Omission of  $g$  and sin/cos interchange are accuracy errors. Mass can be  $m$  or  $5m$  here or just "mass". Use of  $v^2 = u^2 + 2as$  gets M0

A1 for a fully correct equation  $\frac{1}{2} \times (5m)v^2 - \frac{1}{2} \times (5m) \times \frac{9ag}{5} = (5m)ga \cos \theta$

M1dep for eliminating  $v^2$  between the 2 equations. Dependent on both previous M marks.

A1cso for  $T = 3mg(5 \cos \theta + 3)$  \*

**(b)**

B1 for obtaining  $\cos \theta = -\frac{3}{5}$

M1 for using **their** value for  $\cos \theta$  - must be numerical - in the energy equation to get  $v^2 = \dots$  (no need to simplify) Accept with  $5m$  or  $m$ .

**OR** making  $T = 0$  and  $\cos \theta = -\frac{3}{5}$  (their value) in  $T - 5mg \cos \theta = \frac{5mv^2}{a}$

A1cao for  $v = \sqrt{\frac{3ag}{5}}$  oe Check square root is applied correctly.

**(c)**

M1 for resolving **their**  $v$  to get the horizontal component of the speed at  $B$ . May not be seen explicitly, but seen in their attempt at  $x$ .

M1 for resolving **their**  $v$  to get the vertical component of the speed at  $B$

Both of these M marks can be given if sin and cos are interchanged or numerical substitutions not made.

M1dep for attempting to obtain  $x$  by using the distance from  $B$  to the  $y$ -axis with the horizontal distance travelled (found using their horizontal component, so dependent on the first M1 of (c))

A1cso for  $x = -\frac{4a}{5} + \frac{3}{5} \sqrt{\frac{3ag}{5}} t$

### Notes for Question 7 Continued

M1dep for attempting to obtain  $y$  by using  $s = ut + \frac{1}{2}at^2$  with **their** vertical component and using the initial vertical distance above the  $x$ -axis. Dependent on the second M mark of (c)

A1ft for  $y - \frac{3a}{5} = \frac{4}{5}\sqrt{\frac{3ag}{5}}t - \frac{1}{2}gt^2$  Follow through their initial vertical component

A1cao for  $y = \frac{4}{5}\sqrt{\frac{3ag}{5}}t - \frac{1}{2}gt^2 + \frac{3a}{5}$