Paper Reference(s)

6678

Edexcel GCE

Mechanics M2

Advanced

Friday 22 May 2009 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Orange or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A particle of mass 0.25 kg is moving with velocity $(3\mathbf{i} + 7\mathbf{j})$ m s^{-1} when it receives the impulse $(5\mathbf{i} - 3\mathbf{j})$ N s.

Find the speed of the particle immediately after the impulse.

(5)

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2. At time $t = 0$ a particle P leaves the origin O and moves along the x-axis. At time t seconds the velocity of P is v m s^{-1}, where

$$v = 8t - t^2 .$$

(a) Find the maximum value of v.

(4)

(b) Find the time taken for P to return to O.

(5)

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3. A truck of mass of 300 kg moves along a straight horizontal road with a constant speed of 10 m s^{-1}. The resistance to motion of the truck has magnitude 120 N.

(a) Find the rate at which the engine of the truck is working.

(2)

On another occasion the truck moves at a constant speed up a hill inclined at θ to the horizontal, where $\sin\theta = \frac{1}{14}$. The resistance to motion of the truck from non-gravitational forces remains of magnitude 120 N. The rate at which the engine works is the same as in part (a).

(b) Find the speed of the truck.

(4)

4.

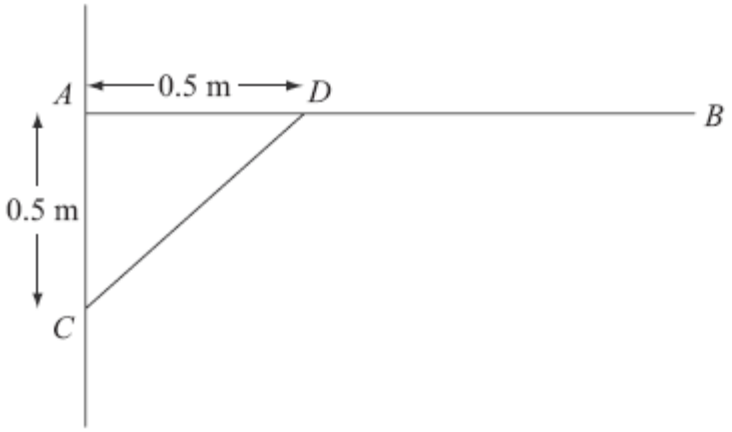

Figure 1

A uniform rod *AB*, of length 1.5 m and mass 3 kg, is smoothly hinged to a vertical wall at *A*. The rod is held in equilibrium in a horizontal position by a light strut *CD* as shown in Figure 1. The rod and the strut lie in the same vertical plane, which is perpendicular to the wall. The end *C* of the strut is freely jointed to the wall at a point 0.5 m vertically below *A*. The end *D* is freely joined to the rod so that *AD* is 0.5 m.

(*a*) Find the thrust in *CD*.

(4)

(*b*) Find the magnitude and direction of the force exerted on the rod *AB* at *A*.

(7)

5.

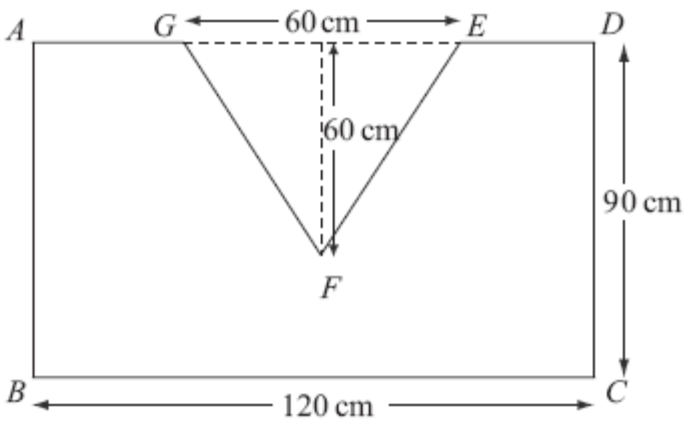

Figure 2

A shop sign *ABCDEFG* is modelled as a uniform lamina, as illustrated in Figure 2. *ABCD* is a rectangle with *BC* = 120 cm and *DC* = 90 cm. The shape *EFG* is an isosceles triangle with *EG* = 60 cm and height 60 cm. The mid-point of *AD* and the mid-point of *EG* coincide.

(*a*) Find the distance of the centre of mass of the sign from the side *AD*.

(5)

The sign is freely suspended from *A* and hangs at rest.

(*b*) Find the size of the angle between *AB* and the vertical.

(4)

6.

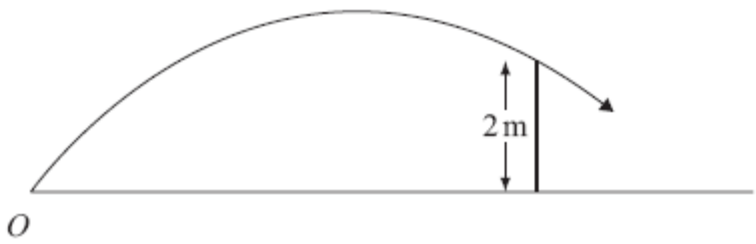

Figure 3

A child playing cricket on horizontal ground hits the ball towards a fence 10 m away. The ball moves in a vertical plane which is perpendicular to the fence. The ball just passes over the top of the fence, which is 2 m above the ground, as shown in Figure 3.

The ball is modelled as a particle projected with initial speed u m s^{-1} from point O on the ground at an angle α to the ground.

(*a*) By writing down expressions for the horizontal and vertical distances, from O of the ball t seconds after it was hit, show that

$$2 = 10 \tan \alpha - \frac{50g}{u^2 \cos^2 \alpha}.$$

(6)

Given that $\alpha = 45°$,

(*b*) find the speed of the ball as it passes over the fence.

(6)

7.

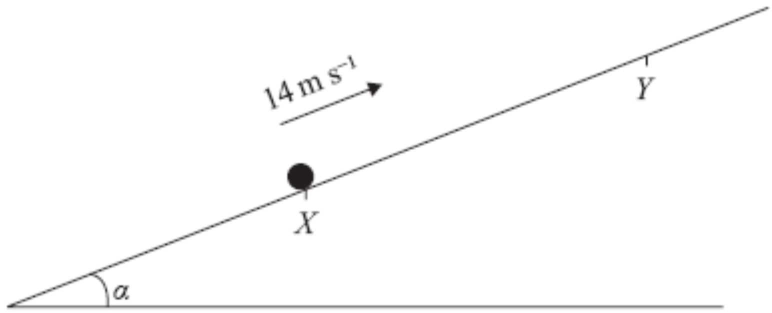

Figure 4

A particle P of mass 2 kg is projected up a rough plane with initial speed 14 m s^{-1}, from a point X on the plane, as shown in Figure 4. The particle moves up the plane along the line of greatest slope through X and comes to instantaneous rest at the point Y. The plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{7}{24}$. The coefficient of friction between the particle and the plane is $\frac{1}{8}$.

(*a*) Use the work-energy principle to show that $XY = 25$ m.

(7)

After reaching Y, the particle P slides back down the plane.

(*b*) Find the speed of P as it passes through X.

(4)

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8. Particles A, B and C of masses $4m$, $3m$ and m respectively, lie at rest in a straight line on a smooth horizontal plane with B between A and C. Particles A and B are projected towards each other with speeds u m s^{-1} and v m s^{-1} respectively, and collide directly. As a result of the collision, A is brought to rest and B rebounds with speed kv m s^{-1}. The coefficient of restitution between A and B is $\frac{3}{4}$.

(*a*) Show that $u = 3v$.

(6)

(*b*) Find the value of k.

(2)

Immediately after the collision between A and B, particle C is projected with speed $2v$ m s^{-1} towards B so that B and C collide directly.

(*c*) Show that there is no further collision between A and B.

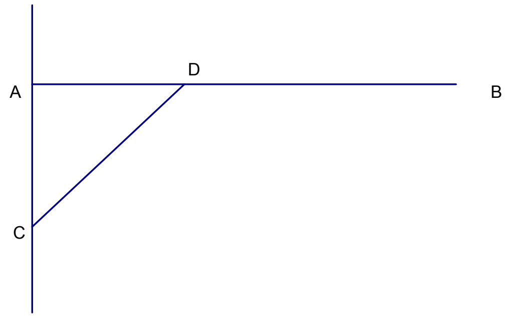
(4)

TOTAL FOR PAPER: 75 MARKS

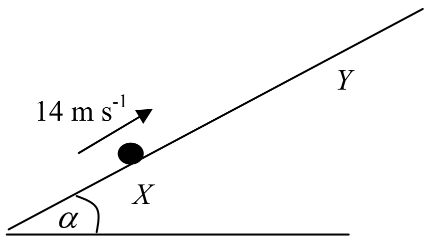
END

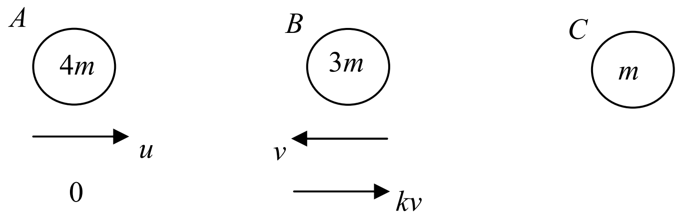
June 2009
6678 Mechanics M2
Mark Scheme

Question Number	Scheme	Marks
Q1	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	
	$5\mathbf{i} - 3\mathbf{j} = \frac{1}{4}\ \mathbf{v} - \frac{1}{4}(3\mathbf{i} + 7\mathbf{j})$	M1A1
	$\mathbf{v} = 23\mathbf{i} - 5\mathbf{j}$	A1
	$\lvert\mathbf{v}\rvert = \sqrt{23^2 + 5^2} = 23.5$	M1A1
		[5]
Q2 (a)	$\frac{dv}{dt} = 8 - 2t$	M1
	$8 - 2t = 0$	M1
	Max $v = 8 \times 4 - 4^2 = 16\ (\text{ms}^{-1})$	M1A1
		(4)
(b)	$\int 8t - t^2 dt = 4t^2 - \frac{1}{3}t^3 (+c)$	M1A1
	(t=0, displacement = 0 $\Rightarrow$ c=0)	
	$4T^2 - \frac{1}{3}T^3 = 0$	DM1
	$T^2(4 - \frac{T}{3}) = 0 \Rightarrow T = 0, 12$	DM1
	T = 12 (seconds)	A1
		(5)
		[9]
Q3 (a)	Constant v $\Rightarrow$ driving force = resistance	
	$\Rightarrow$ F=120 (N)	M1
	$\Rightarrow$ P=120 x 10 = 1200W	M1
		(2)
(b)	Resolving parallel to the slope, zero acceleration:	
	$\frac{P}{v} = 120 + 300g\sin\theta (= 330)$	M1A1A1
	$\Rightarrow \text{v} = \frac{1200}{330} = 3.6\ (\text{ms}^{-1})$	A1
		(4)
		[6]

Question Number	Scheme	Marks
Q4 (a)	 Taking moments about A: $3g \times 0.75 = \frac{T}{\sqrt{2}} \times 0.5$	M1A1A1
	$T = 3\sqrt{2}g \times \frac{7.5}{5} = \frac{9\sqrt{2}g}{2} (= 62.4N)$	A1
		(4)
(b)	$\leftarrow \pm H = \frac{T}{\sqrt{2}} (= \frac{9g}{2} \approx 44.1N)$	B1
	$\uparrow \; \pm V + \frac{T}{\sqrt{2}} = 3g \quad (\Rightarrow V = 3g - \frac{9g}{2} = \frac{-3g}{2} \approx -14.7\,\text{N})$	M1A1
	$\Rightarrow \; \lvert R \rvert = \sqrt{81+9} \times \frac{g}{2} \approx 46.5(N)$	M1A1
	at angle $\tan^{-1}\frac{1}{3}$ = 18.4° (0.322 radians) below the line of BA 161.6° (2.82 radians) below the line of AB (108.4° or 1.89 radians to upward vertical)	M1A1
		(7) **[11]**
Q5 (a)	Ratio of areas triangle:sign:rectangle = 1 : 5 : 6 (1800:9000:10800)	B1
	Centre of mass of the triangle is 20cm down from *AD* (seen or implied)	B1
	$\Rightarrow 6 \times 45 - 1 \times 20 = 5 \times \bar{y}$	M1A1
	$\bar{y} = 50cm$	A1
		(5)
(b)	Distance of centre of mass from *AB* is 60cm	B1
	Required angle is $\tan^{-1}\frac{60}{50}$ (their values)	M1A1ft
	$= 50.2^{\circ}$ (0.876 rads)	A1
		(4) **[9]**

Question Number	Scheme	Marks
Q6 (a)	$\rightarrow \quad x = u\cos\alpha t = 10$	M1A1
	$\uparrow \quad y = u\sin\alpha t - \frac{1}{2}gt^2 = 2$	M1A1
	$\Rightarrow t = \frac{10}{u\cos\alpha}$	
	$2 = u\sin\alpha \times \frac{10}{u\cos\alpha} - \frac{g}{2} \times \frac{100}{u^2\cos^2\alpha}$	M1
	$= 10\tan\alpha - \frac{50g}{u^2\cos^2\alpha}$ (given answer)	A1
		(6)
(b)	$2 = 10 \times 1 - \frac{100g \times 2}{2u^2 \times 1}$	M1A1
	$u^2 = \frac{100g}{8}$, $u = \sqrt{\frac{100g}{8}} = 11.1$ (m s^{-1})	A1
	$\frac{1}{2}mu^2 = m \times 9.8 \times 2 + \frac{1}{2}mv^2$	M1A1
	$v = 9.1ms^{-1}$	A1
		(6)
		[12]

Question Number	Scheme	Marks
Q7 (a)	 KE at $X = \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times 14^2$	B1
	GPE at $Y =$ $mgd\sin\alpha \left(= 2 \times g \times d \times \frac{7}{25}\right)$	B1 B1
	Normal reaction $R = mg\cos\alpha$	M1
	Friction $= \mu \times R = \frac{1}{8} \times 2g \times \frac{24}{25}$	M1A1
	Work Energy: $\frac{1}{2}mv^2 - mgd\sin\alpha = \mu \times R \times d$ or equivalent	
	$196 = \frac{14gd}{25} + \frac{6gd}{25} = \frac{20gd}{25}$	A1
	$d = 25$ m	(7)
(b)	Work Energy	
	First time at X: $\frac{1}{2}mv^2 = \frac{1}{2}m14^2$	
	Work done $= \mu \times R \times 2d = \frac{1}{8} \times 2g \times \frac{24}{25} \times 2d$	
	Return to X: $\frac{1}{2}mv^2 = \frac{1}{2}m14^2 - \frac{1}{8} \times 2g \times \frac{24}{25} \times 50$	M1A1
	$v = 8.9 \text{ ms}^{-1}$ (accept 8.85 ms^{-1})	DM1A1
		(4)
	OR: Resolve parallel to XY to find the acceleration and use of $v^2 = u^2 + 2as$	
	$2a = 2g\sin\alpha - F_{max} = 2g \times \frac{7}{25} - \frac{6g}{25} = \frac{8g}{25}$	M1A1
	$v^2 = (0+)2 \times a \times s = 8g$; $v = 8.9$ (accept 8.85 ms^{-1})	DM1;A1
		[11]

Question Number	Scheme	Marks
Q8 (a)		
	Conservation of momentum: $4mu - 3mv = 3mkv$	M1A1
	Impact law: $kv = \frac{3}{4}(u+v)$	M1A1
	Eliminate k: $4mu - 3mv = 3m \times \frac{3}{4}(u+v)$	DM1
	$u = 3v$ (Answer given)	A1
		(6)
(b)	$kv = \frac{3}{4}(3v+v)$, $k = 3$	M1,A1
		(2)
(c)	Impact law: $(kv+2v)e = v_C - v_B$ $(5ve = v_C - v_B)$	B1
	Conservation of momentum : $3 \times kv - 1 \times 2v = 3v_B + v_c$ $(7v = 3v_B + v_c)$	B1
	Eliminate v_C : $v_B = \frac{v}{4}(7-5e) > 0$ hence no further collision with *A*.	M1 A1
		(4)
		[12]