Paper Reference(s)

6678

Edexcel GCE

Mechanics M2

Advanced Level

Thursday 29 January 2009 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. A car of mass 1500 kg is moving up a straight road, which is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{14}$. The resistance to the motion of the car from non-gravitational forces is constant and is modelled as a single constant force of magnitude 650 N. The car's engine is working at a rate of 30 kW.

Find the acceleration of the car at the instant when its speed is 15 m s^{-1}.

(5)

2.

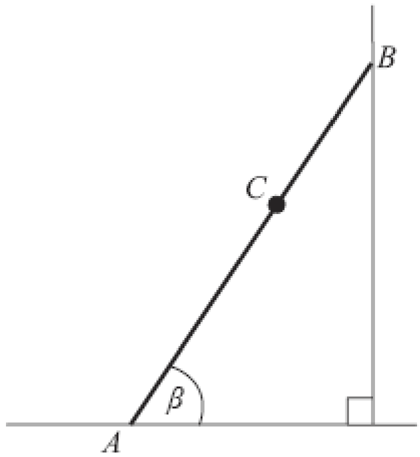

Figure 1

Figure 1 shows a ladder *AB*, of mass 25 kg and length 4 m, resting in equilibrium with one end *A* on rough horizontal ground and the other end *B* against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is $\frac{11}{25}$. The ladder makes an angle β with the ground. When Reece, who has mass 75 kg, stands at the point *C* on the ladder, where *AC* = 2.8 m, the ladder is on the point of slipping. The ladder is modelled as a uniform rod and Reece is modelled as a particle.

(*a*) Find the magnitude of the frictional force of the ground on the ladder.

(3)

(*b*) Find, to the nearest degree, the value of β.

(6)

(*c*) State how you have used the modelling assumption that Reece is a particle.

(1)

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3. A block of mass 10 kg is pulled along a straight horizontal road by a constant horizontal force of magnitude 70 N in the direction of the road. The block moves in a straight line passing through two points A and B on the road, where AB = 50 m. The block is modelled as a particle and the road is modelled as a rough plane. The coefficient of friction between the block and the road is $\frac{4}{7}$.

(*a*) Calculate the work done against friction in moving the block from A to B.

(4)

The block passes through A with a speed of 2 m s^{-1}.

(*b*) Find the speed of the block at B.

(4)

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4. A particle P moves along the x-axis in a straight line so that, at time t seconds, the velocity of P is v m s^{-1}, where

$$v = \begin{cases} 10t - t^2, & 0 \le t \le 6, \\ \dfrac{-432}{t^2}, & t > 6. \end{cases}$$

At $t = 0$, P is at the origin O. Find the displacement of P from O when

(*a*) $t = 6$,

(3)

(*b*) $t = 10$.

(5)

5.

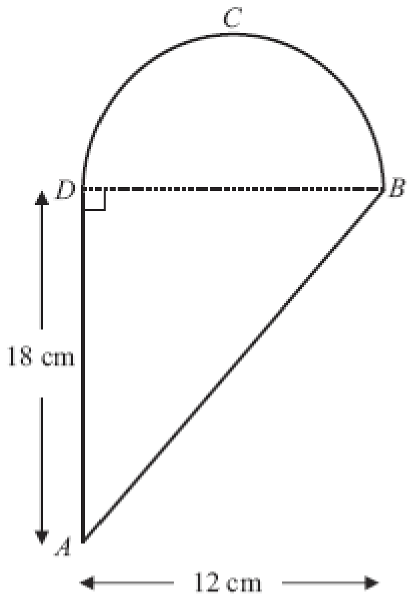


Figure 2

A uniform lamina *ABCD* is made by joining a uniform triangular lamina *ABD* to a uniform semi-circular lamina *DBC*, of the same material, along the edge *BD*, as shown in Figure 2. Triangle *ABD* is right-angled at *D* and *AD* = 18 cm. The semi-circle has diameter *BD* and *BD* = 12 cm.

(*a*) Show that, to 3 significant figures, the distance of the centre of mass of the lamina *ABCD* from *AD* is 4.69 cm.

(4)

Given that the centre of mass of a uniform semicircular lamina, radius r, is at a distance $\frac{4r}{3\pi}$ from the centre of the bounding diameter,

(*b*) find, in cm to 3 significant figures, the distance of the centre of mass of the lamina *ABCD* from *BD*.

(4)

The lamina is freely suspended from *B* and hangs in equilibrium.

(*c*) Find, to the nearest degree, the angle which *BD* makes with the vertical.

(4)

6.

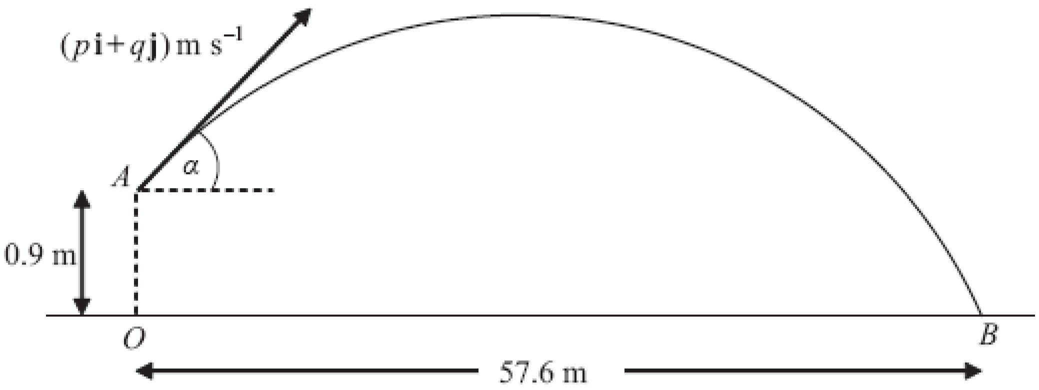


Figure 3

A cricket ball is hit from a point A with velocity of $(p\mathbf{i} + q\mathbf{j})$ m s^{-1}, at an angle α above the horizontal. The unit vectors **i** and **j** are respectively horizontal and vertically upwards. The point A is 0.9 m vertically above the point O, which is on horizontal ground.

The ball takes 3 seconds to travel from A to B, where B is on the ground and OB = 57.6 m, as shown in Figure 3. By modelling the motion of the cricket ball as that of a particle moving freely under gravity,

(*a*) find the value of p,

(2)

(*b*) show that $q = 14.4$,

(3)

(*c*) find the initial speed of the cricket ball,

(2)

(*d*) find the exact value of $\tan \alpha$.

(1)

(*e*) Find the length of time for which the cricket ball is at least 4 m above the ground.

(6)

(*f*) State an additional physical factor which may be taken into account in a refinement of the above model to make it more realistic.

(1)

7. A particle P of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal table. It collides directly with another particle Q of mass $2m$ which is moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(*a*) Show that the speed of Q immediately after the collision is $\frac{1}{5}(9e + 4)u$.

(5)

The speed of P immediately after the collision is $\frac{1}{2}u$.

(*b*) Show that $e = \frac{1}{4}$.

(4)

The collision between P and Q takes place at the point A. After the collision Q hits a smooth fixed vertical wall which is at right-angles to the direction of motion of Q. The distance from A to the wall is d.

(*c*) Show that P is a distance $\frac{3}{5}d$ from the wall at the instant when Q hits the wall.

(4)

Particle Q rebounds from the wall and moves so as to collide directly with particle P at the point B. Given that the coefficient of restitution between Q and the wall is $\frac{1}{5}$,

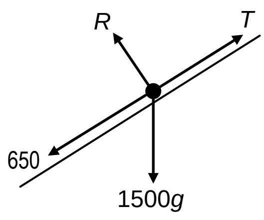
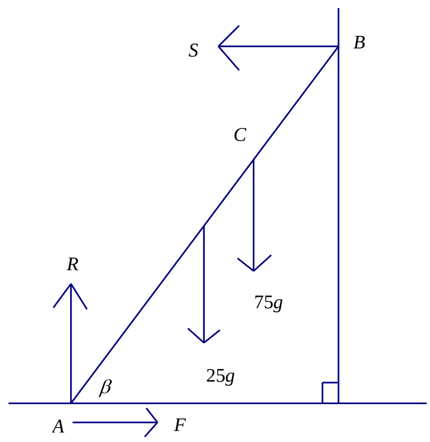
(*d*) find, in terms of d, the distance of the point B from the wall.

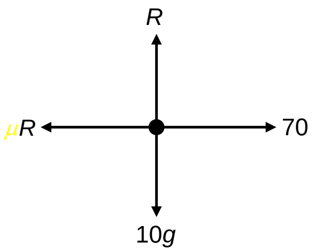
(4)

TOTAL FOR PAPER: 75 MARKS

END

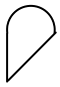
January 2009
6678 Mechanics M2
Mark Scheme

Question Number	Scheme	Marks
1		
	F = ma parallel to the slope,	M1*
	$T - 1500g\sin\theta - 650 = 1500a$	A1
	Tractive force, 30000 = T x 15	M1*
	$a = \dfrac{\frac{30000}{15} - 1500(9.8)(\frac{1}{14}) - 650}{1500}$	d*M1
	<u>0.2</u> (m s^{-2})	A1
		(5) **[5]**
2 (a)		
	$R(\uparrow): R = 25g + 75g (= 100g)$	B1
	$F = \mu R \Rightarrow F = \frac{11}{25} \times 100g$	M1
	= 44g (=431)	A1
		(3)
(b)	M(A): $25g \times 2\cos\beta + 75g \times 2.8\cos\beta$ $= S \times 4\sin\beta$	M1 A2,1,0
	$R(\leftrightarrow): F = S$	
	$176g\sin\beta = 260g\cos\beta$	M1A1
	$\beta = 56(°)$	A1
		(6)
(c)	So that Reece's weight acts directly at the point C.	B1 **[10]**

Question Number	Scheme	Marks
3 (a)		
	$R(\updownarrow): R = 10g$	B1
	$F = \mu R \Rightarrow F = \frac{4}{7}(10g) = 56$	B1
	$\therefore$ WD against friction $= \frac{4}{7}(10g)(50)$	M1
	2800(J)	A1
		(4)
(b)	$70(50) - \text{"}2800\text{"} = \frac{1}{2}(10)v^2 - \frac{1}{2}(10)(2)^2$	M1* A1ft
	$700 = 5v^2 - 20,\ 5v^2 = 720 \Rightarrow v^2 = 144$	d*M1
	Hence, $v = \underline{12}$ (m s^{-1})	A1 cao
		(4)
Or (b)	N2L $(\rightarrow)$: $70 - \frac{4}{7}R = 10a$	M1*
	$70 - \frac{4}{7} \times 10g = 10a,\ (a = 1.4)$	A1ft
	$AB\ (\rightarrow)$: $v^2 = (2)^2 + 2(1.4)(50)$	d*M1
	Hence, $v = \underline{12}$ (m s^{-1})	A1 cao
		(4)
		[8]
4 (a)	$v = 10t - 2t^2,\ s = \int v\,dt$	M1
	$= 5t^2 - \frac{2t^3}{3}(+C)$	A1
	$t = 6 \Rightarrow s = 180 - 144 = \underline{36}$ (m)	A1
		(3)
(b)	$\underline{s} = \int v\,dt = \underline{\frac{-432t^{-1}}{-1}}(+K) = \underline{\frac{432}{t}}(+K)$	$\underline{\text{B1}}$
	t = 6, s = "36" $\Rightarrow 36 = \frac{432}{6} + K$	M1*
	$\Rightarrow K = -36$	A1
	At $t = 10,\ s = \frac{432}{10} - 36 = \underline{7.2}$ (m)	d*M1 $\underline{\text{A1}}$
		(5)
		[8]

Question Number	Scheme	Marks

5 (a)

MR	 108	 18π	 $108 + 18\pi$
x_i $(\rightarrow)$ from *AD*	4	6	$\bar{x}$
y_i $(\downarrow)$ from *BD*	6	$-\frac{8}{\pi}$	$\bar{y}$

B1
B1

$AD(\rightarrow)$: $108(4) + 18\pi(6) = (108 + 18\pi)\bar{x}$ — M1

$\bar{x} = \dfrac{432 + 108\pi}{108 + 18\pi} = 4.68731... = \underline{4.69}$ (cm) (3 sf) **AG** — A1

(4)

(b)

y_i $(\downarrow)$ from *BD*	6	$-\dfrac{8}{\pi}$	$\bar{y}$

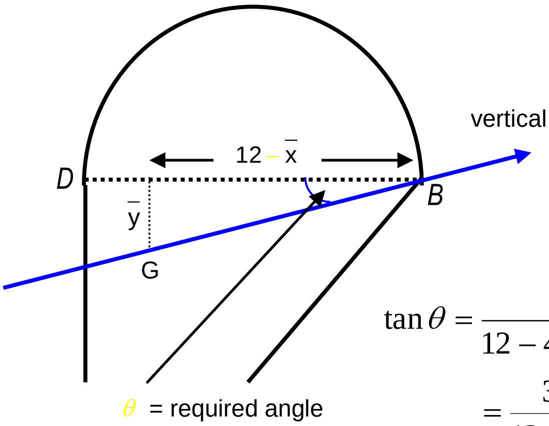
B1 oe

$BD(\downarrow)$: $108(6) + 18\pi(-\frac{8}{\pi}) = (108 + 18\pi)\bar{y}$ — M1 A1ft

$\bar{y} = \dfrac{504}{108 + 18\pi} = 3.06292... = 3.06$ (cm) (3 sf) — A1

(4)

(c)



M1

$\tan\theta = \dfrac{\bar{y}}{12 - 4.68731..}$ — dM1

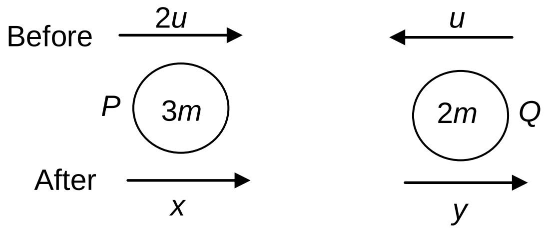
$= \dfrac{3.06392..}{12 - 4.68731..}$ — A1

$\theta = 22.72641... = \underline{23}$ (nearest degree) — A1

(4)

[12]

Question Number	Scheme	Marks
6 (a)	Horizontal distance: $57.6 = p \times 3$ $p = 19.2$	M1 A1 (2)
(b)	Use $s = ut + \frac{1}{2}at^2$ for vertical displacement. $-0.9 = q \times 3 - \frac{1}{2}g \times 3^2$ $-0.9 = 3q - \frac{9g}{2} = 3q - 44.1$ $q = \frac{43.2}{3} = 14.4$ ***AG***	M1 A1 A1 cso (3)
(c)	initial speed $\sqrt{p^2 + 14.4^2}$ (with their p) $= \sqrt{576} = \underline{24}$ (m s^{-1})	M1 A1 cao (2)
(d)	$\tan\alpha = \frac{14.4}{p} (= \frac{3}{4})$ (with their p)	B1 (1)
(e)	When the ball is 4 m above ground: $3.1 = ut + \frac{1}{2}at^2$ used $3.1 = 14.4t - \frac{1}{2}gt^2$ o.e $(4.9t^2 - 14.4t + 3.1 = 0)$ $\Rightarrow t = \frac{14.4 \pm \sqrt{(14.4)^2 - 4(4.9)(3.1)}}{2(4.9)}$ seen or implied $t = \frac{14.4 \pm \sqrt{146.6}}{9.8} = 0.023389\ldots$ or $2.70488\ldots$ awrt 0.23 and 2.7 duration $= 2.70488... - 0.23389...$ $= 2.47$ or 2.5 (seconds)	 M1 A1 M1 A1 M1 A1 (6)
or 6 (e)	M1A1M1 as above $t = \frac{14.4 \pm \sqrt{146.6}}{9.8}$ Duration $2 \times \frac{\sqrt{146.6}}{9.8}$ o.e. $= 2.47$ or 2.5 (seconds)	 A1 M1 A1 (6)
(f)	Eg. : Variable 'g', Air resistance, Speed of wind, Swing of ball, The ball is not a particle.	B1 (1) **[15]**

Question Number	Scheme	Marks
7 (a)	 Correct use of NEL	M1*
	$y - x = e(2u + u)$ o.e.	A1
	CLM (→): $3m(2u) + 2m(-u) = 3m(x) + 2m(y)$ $(\Rightarrow 4u = 3x + 2y)$	B1
	Hence $x = y - 3eu$, $4u = 3(y - 3eu) + 2y$, $(u(9e + 4) = 5y)$	d*M1
	Hence, speed of $Q = \frac{1}{5}(9e + 4)u$ **AG**	A1 cso
		(5)
(b)	$x = y - 3eu = \frac{1}{5}(9e + 4)u - 3eu$	M1[#]
	Hence, speed P $= \frac{1}{5}(4 - 6e)u = \frac{2u}{5}(2 - 3e)$ o.e.	A1
	$x = \frac{1}{2}u = \frac{2u}{5}(2 - 3e) \Rightarrow 5u = 8u - 12eu, \Rightarrow 12e = 3$ & solve for e	d[#]M1
	gives, $e = \frac{3}{12} \Rightarrow e = \frac{1}{4}$ **AG**	A1
		(4)
Or (b)	Using NEL correctly with given speeds of P and Q	M1[#]
	$3eu = \frac{1}{5}(9e + 4)u - \frac{1}{2}u$	A1
	$3eu = \frac{9}{5}eu + \frac{4}{5}u - \frac{1}{2}u$, $3e - \frac{9}{5}e = \frac{4}{5} - \frac{1}{2}$ & solve for e	d[#]M1
	$\frac{6}{5}e = \frac{3}{10} \Rightarrow e = \frac{15}{60} \Rightarrow e = \frac{1}{4}$.	A1
		(4)
(c)	Time taken by Q from A to the wall $= \frac{d}{y} = \left\{\frac{4d}{5u}\right\}$	M1†
	Distance moved by P in this time $= \frac{u}{2} \times \frac{d}{y}$ $\left(= \frac{u}{2}\left(\frac{4d}{5u}\right) = \frac{2}{5}d\right)$	A1
	Distance of P from wall $= d - x\left(\frac{d}{y}\right)$; $= d - \frac{2}{5}d = \frac{3}{5}d$ **AG**	d†M1; A1 cso
		(4)
or (c)	Ratio speed P:speed Q = x:y = $\frac{1}{2}u : \frac{1}{5}(\frac{9}{4} + 4)u = \frac{1}{2}u : \frac{5}{4}u$ = 2:5	M1†
	So if Q moves a distance d, P will move a distance $\frac{2}{5}d$	A1
	Distance of P from wall $= d - \frac{2}{5}d$; $= \frac{3}{5}d$ **AG** cso	d†M1; A1
		(4)

Question Number	Scheme	Marks
(d)	After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their *y*	B1ft
	Time for *P*, $T_{AB} = \dfrac{\frac{3d}{5} - x}{\frac{1}{2}u}$, Time for *Q*, $T_{WB} = \dfrac{x}{\frac{1}{4}u}$ from their *y*	B1ft
	Hence $T_{AB} = T_{WB} \Rightarrow \dfrac{\frac{3d}{5} - x}{\frac{1}{2}u} = \dfrac{x}{\frac{1}{4}u}$	M1
	gives, $2\left(\frac{3d}{5} - x\right) = 4x \Rightarrow \frac{3d}{5} - x = 2x$, $3x = \frac{3d}{5} \Rightarrow x = \underline{\frac{1}{5}d}$	A1 **cao**
		(4)
or (d)	After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their *y*	B1ft
	speed $P = x = \frac{1}{2}u$, speed *P*: new speed $Q = \frac{1}{2}u : \frac{1}{4}u = 2:1$ from their *y*	B1ft
	Distance of *B* from wall $= \dfrac{1}{3} \times \dfrac{3d}{5}; = \dfrac{d}{5}$ their $\dfrac{1}{2+1}$	M1; A1
		(4)
2nd or (d)	After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their *y*	B1ft
	Combined speed of *P* and $Q = \frac{1}{2}u + \frac{1}{4}u = \frac{3}{4}u$	
	Time from wall to 2nd collision $= \dfrac{\frac{3d}{5}}{\frac{3u}{4}} = \frac{3d}{5} \times \frac{4}{3u} = \frac{4d}{5u}$ from their *y*	B1ft
	Distance of *B* from wall = (their speed)x(their time) $= \dfrac{u}{4} \times \dfrac{4d}{5u}$; $= \underline{\frac{1}{5}d}$	M1; A1
		(4) **[17]**