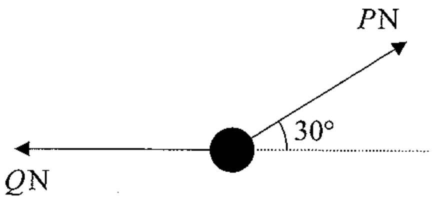
M1 January 2007

1.

Figure 1



A particle of weight 24 N is held in equilibrium by two light inextensible strings. One string is horizontal. The other string is inclined at an angle of 30° to the horizontal, as shown in Figure 1. The tension in the horizontal string is Q newtons and the tension in the other string is P newtons. Find

(a) the value of P,

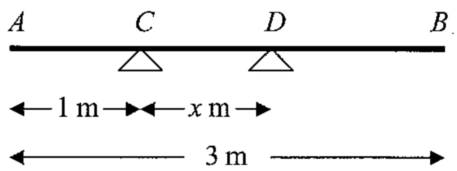
(3)

(b) the value of Q.

(3)

2.

Figure 2



A uniform plank AB has weight 120 N and length 3 m. The plank rests horizontally in equilibrium on two smooth supports C and D, where $AC = 1$ m and $CD = x$ m, as shown in Figure 2. The reaction of the support on the plank at D has magnitude 80 N. Modelling the plank as a rod,

(a) show that $x = 0.75$

(3)

A rock is now placed at B and the plank is on the point of tilting about D. Modelling the rock as a particle, find

(b) the weight of the rock,

(4)

(c) the magnitude of the reaction of the support on the plank at D.

(2)

(d) State how you have used the model of the rock as a particle.

(1)

---

3. A particle P of mass 2 kg is moving under the action of a constant force **F** newtons. When $t = 0$, P has velocity $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1} and at time $t = 4$ s, P has velocity $(15\mathbf{i} - 4\mathbf{j})$ m s^{-1}. Find

(a) the acceleration of P in terms of **i** and **j**,

(2)

(b) the magnitude of **F**,

(4)

(c) the velocity of P at time $t = 6$ s.

(3)

---

4. A particle P of mass 0.3 kg is moving with speed u m s^{-1} in a straight line on a smooth horizontal table. The particle P collides directly with a particle Q of mass 0.6 kg, which is at rest on the table. Immediately after the particles collide, P has speed 2 m s^{-1} and Q has speed 5 m s^{-1}. The direction of motion of P is reversed by the collision. Find

(a) the value of u,

(4)

(b) the magnitude of the impulse exerted by P on Q.

(2)

Immediately after the collision, a constant force of magnitude R newtons is applied to Q in the direction directly opposite to the direction of motion of Q. As a result Q is brought to rest in 1.5 s.

(c) Find the value of R.

(4)

---

5. A ball is projected vertically upwards with speed 21 m s^{-1} from a point A, which is 1.5 m above the ground. After projection, the ball moves freely under gravity until it reaches the ground. Modelling the ball as a particle, find

(a) the greatest height above A reached by the ball,

(3)

(b) the speed of the ball as it reaches the ground,

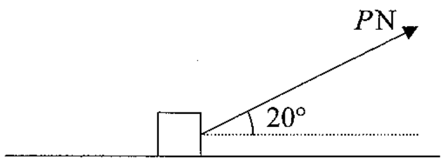
(3)

(c) the time between the instant when the ball is projected from A and the instant when the ball reaches the ground.

(4)

6.

Figure 3



A box of mass 30 kg is being pulled along rough horizontal ground at a constant speed using a rope. The rope makes an angle of 20° with the ground, as shown in Figure 3. The coefficient of friction between the box and the ground is 0.4. The box is modelled as a particle and the rope as a light, inextensible string. The tension in the rope is P newtons.

(a) Find the value of P.

(8)

The tension in the rope is now increased to 150 N.

(b) Find the acceleration of the box.

(6)

7.

Figure 4

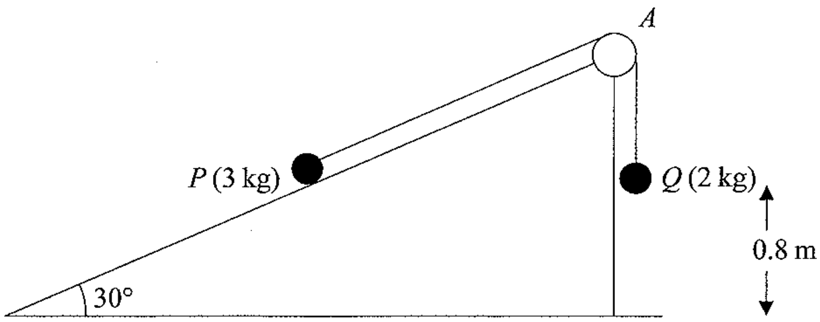


Figure 4 shows two particles P and Q, of mass 3 kg and 2 kg respectively, connected by a light inextensible string. Initially P is held at rest on a fixed smooth plane inclined at 30° to the horizontal. The string passes over a small smooth light pulley A fixed at the top of the plane. The part of the string from P to A is parallel to a line of greatest slope of the plane. The particle Q hangs freely below A. The system is released from rest with the string taut.

(a) Write down an equation of motion for P and an equation of motion for Q.

(4)

(b) Hence show that the acceleration of Q is 0.98 m s^{-2}.

(2)

(c) Find the tension in the string.

(2)

(d) State where in your calculations you have used the information that the string is inextensible.

(1)

On release, Q is at a height of 0.8 m above the ground. When Q reaches the ground, it is brought to rest immediately by the impact with the ground and does not rebound. The initial distance of P from A is such that in the subsequent motion P does not reach A. Find

(e) the speed of Q as it reaches the ground,

(2)

(f) the time between the instant when Q reaches the ground and the instant when the string becomes taut again.

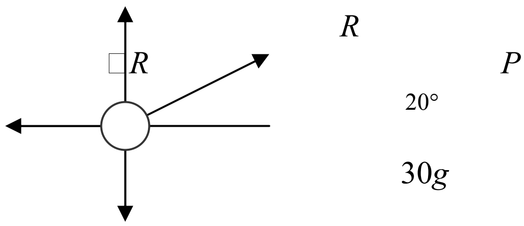
(5)

TOTAL FOR PAPER: 75 MARKS

January 2007
6677 Mechanics M1
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $P\sin 30° = 24$	M1 A1
	$P = 48$	A1 <u>3</u>
	(b) $Q = P\cos 30°$	M1 A1
	≈ 41.6 accept $24\sqrt{3}$, awrt 42	A1 <u>3</u> **6**
2.	(a) M(C) $80 \times x = 120 \times 0.5$	M1 A1
	$x = 0.75$ ∗ cso	A1 <u>3</u>
	(b) Using reaction at $C = 0$	B1
	M(D) $120 \times 0.25 = W \times 1.25$ ft their x	M1 A1
	$W = 24$ (N)	A1 <u>4</u>
	(c) i $X = 24 + 120 = 144$ (N) ft their W	M1 A1ft <u>2</u>
	(d) The weight of the rock acts precisely at B.	B1 <u>1</u> **10**
3.	(a) $\mathbf{a} = \dfrac{(15\mathbf{i} - 4\mathbf{j}) - (3\mathbf{i} + 2\mathbf{j})}{4} = 3\mathbf{i} - 1.5\mathbf{j}$	M1 A1 <u>2</u>
	(b) N2L $\mathbf{F} = m\mathbf{a} = 6\mathbf{i} - 3\mathbf{j}$ ft their **a**	M1 A1
	$\lvert\mathbf{F}\rvert = \sqrt{(6^2 + 3^2)} \approx 6.71$ (N) accept $\sqrt{45}$, awrt 6.7	M1 A1 <u>4</u>
	(c) $\mathbf{v}_6 = (3\mathbf{i} + 2\mathbf{j}) + (3\mathbf{i} - 1.5\mathbf{j})6$ ft their **a**	M1 A1ft
	$= 21\mathbf{i} - 7\mathbf{j}$ $(\mathrm{m\,s^{-1}})$	A1 <u>1</u> **9**

Question Number	Scheme	Marks
4.	(a) CLM $0.3u = 0.3\times(-2) + 0.6\times 5$	M1 A1
	$u = 8$	M1 A1 <u>4</u>
	(b) $I = 0.6\times 5 = 3 \quad (\text{N s})$	M1 A1 <u>2</u>
	(c) $v = u + at \Rightarrow 5 = a\times 1.5 \quad \left(a = \frac{10}{3}\right)$	M1 A1
	N2L $R = 0.6\times\frac{10}{3} = 2$	M1 A1 <u>4</u> **10**
5.	(a) $v^2 = u^2 + 2as \Rightarrow 0^2 = 21^2 - 2\times 9.8\times h$	M1 A1
	$h = 22.5 \quad (\text{m})$	A1 <u>3</u>
	(b) $v^2 = u^2 + 2as \Rightarrow v^2 = 0^2 + 2\times 9.8\times 24$ or equivalent	M1 A1
	$(= 470.4)$	
	$v \approx 22 \quad (\text{m s}^{-1})$ accept 21.7	A1 <u>3</u>
	(c) $v = u + at \Rightarrow -\sqrt{470.4} = 21 - 9.8t$ or equivalent	M1 A2 (1, 0)
	− 1 each error	
	$t \approx 4.4 \quad (\text{s})$ accept 4.36	A1 <u>4</u> **10**

Question Number	Scheme	Marks
6.	(a) 	
	Use of $F = \mu R$	B1
	$\rightarrow$ $P\cos 20° = \mu R$	M1 A1
	$\uparrow$ $R + P\sin 20° = 30g$	M1 A1
	$P\cos 20° = \mu(30g - P\sin 20°)$	M1
	$P = \dfrac{0.4 \times 30g}{\cos 20° + 0.4\sin 20°}$	M1
	$\approx 110 \quad (\text{N})$ accept 109	A1 8
	(b) $\uparrow$ $R + 150\sin 20° = 30g$	M1 A1
	$(R \approx 242.7)$	
	N2L $\rightarrow$ $150\cos 20° - \mu R = 30a$	M1 A1
	$a \approx \dfrac{150\cos 20° - 0.4 \times 242.7}{30}$	M1
	$= 1.5\ (\text{ms}^{-2})$ accept 1.46	A1 6 **14**

Question Number	Scheme	Marks
7.	(a) N2L Q $2g - T = 2a$	M1 A1
	N2L P $T - 3g\sin 30° = 3a$	M1 A1 4
	(b) $2g - 3g\sin 30° = 5a$	M1
	$a = 0.98$ (ms^{-2}) ✱ cso	A1 2
	(c) $T = 2(g - a)$ or equivalent	M1
	≈ 18 (N) accept 17.6	A1 2
	(d) The (magnitudes of the) accelerations of P and Q are equal	B1 1
	(e) $v^2 = u^2 + 2as \Rightarrow v^2 = 2 \times 0.98 \times 0.8$ $(= 1.568)$	M1
	$v \approx 1.3$ (m s^{-1}) accept 1.25	A1 2
	(f) N2L for P $-3g\sin 30° = 3a$	
	$a = (-)\frac{1}{2}g$	M1 A1
	$s = ut + \frac{1}{2}at^2 \Rightarrow 0 = \sqrt{1.568}t - \frac{1}{2}4.9t^2$ or equivalent	M1 A1
	$t = 0.51$ (s) accept 0.511	A1 5 **16**
	A maximum of one mark can be lost for giving too great accuracy.	