Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 27 June 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
6669/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. $$\mathbf{A} = \begin{pmatrix} -2 & 1 & -3 \\ k & 1 & 3 \\ 2 & -1 & k \end{pmatrix}, \text{ where } k \text{ is a constant}$$

Given that the matrix **A** is singular, find the possible values of k.

(Total 4 marks)

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2. The curve C has equation

$$y = \frac{x^2}{8} - \ln x, \quad 2 \le x \le 3.$$

Find the length of the curve C giving your answer in the form $p + \ln q$, where p and q are rational numbers to be found.

(Total 7 marks)

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3. (*a*) Prove that

$$\frac{\mathrm{d}(\operatorname{arcoth} x)}{\mathrm{d}x} = \frac{1}{1-x^2}.$$

(3)

Given that $y = (\operatorname{arcoth} x)^2$,

(*b*) show that

$$\left(1-x^2\right)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2x\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{1-x^2},$$

where k is a constant to be determined.

(5)

(Total 8 marks)

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4. (i) Find, without using a calculator,

$$\int_3^5 \frac{1}{\sqrt{15+2x-x^2}}\,dx$$

giving your answer as a multiple of π.

(5)

(ii) (*a*) Show that

$$5\cosh x - 4\sinh x = \frac{e^{2x}+9}{2e^x}.$$

(3)

(*b*) Hence, using the substitution $u = e^x$ or otherwise, find

$$\int \frac{1}{5\cosh x - 4\sinh x}\,dx.$$

(4)

(Total 12 marks)

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5. The hyperbola H has equation

$$\frac{x^2}{16} - \frac{y^2}{9} = 1.$$

The point $P\,(4\sec\theta, 3\tan\theta)$, $0 < \theta < \frac{\pi}{2}$ lies on H.

(*a*) Show that an equation of the normal to H at the point P is

$$3y + 4x\sin\theta = 25\tan\theta.$$

(5)

The line l is the directrix of H for which $x > 0$.

The normal to H at P crosses the line l at the point Q. Given that $\theta = \frac{\pi}{4}$,

(*b*) find the y coordinate of Q, giving your answer in the form $a + b\sqrt{2}$, where a and b are rational numbers to be found.

(6)

(Total 11 marks)

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6. $$\mathbf{M} = \begin{pmatrix} p & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & q \end{pmatrix},$$

where p and q are constants.

Given that $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ is an eigenvector of the matrix **M**,

(*a*) find the eigenvalue corresponding to this eigenvector,

(3)

(*b*) find the value of p and the value of q.

(3)

Given that 6 is another eigenvalue of **M**,

(*c*) find a corresponding eigenvector.

(2)

Given that $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ is a third eigenvector of **M** with eigenvalue 3,

(*d*) find a matrix **P** and a diagonal matrix **D** such that

$$\mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P} = \mathbf{D}.$$

(3)

(Total 11 marks)

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7. Given that

$$I_n = \int \frac{\sin nx}{\sin x}\,dx, \qquad n \geq 1,$$

(*a*) prove that, for $n \geq 3$

$$I_n - I_{n-2} = \int 2\cos(n-1)x\;dx.$$

(3)

(*b*) Hence, showing each step of your working, find the exact value of

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \frac{\sin 5x}{\sin x}\,dx,$$

giving your answer in the form $\frac{1}{12}(a\pi + b\sqrt{3} + c)$, where a, b and c are integers to be found.

(7)

(Total 10 marks)

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8. The plane Π_1 has equation

$$x - 5y - 2z = 3.$$

The plane Π_2 has equation

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k}),$$

where λ and μ are scalar parameters.

(*a*) Show that Π_1 is perpendicular to Π_2.

(4)

(*b*) Find a cartesian equation for Π_2.

(2)

(*c*) Find an equation for the line of intersection of Π_1 and Π_2 giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors to be found.

(6)

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

Question Number	Scheme	Notes	Marks
1.	$\mathbf{A}=\begin{pmatrix}-2 & 1 & -3\\ k & 1 & 3\\ 2 & -1 & k\end{pmatrix}$		
	$\det\mathbf{A}=-2(k+3)-(k^2-6)-3(-k-2)\ row1$ or e.g. $\det\mathbf{A}=-k(k-3)+(-2k+6)-3(2-2)\ row2$ $\det\mathbf{A}=2(3+3)+(-6+3k)+k(-2-k)\ row3$ $\det\mathbf{A}=-2(k+3)-k(k-3)+2(3+3)\ col1$ $\det\mathbf{A}=-(k^2-6)+(-2k+6)+(-6+3k)\ col2$ $\det\mathbf{A}=-3(-2-k)-3(2-2)+k(-2-k)\ col3$	M1: Correct attempt at determinant (3 'elements' (may be implied if one is zero) with at least two elements correct). **Note that there are various alternatives depending on the choice of row or column.** A1: Correct determinant **in any form**	M1A1
	Note that e.g. $\det\mathbf{A}=-2\begin{vmatrix}1 & 3\\ -1 & k\end{vmatrix}-\begin{vmatrix}k & 3\\ 2 & k\end{vmatrix}-3\begin{vmatrix}k & 1\\ 2 & -1\end{vmatrix}$ scores no marks until the determinants are 'extracted'.		
	$-2(k+3)-(k^2-6)-3(-k-2)=0\Rightarrow k=...$	Sets their det**A** = 0 (= 0 may be implied) and attempts to solve a 3 term quadratic (see general guidance) as far as k = ... NB Correct quadratic is $k^2-k-6=0$	M1
	$(k+2)(k-3)=0\Rightarrow k=-2,\ 3$	Both values correct	A1
			(4)
			Total 4

Question Number	Scheme	Notes	Marks
2.	$y=\frac{x^2}{8}-\ln x,\quad 2\le x\le 3$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x}{4}-\frac{1}{x}$	Correct derivative. Allow any correct equivalent e.g. $\frac{2x}{8}-\frac{1}{x}$	B1
	$L=\int\sqrt{\left(1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)}\,\mathrm{d}x=\int\sqrt{\left(1+\left(\frac{x}{4}-\frac{1}{x}\right)^2\right)}\,\mathrm{d}x$	Use of a correct formula using their derivative and not the given *y*.	M1
	$=\int\sqrt{\left(1+\frac{x^2}{16}-\frac{1}{2}+\frac{1}{x^2}\right)}\,\mathrm{d}x=\int\sqrt{\left(\frac{x^2}{16}+\frac{1}{2}+\frac{1}{x^2}\right)}\,\mathrm{d}x=\int\sqrt{\left(\frac{x}{4}+\frac{1}{x}\right)^2}\,\mathrm{d}x=\int\left(\frac{x}{4}+\frac{1}{x}\right)\mathrm{d}x$ M1: Squares their derivative to obtain $ax^2+bx^{-2}+c$, where none of *a*, *b* or *c* are zero – this may be implied by e.g. $\frac{ax^4+bx^2+c}{dx^2}$ **and** adds 1 to their constant term. A1: Correct integrand $\frac{x}{4}+\frac{1}{x}$ or equivalent e.g. $\frac{x^2+4}{4x}$ (integral sign not needed)		M1 A1
	$=\frac{x^2}{8}+\ln kx$	Correct integration	A1
	$\left[\frac{x^2}{8}+\ln x\right]_2^3=\left(\frac{3^2}{8}+\ln 3\right)-\left(\frac{2^2}{8}+\ln 2\right)$	Substitutes 2 and 3 into an expression of the form $px^2+q\ln x\ (p,q\ne 0)$ and subtracts the right way round. Must be seen explicitly or may be implied by a correct exact answer for their integration. If the candidate gives the **final single answer** in decimals with no substitution shown, e.g. 1.030…this is M0.	M1
	$\frac{5}{8}+\ln\frac{3}{2}$	Cao and cso (oe e.g $0.625+\ln\frac{3}{2}$)	A1
			(7)
			Total 7

Question Number	Scheme	Notes	Marks
3(a)	$y=\text{arcoth}\,x \Rightarrow \coth y = x$ or e.g. $u=\text{arcoth}\,x \Rightarrow \coth u = x$	Changes from arcoth to coth correctly. This may be implied by e.g. $\tanh y=\frac{1}{x}$	B1
	$x=\frac{\cosh y}{\sinh y}\Rightarrow\frac{dx}{dy}=\frac{\sinh^2 y-\cosh^2 y}{\sinh^2 y}\left(=-\frac{1}{\sinh^2 y}\right)$	Uses $\coth y=\frac{\cosh y}{\sinh y}$ and attempts product or quotient rule	M1
	$\frac{dx}{dy}=-\text{cosech}^2 y=1-\coth^2 y$ $\Rightarrow\frac{dy}{dx}=\frac{1}{1-\coth^2 y}=\frac{1}{1-x^2}$ *	Correct completion with no errors seen and an intermediate step shown.	A1*
			(3)
	(a) Alternative 2		
	$y=\text{arcoth}\,x \Rightarrow \coth y = x$	Changes from arcoth to coth correctly. This may be implied by e.g. $\tanh y=\frac{1}{x}$	B1
	$-\text{cosech}^2 y\frac{dy}{dx}=1$ or $-\text{cosech}^2 y=\frac{dx}{dy}$ $\left(\Rightarrow\frac{dy}{dx}=-\frac{1}{\text{cosech}^2 y}\right)$	$\pm\text{cosech}^2 y\frac{dy}{dx}=1$ or $\pm\text{cosech}^2 y=\frac{dx}{dy}$	M1
	$\coth^2 y-1=\text{cosech}^2 y\Rightarrow\frac{dy}{dx}=\frac{1}{1-x^2}$ *	Correct completion with no errors seen and an intermediate step shown.	A1*
	(a) Alternative 3		
	$y=\text{arcoth}\,x \Rightarrow \coth y = x$	Changes from arcoth to coth correctly This may be implied by e.g. $\tanh y=\frac{1}{x}$	B1
	$x=\coth y=\frac{e^{2y}+1}{e^{2y}-1}\Rightarrow\frac{dx}{dy}=\frac{(e^{2y}-1)2e^{2y}-(e^{2y}+1)2e^{2y}}{(e^{2y}-1)^2}$	Expresses $\coth y$ in terms of exponentials and differentiates	M1
	$\frac{dx}{dy}=\frac{-4e^{2y}}{(e^{2y}-1)^2}\Rightarrow\frac{dy}{dx}=\frac{e^{4y}-2e^{2y}+1}{-4e^{2y}}=\frac{e^{2y}-2+e^{-2y}}{-4}=-\left(\frac{e^{y}-e^{-y}}{2}\right)^2=-\sinh^2 y=-\frac{1}{\text{cosech}^2 y}$ $=\frac{1}{1-\coth^2 y}=\frac{1}{1-x^2}$ * Completes correctly with no errors		A1*

	(a) Alternative 4		
	$y=\text{arcoth}\,x\Rightarrow\coth y=x$	Changes from arcoth to coth correctly This may be implied by e.g. $\tanh y=\frac{1}{x}$	B1
	$x=\coth y=\frac{e^y+e^{-y}}{e^y-e^{-y}}\Rightarrow\frac{dx}{dy}=\frac{\left(e^y-e^{-y}\right)^2-\left(e^y+e^{-y}\right)^2}{\left(e^y-e^{-y}\right)^2}$	Expresses coth*y* in terms of exponentials and differentiates	M1
	$\frac{dx}{dy}=\frac{-4}{\left(e^y-e^{-y}\right)^2}\Rightarrow\frac{dy}{dx}=-\frac{1}{\text{cosech}^2 y}$		
	$\coth^2 y-1=\text{cosech}^2 y\Rightarrow\frac{dy}{dx}=\frac{1}{1-x^2}$ *	Correct completion with no errors seen and an intermediate step shown.	A1*

	(a) Alternative 5		
	$y=\text{arcoth}\,x=\frac{1}{2}\ln\left(\frac{1+x}{x-1}\right)$	**Correct** ln form for arcoth	B1
	$\frac{dy}{dx}=\frac{1}{2}\left[\frac{x-1}{x+1}\times\frac{(x-1)-(x+1)}{(x-1)^2}\right]$ or $\frac{1}{2}\ln\left(\frac{1+x}{x-1}\right)=\frac{1}{2}\ln(1+x)-\frac{1}{2}\ln(x-1)$ $\Rightarrow\frac{dy}{dx}=\frac{1}{2(x+1)}-\frac{1}{2(x-1)}$	Attempts to differentiate using the chain rule and quotient rule or writes as two logarithms and differentiates.	M1
	$\frac{dy}{dx}=\frac{1}{1-x^2}$	Correct completion with no errors seen.	A1
	Note that use of $\text{arcoth}\,x=\frac{1}{\text{artanh}x}\left(=\frac{1}{\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)}\right)$ **scores no marks**		

	(a) Alternative 6		
	$y = \text{arcoth}\, x \Rightarrow \coth y = x$	Changes from arcoth to coth correctly This may be implied by e.g. $\tanh y = \frac{1}{x}$	B1
	$\tanh y = \frac{1}{x} \Rightarrow -\frac{1}{x^2} = \text{sech}^2\, y \frac{dy}{dx}$	$\pm\frac{1}{x^2} = \pm\text{sech}^2\, y \frac{dx}{dy}$	M1
	$-\frac{1}{x^2} = \left(1 - \frac{1}{x^2}\right)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{1-x^2}$	Correct completion with no errors seen.	A1*
	(a) Alternative 7		
	$y = \text{arcoth}\, x = \text{artanh}\left(\frac{1}{x}\right)$	Expresses arcoth in terms of artanh correctly	B1
	$\frac{dy}{dx} = \frac{1}{1-\left(\frac{1}{x}\right)^2} \times -x^{-2}$	Differentiates using the chain rule	M1
	$= \frac{-1}{x^2-1} = \frac{1}{1-x^2}$	Correct completion with no errors seen.	A1*

(b)	$y=(\text{arcoth}\,x)^2 \Rightarrow \frac{dy}{dx}=2(\text{arcoth}\,x)\times\frac{1}{1-x^2}$	Correct first derivative	B1
	$\frac{d^2y}{dx^2}=\frac{2}{1-x^2}(1-x^2)^{-1}+4x\text{arcoth}\,x\times(1-x^2)^{-2}$ $\frac{d^2y}{dx^2}=\frac{2(1-x^2)\times\frac{1}{1-x^2}+2\text{arcoth}\,x\times 2x}{(1-x^2)^2}\left(=\frac{4x\text{arcoth}x+2}{(1-x^2)^2}\right)$		M1A1
	M1: Attempts product or quotient rule on an expression of the form $\frac{k\text{arcoth}\,x}{1-x^2}$ Product rule requires $\pm P(1-x^2)^{-2}\pm Qx\,\text{arcoth}x\,(1-x^2)^{-2}$ oe Quotient rule requires $\frac{\pm P\pm Qx\,\text{arcoth}x}{(1-x^2)^2}$ oe		
	$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}=(1-x^2)\left(\frac{4x\text{arcoth}x+2}{(1-x^2)^2}\right)-2x\times\left(\frac{2\text{arcoth}\,x}{1-x^2}\right)$ or $(1-x^2)\frac{d^2y}{dx^2}=\frac{2}{1-x^2}+2x\times\left(\frac{dy}{dx}\right)$		M1
	M1: Substitutes their first and second derivatives into the lhs of the differential equation **or** multiplies through by $(1-x^2)$ and replaces $2(\text{arcoth}\,x)\times\frac{1}{1-x^2}$ by $\frac{dy}{dx}$		
	$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}=\frac{2}{1-x^2}$	Correct conclusion with no errors	A1cso
			(5)

	(b) Alternative 1		
	$y=(\text{arcoth}\,x)^2 \Rightarrow \frac{dy}{dx}=2(\text{arcoth}\,x)\times\frac{1}{1-x^2}$	Correct first derivative	B1
	$(1-x^2)\frac{dy}{dx}=2\text{arcoth}\,x\Rightarrow(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}=...$	M1: Multiplies through by $1-x^2$ and attempts product rule on $(1-x^2)\frac{dy}{dx}$. Requires $(1-x^2)\frac{d^2y}{dx^2}\pm Px\frac{dy}{dx}$ oe A1: Correct differentiation	M1A1
	$\frac{d(2\text{arcoth}\,x)}{dx}=\frac{2}{1-x^2}$	Differentiates rhs using the result from part (a)	M1
	$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}=\frac{2}{1-x^2}$	Correct conclusion with no errors	A1cso

	(b) Alternative 2		
	$y=(\text{arcoth}\,x)^2 \Rightarrow y^{\frac{1}{2}}=\text{arcoth}\,x \Rightarrow \frac{1}{2}y^{-\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1-x^2}$	Correct differentiation	B1
	$\frac{1}{2}y^{-\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\frac{1}{4}y^{-\frac{3}{2}}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{2x}{\left(1-x^2\right)^2}$	M1: Correct use of product rule to give $py^{-\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-qy^{-\frac{3}{2}}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ A1: $\frac{1}{2}y^{-\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\frac{1}{4}y^{-\frac{3}{2}}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{2x}{\left(1-x^2\right)^2}$	M1A1
	Then substitute as before to obtain $\frac{2}{1-x^2}$		M1A1cso
			Total 8

Question Number	Scheme	Notes	Marks
4(i)	$15+2x-x^2=16-(x-1)^2$	Correct completion of the square. Allow e.g. $15+2x-x^2=-\left[(x-1)^2-16\right]$ Allow 4^2 for 16	B1
	$\int \frac{1}{\sqrt{16-(x-1)^2}}\,dx=\arcsin\left(\frac{x-1}{4}\right)$	M1: $k\arcsin(f(x))$ A1: Correct integration	M1A1
	$\left[\arcsin\left(\frac{x-1}{4}\right)\right]_3^5=\arcsin 1-\arcsin\frac{1}{2}$	Correct use of correct limits	**d**M1
	$=\frac{\pi}{3}$		A1
	May see:		
	$15+2x-x^2=16-(1-x)^2$	Correct completion of the square. Allow e.g. $15+2x-x^2=-\left[(1-x)^2-16\right]$ Allow 4^2 for 16	B1
	$\int \frac{1}{\sqrt{16-(1-x)^2}}\,dx=-\arcsin\left(\frac{1-x}{4}\right)$	M1: $k\arcsin(f(x))$ A1: Correct integration	M1A1
	$\left[-\arcsin\left(\frac{1-x}{4}\right)\right]_3^5=-\arcsin(-1)+\arcsin\left(-\frac{1}{2}\right)$	Correct use of correct limits	**d**M1
	$=\frac{\pi}{3}$		A1
	By substitution 1:		
	$15+2x-x^2=16-(x-1)^2$	Correct completion of the square. Allow e.g. $15+2x-x^2=-\left[(1-x)^2-16\right]$ Allow 4^2 for 16	B1
	$x-1=4\sin\theta \Rightarrow \int \frac{1}{\sqrt{16-(x-1)^2}}\,dx=\int \frac{1}{\sqrt{16-(4\sin\theta)^2}}4\cos\theta\,d\theta$		
	$=\int d\theta=\theta$	M1: A full substitution leading to $k\theta$ or $k\times$ their variable A1: Correct integration	M1A1
	$[\theta]_{\frac{\pi}{6}}^{\frac{\pi}{2}}=\frac{\pi}{2}-\frac{\pi}{6}$	Correct use of correct limits	**d**M1
	$=\frac{\pi}{3}$		A1

	By substitution 2:		
	$15+2x-x^2=16-(x-1)^2$	Correct completion of the square. Allow e.g. $15+2x-x^2=-\left[(1-x)^2-16\right]$ Allow 4^2 for 16	B1
	$x-1=u\Rightarrow\int\frac{1}{\sqrt{16-(x-1)^2}}\,\mathrm{d}x=\int\frac{1}{\sqrt{16-u^2}}\,\mathrm{d}u$		
	$\int\frac{1}{\sqrt{16-u^2}}\,\mathrm{d}x=\arcsin\left(\frac{u}{4}\right)$	M1: $k$arcsin(f(u)) A1: Correct integration	M1A1
	$\left[\arcsin\left(\frac{u}{4}\right)\right]_2^4=\arcsin 1-\arcsin\frac{1}{2}$	Correct use of correct limits	**d**M1
	$=\frac{\pi}{3}$		A1
	By substitution 3:		
	$15+2x-x^2=16-(x-1)^2$	Correct completion of the square. Allow e.g. $15+2x-x^2=-\left[(1-x)^2-16\right]$ Allow 4^2 for 16	B1
	$x-1=4\cos\theta\Rightarrow\int\frac{1}{\sqrt{16-(x-1)^2}}\,\mathrm{d}x=\int\frac{1}{\sqrt{16-(4\cos\theta)^2}}-4\sin\theta\,\mathrm{d}\theta$		
	$=\int-\mathrm{d}\theta=-\theta$	M1: A full substitution leading to $k\theta$ or $k\times$ their variable A1: Correct integration	M1A1
	$[-\theta]_{\frac{\pi}{3}}^{0}=0+\frac{\pi}{3}$	Correct use of correct limits	**d**M1
	$=\frac{\pi}{3}$		A1
			(5)
(ii)(a)	$5\cosh x-4\sinh x=5\left(\frac{e^x+e^{-x}}{2}\right)-4\left(\frac{e^x-e^{-x}}{2}\right)$	Substitutes correct exponential forms	B1
	$=\frac{e^x+9e^{-x}}{2}$ or $\frac{e^x}{2}+\frac{9e^{-x}}{2}$	Expands and collects terms in e^x and e^{-x}	M1
	$=\frac{e^{2x}+9}{2e^x}$ *	Correct completion with no errors	A1*
	More working may be shown but allow e.g. $\frac{e^x+9e^{-x}}{2}=\frac{e^{2x}+9}{2e^x}$ or $\frac{e^x}{2}+\frac{9e^{-x}}{2}=\frac{e^{2x}+9}{2e^x}$		
			(3)

(b)	$u = e^x \Rightarrow \frac{du}{dx} = e^x$	Correct derivative. Allow equivalents e.g. $\frac{dx}{du} = \frac{1}{u}$, $du = e^x dx$	B1
	$\int \frac{2e^x}{e^{2x}+9} dx = \int \frac{2u}{u^2+9} . \frac{du}{u}$	Complete substitution into $\int \frac{2e^x}{e^{2x}+9} dx$. Condone omission of "d$u$" provided the substitution is otherwise complete apart from this. May be implied by e.g. $\int \frac{2}{u^2+9} du$	M1
	$= \frac{2}{3} \arctan\left(\frac{u}{3}\right)(+c)$	$k$arctan(f(u)) only. **Dependent on the first method mark.**	dM1
	$= \frac{2}{3} \arctan\left(\frac{e^x}{3}\right)(+c)$	Cao (+c not required)	A1
			(4)
			Total 12

Question Number	Scheme	Notes	Marks
5	$\frac{x^2}{16}-\frac{y^2}{9}=1 \quad P(4\sec\theta, 3\tan\theta)$		
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3\sec^2\theta}{4\sec\theta\tan\theta}\left(=\frac{3}{4\sin\theta}\right)$ or $\frac{2x}{16}-\frac{2y}{9}\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8\sec\theta}{16}\times\frac{9}{6\tan\theta}$ or $y=3\left(\frac{x^2}{16}-1\right)^{\frac{1}{2}}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2}\left(\frac{x^2}{16}-1\right)^{-\frac{1}{2}}\frac{x}{8}$ $=\frac{3}{2}\left(\frac{(4\sec\theta)^2}{16}-1\right)^{-\frac{1}{2}}\frac{4\sec\theta}{8}$	M1: Correct gradient method. Finds $\frac{\mathrm{d}y}{\mathrm{d}\theta}=p\sec^2\theta$ and $\frac{\mathrm{d}x}{\mathrm{d}\theta}=q\sec\theta\tan\theta$ and uses $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}\theta}\times\frac{\mathrm{d}\theta}{\mathrm{d}x}$ or differentiates implicitly to give $px+qy\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and substitutes for y and x to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ or differentiates explicitly to give $\frac{\mathrm{d}y}{\mathrm{d}x}=px\left(qx^2-1\right)^{-\frac{1}{2}}$ and substitutes for x A1: Correct derivative in terms of trig. functions, e.g. $\frac{3\sec^2\theta}{4\sec\theta\tan\theta}, \frac{8\sec\theta}{16}\times\frac{9}{6\tan\theta}$ Does not need to be simplified.	M1 A1
	Normal gradient $-\frac{4\sin\theta}{3}$	Correct perpendicular gradient rule. Does not need to be simplified.	M1
	$y-3\tan\theta=-\frac{4\sin\theta}{3}(x-4\sec\theta)$	Correct straight line method using a gradient (does not need to be simplified) in terms of θ that has come from calculus and is not the tangent gradient. If they use $y = mx + c$ then they must reach as far as finding c.	M1
	$3y+4x\sin\theta=25\tan\theta$*	Correct proof with no errors and one intermediate step from the previous line. Allow $25\tan\theta=3y+4x\sin\theta$	A1*
			(5)

(b)	$b^2 = a^2\left(e^2 - 1\right) \Rightarrow 9 = 16\left(e^2 - 1\right) \Rightarrow e = \frac{5}{4}$	M1: Use of the correct eccentricity formula to obtain a value for *e* A1: Correct value for *e*. Ignore ±	M1A1
	$x = \frac{a}{e} \Rightarrow x = \frac{16}{5}$ or $\frac{4}{5/4}$ etc.	Correct value for $\frac{a}{e}$ Ignore ±	A1
	$\theta = \frac{\pi}{4},\ x = \frac{16}{5} \Rightarrow 3y + 2\sqrt{2} \times \frac{16}{5} = 25$	Substitutes $\theta = \frac{\pi}{4}$ into the given normal equation and uses their **positive** directrix equation to obtain an equation in *y* or in *y* and e only.	M1
	$y = \frac{25}{3} - \frac{32}{15}\sqrt{2}$	B1: $a = \frac{25}{3}$ oe or $b = -\frac{32}{15}$ oe B1: $a = \frac{25}{3}$ oe and $b = -\frac{32}{15}$ oe	B1B1 (A marks on EPEN)
	Special Case: If the correct form of the answer is never seen but it appears correctly as a single fraction, allow B1B0 e.g. $y = \frac{125 - 32\sqrt{2}}{15}$		
			(6)
			Total 11

Question Number	Scheme	Notes	Marks
6(a)	$\begin{pmatrix} p & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & q \end{pmatrix}\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \lambda\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} p-\lambda & -2 & 0 \\ -2 & 6-\lambda & -2 \\ 0 & -2 & q-\lambda \end{pmatrix}\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	This statement is sufficient for this mark. May be implied by one correct equation e.g. $2p+4=2\lambda$, $-4-12-2=-2\lambda$, $4+q=\lambda$	M1
	$-4-12-2=-2\lambda \Rightarrow \lambda=9$	M1: Compares *y*-components to obtain a value for λ. Note that $-4-12-2=-2\lambda$ leading to a value for λ scores both method marks. If working is not clear, at least 2 terms of $"-4-12-2"$ should be correct.	M1A1
		A1: Correct eigenvalue	
			(3)
(b)	$\lambda=9 \Rightarrow 2p+4=18 \Rightarrow p=7$ $\lambda=9 \Rightarrow 4+q=9 \Rightarrow q=5$	M1: Uses their eigenvalue to form an equation in *p* or *q*	M1A1A1
		A1: Either *p* = 7 or *q* = 5	
		A1: Both *p* = 7 and *q* = 5	
			(3)
(c)	$\begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 6\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{matrix} 7x-2y=6x \\ -2x+6y-2z=6y \\ -2y+5z=6z \end{matrix}$		M1
	Uses the eigenvalue 6 and their value of *p* or *q* correctly to obtain at least 2 equations.		
	$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ or e.g. $\begin{pmatrix} 1 \\ \frac{1}{2} \\ -1 \end{pmatrix}$	This vector or any multiple of this vector.	A1
	Note that an eigenvector can be found from the cross product of any 2 rows of $\mathbf{M}-6\mathbf{I}$ e.g. $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 0 & -2 \\ 0 & -2 & -1 \end{vmatrix} = \begin{pmatrix} -4 \\ -2 \\ 4 \end{pmatrix}$		
			(2)

(d)	$\mathbf{P} = \begin{pmatrix} 2 & "2" & 1 \\ -2 & "1" & 2 \\ 1 & "-2" & 2 \end{pmatrix}$	Correct ft **P**. This should be a matrix of eigenvectors two of which are given in the question together with their eigenvector found from part (c). If an attempt is made to normalise the eigenvectors then allow the ft if slips are made when normalising.	B1ft
	$\mathbf{D} = \begin{pmatrix} "9" & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	Forms the matrix **D** by writing the eigenvalues 6, 3 and their λ on the leading diagonal and zeros elsewhere **or** attempts to calculate $\mathbf{P}^{\mathrm{T}}\mathbf{MP}$ to obtain a single 3 by 3 matrix. Consistency not needed for this mark.	M1
	$\left(\mathbf{P} = \frac{1}{3}\begin{pmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 9 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{pmatrix}\right)$ or $\left(\mathbf{P} = \begin{pmatrix} 2 & -2 & 1 \\ -2 & -1 & 2 \\ 1 & 2 & 2 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 81 & 0 & 0 \\ 0 & 54 & 0 \\ 0 & 0 & 27 \end{pmatrix}\right)$ Fully correct and consistent matrices		A1
	Note that the answers to part (d) may be implied e.g. $\mathbf{D} = \mathbf{P}^{\mathrm{T}}\mathbf{MP} = \begin{pmatrix} 2 & -2 & 1 \\ -2 & -1 & 2 \\ 1 & 2 & 2 \end{pmatrix}\begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}\begin{pmatrix} 2 & -2 & 1 \\ -2 & -1 & 2 \\ 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 81 & 0 & 0 \\ 0 & 54 & 0 \\ 0 & 0 & 27 \end{pmatrix}$ Would score all 3 marks by implication.		
			(3)
			Total 11

Question Number	Scheme	Notes	Marks
7(a)	$\frac{\sin nx}{\sin x} - \frac{\sin(n-2)x}{\sin x} = \frac{\sin nx - \sin nx \cos 2x + \cos nx \sin 2x}{\sin x}$ Expands $\sin(n-2)x$ correctly		M1
	$= \frac{\sin nx - \sin nx(1-2\sin^2 x) + 2\sin x\cos x\cos nx}{\sin x}$ Replaces $\cos 2x$ and $\sin 2x$ by the correct trigonometric identities		M1
	$= 2\sin nx \sin x + 2\cos nx \cos x$		
	$= 2\cos(n-1)x$		
	$(\therefore I_n - I_{n-2}) = \int 2\cos(n-1)x\,dx$ *	Correct completion with no errors. The $I_n - I_{n-2}$ does not need to be seen explicitly but $\int 2\cos(n-1)x\,dx$ must seen, including the integral sign.	A1*
			(3)
	(a) Way 2 (factor formula)		
	$\frac{\sin nx}{\sin x} - \frac{\sin(n-2)x}{\sin x} = \frac{2\cos\left(\frac{nx+nx-2x}{2}\right)\sin\left(\frac{nx-nx+2x}{2}\right)}{\sin x}$ Use of the correct factor formula		M1
	$= \frac{2\cos(nx-x)\sin x}{\sin x}$	Attempts to replaces $nx+nx-2x$ with $2nx-2x$ and attempts to replace $nx-nx+2x$ with $2x$	M1
	$= 2\cos(n-1)x$		
	$(I_n - I_{n-2}) = \int 2\cos(n-1)x\,dx$ *	Correct completion with no errors. The $I_n - I_{n-2}$ does not need to be seen explicitly but $\int 2\cos(n-1)x\,dx$ must seen, including the integral sign.	A1*
	(a) Way 3		
	$I_n = \int \frac{\sin((n-1)x+x)}{\sin x}\,dx$	Uses $\sin nx = \sin((n-1)x+x)$	M1
	$= \int \frac{\sin(n-1)x\cos x + \sin x\cos(n-1)x}{\sin x}\,dx$	Expands $\sin((n-1)x+x)$ correctly	M1
	$= \frac{1}{2}\int \frac{\sin nx + \sin(n-2)x}{\sin x}\,dx + \int \cos(n-1)x\,dx$		
	$= \frac{1}{2}I_n + \frac{1}{2}I_{n-2} + \int \cos(n-1)x\,dx$		
	$\therefore I_n - I_{n-2} = \int 2\cos(n-1)x\,dx$ *	Correct completion with no errors.	A1*

	(a) Way 4		
	$\frac{\sin nx}{\sin x} = \frac{\sin((n-2)x+2x)}{\sin x}$	Uses $\sin nx = \sin((n-2)x+2x)$	M1
	$= \frac{\sin(n-2)x(1-2\sin^2 x)+2\sin x\cos x\cos(n-2)x}{\sin x}$ Replaces $\cos 2x$ and $\sin 2x$ by the correct trigonometric identities		M1
	$= \frac{\sin(n-2)x}{\sin x} - 2\sin x\sin(n-2)x + 2\cos x\cos(n-2)x$		
	$= \frac{\sin(n-2)x}{\sin x} + 2\cos((n-2)x+x)$		
	$I_n = I_{n-2} + 2\int \cos(n-1)x\,dx$		
	$\therefore I_n - I_{n-2} = \int 2\cos(n-1)x\,dx$ *	Correct completion with no errors.	A1*
	(a) Way 5		
	$\sin nx = \sin((n-1)x+x)$ and $\sin(n-2)x = \sin((n-1)x-x)$		M1
	$\frac{\sin nx}{\sin x} - \frac{\sin(n-2)x}{\sin x} = \frac{\sin(n-1)x\cos x + \cos(n-1)x\sin x - (\sin(n-1)x\cos x - \sin x\cos(n-1)x)}{\sin x}$ Replaces $\sin((n-1)x+x)$ with $\sin(n-1)x\cos x + \cos(n-1)x\sin x$ and Replaces $\sin((n-1)x-x)$ with $\sin(n-1)x\cos x - \cos(n-1)x\sin x$		M1
	$\frac{\sin nx}{\sin x} - \frac{\sin(n-2)x}{\sin x} = \frac{2\sin x\cos(n-1)x}{\sin x}$		
	$(\therefore I_n - I_{n-2}) = \int 2\cos(n-1)x\,dx$ *	Correct completion with no errors. The $I_n - I_{n-2}$ does not need to be seen explicitly but $\int 2\cos(n-1)x\,dx$ must seen, including the integral sign.	A1

(b)	$\int \cos 4x\,dx = k\sin 4x$ or $\int \cos 2x\,dx = k\sin 2x$	$\cos 4x$ integrated to $\pm k\sin 4x$ or $\cos 2x$ integrated to $\pm k\sin 2x$	M1
	$2\int \cos 4x\,dx = \frac{1}{2}\sin 4x$ and $2\int \cos 2x\,dx = \sin 2x$	Both $2\cos 4x$ and $2\cos 2x$ integrated correctly with the correct (possibly un-simplified) coefficients	A1
	$\int \frac{\sin 5x}{\sin x}dx = \frac{2\sin(4x)}{4} + I_3$ or $\int \frac{\sin 3x}{\sin x}dx = \frac{2\sin(2x)}{2} + I_1$	One application of reduction formula. This may appear in any form and there does not need to be any integration e.g. $I_5 = \int 2\cos 4x\,dx + I_3$ **or** e.g. $I_3 = \int 2\cos 2x\,dx + I_1$	M1
	$\int \frac{\sin 5x}{\sin x}dx = \frac{2\sin(4x)}{4} + I_3$ and $\int \frac{\sin 3x}{\sin x}dx = \frac{2\sin(2x)}{2} + I_1$	Two applications of reduction formula. This may appear in any form and there does not need to be any integration e.g. $I_5 = \int 2\cos 4x\,dx + I_3$ and e.g. $I_3 = \int 2\cos 2x\,dx + I_1$ **Note that** $\int \frac{\sin 3x}{\sin x}dx$ **may be attempted using trig. Identities and can score full marks as long as use of the reduction formula is seen at least once.**	M1
	$I_1 = \frac{\pi}{12}$	(Could be implied by their final answer)	B1
	$\left[\frac{2\sin(4x)}{4} + \frac{2\sin(2x)}{2}\right]_{\frac{\pi}{12}}^{\frac{\pi}{6}} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4} - \frac{1}{2}$	Correct use of the given limits at least once on an expression of the form $\pm k\sin 4x$ or $\pm k\sin 2x$	M1
	$\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \frac{\sin 5x}{\sin x}dx = \frac{1}{12}\left(\pi + 6\sqrt{3} - 6\right)$	cao	A1
			(7)
	Note that correct work leading to $\left[\frac{2\sin(4x)}{4} + \frac{2\sin(2x)}{2} + x\right]$ or equivalent could score the first 4 marks		
			Total 10

Question Number	Scheme	Notes	Marks
8			
(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & 3 \\ 2 & -1 & 1 \end{vmatrix} = \begin{pmatrix} 7 \\ 5 \\ -9 \end{pmatrix}$	M1: Attempt cross product between direction vectors or any 2 vectors **in the plane**. If working is not shown or is unclear, 2 elements should be correct for their vectors for this mark. A1: Correct vector	M1A1
	$\begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 7 \\ 5 \\ -9 \end{pmatrix} (= 7 - 25 + 18)$	Attempts $\begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix} \bullet$ their vector product	M1
	$\begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 7 \\ 5 \\ -9 \end{pmatrix} = 7 - 25 + 18 = 0 \therefore$ perpendicular	Correctly obtains = 0 and gives a conclusion.	A1
	Note: $\begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 7 \\ 5 \\ -9 \end{pmatrix} = 0 \quad \therefore$ perpendicular scores M1A0 here. However $\begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 7 \\ 5 \\ -9 \end{pmatrix} = 7 - 25 + 18 = 0 \quad \therefore$ perpendicular scores M1A1 **BUT** If $\begin{pmatrix} 7 \\ 5 \\ -9 \end{pmatrix}$ is incorrect then $\begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} a \\ b \\ c \end{pmatrix} = a - 5b - 2c$ needs to be seen to score the M mark		
			(4)
(b)	$\begin{pmatrix} 7 \\ 5 \\ -9 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 8 \Rightarrow 7x + 5y - 9z = 8$	M1: Uses $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and their vector product to find the cartesian equation of Π_2. You may need to check their "8" if no working is shown but it must be clear that $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ (or a point on the plane) is being used. A1: Correct equation (any multiple or equivalent equation)	M1A1
	Note that part (b) is possible without part (a): e.g. $x = 1 + \lambda + 2\mu, \quad y = 2 + 4\lambda - \mu, \quad z = 1 + 3\lambda + \mu$ $. \Rightarrow y + z = 3 + 7\lambda$ and $x + 2y = 5 + 9\lambda \Rightarrow 9(y + z) - 7(x + 2y) = -8$ $\therefore 7x + 5y - 9z = 8$ Score as M1: Full method leading to a Cartesian equation, A1: Correct equation		
			(2)

(c)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 5 & -9 \\ 1 & -5 & -2 \end{vmatrix} = \begin{pmatrix} -55 \\ 5 \\ -40 \end{pmatrix}$	M1: Attempt cross product of normal vectors. A1: $k(11\mathbf{i}-\mathbf{j}+8\mathbf{k})$	M1A1
	$x=0:(0, -\frac{1}{5}, -1)$, $y=0:(-\frac{11}{5}, 0, -\frac{13}{5})$, $z=0:(\frac{11}{8}, -\frac{13}{40}, 0)$ Note that points on the line satisfy $(11t, -\frac{1}{5}-t, -1+8t)$		M1A1
	M1: Attempt point on the line (*x*, *y* and *z*). A1: Correct coordinates		
	$(\mathbf{r}-(-\frac{1}{5}\mathbf{j}-\mathbf{k}))\times(11\mathbf{i}-\mathbf{j}+8\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	**dd**M1A1
			(6)
			12 marks

	Alternatives for part (c) by simultaneous equations		
	Case 1: Eliminates y then obtains f(x) = g(y) = z		
	$x-5y-2z=3,\ 7x+5y-9z=8 \Rightarrow 8x-11z=11$		
	$z=\dfrac{8x-11}{11}, x=\dfrac{11+11z}{8} \Rightarrow \dfrac{11+11z}{8}-5y-2z=3 \Rightarrow z=\dfrac{-40y-13}{5}$		
	$\dfrac{8x-11}{11}=\dfrac{-40y-13}{5}=z$	M1: Obtains f(x) = g(y) = z A1: Correct expressions	M1A1
	$\dfrac{x-\frac{11}{8}}{\frac{11}{8}}=\dfrac{y+\frac{13}{40}}{-\frac{1}{8}}=\dfrac{z(-0)}{(1)}$	M1: Correct processing on at least one expression (not z) to enable identification of position and direction. A1: Correct equations	M1A1
	$(\mathbf{r}-(\frac{11}{8}\mathbf{i}-\frac{13}{40}\mathbf{j}))\times(\frac{11}{8}\mathbf{i}-\frac{1}{8}\mathbf{j}+\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	**dd**M1A1
	Case 2: Eliminates x then obtains f(x) = y = g(z)		
	$x-5y-2z=3,\ 7x+5y-9z=8 \Rightarrow 40y+5z=-13$		
	$y=\dfrac{-13-5z}{40}, z=\dfrac{-13-40y}{5} \Rightarrow x-5y+2\left(\dfrac{13+40y}{5}\right)=3 \Rightarrow y=\dfrac{-5x-11}{55}$		
	$\dfrac{-5x-11}{55}=y=\dfrac{-13-5z}{40}$	M1: Obtains f(x) = y = g(z) A1: Correct expressions	M1A1
	$\dfrac{x+\frac{11}{5}}{-11}=\dfrac{y(-0)}{(1)}=\dfrac{z+\frac{13}{5}}{-8}$	M1: Correct processing on at least one expression (not y) to enable identification of position and direction. A1: Correct equations	M1A1
	$(\mathbf{r}-(-\frac{11}{5}\mathbf{i}-\frac{13}{5}\mathbf{k}))\times(-11\mathbf{i}+\mathbf{j}-8\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	**dd**M1A1
	Case 3: Eliminates z then obtains x = f(y) = g(z)		
	$x-5y-2z=3,\ 7x+5y-9z=8 \Rightarrow 5x+55y=-11$		
	$x=\dfrac{-55y-11}{5}, y=\dfrac{-11-5x}{55} \Rightarrow x+5\left(\dfrac{11+5x}{55}\right)-2x=3 \Rightarrow x=\dfrac{11z+11}{8}$		
	$x=\dfrac{-55y-11}{5}=\dfrac{11z+11}{8}$	M1: Obtains x = f(y) = g(z) A1: Correct expressions	M1A1
	$\dfrac{x(-0)}{(1)}=\dfrac{y+\frac{1}{5}}{-\frac{1}{11}}=\dfrac{z+1}{\frac{8}{11}}$	M1: Correct processing on at least one expression (not z) to enable identification of position and direction. A1: Correct equations	M1A1
	$(\mathbf{r}-(-\frac{1}{5}\mathbf{j}-\mathbf{k}))\times(\mathbf{i}-\frac{1}{11}\mathbf{j}+\frac{8}{11}\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	**dd**M1A1

	Alternatives for part (c) by parameters		
	Case 1: Eliminates *x*		
	$x-5y-2z=3,\ 7x+5y-9z=8\ \Rightarrow 8x-11z=11$		
	$x=t\Rightarrow z=-1+\frac{8}{11}t,\ y=-\frac{1}{5}-\frac{1}{11}t$	M1: Obtains *x*, *y* and *z* in terms of λ A1: Correct expressions	M1A1
	$Pos:-\frac{1}{5}\mathbf{j}-\mathbf{k}\quad Dir:\mathbf{i}-\frac{1}{11}\mathbf{j}+\frac{8}{11}\mathbf{k}$	M1: Uses their equations to obtain position and direction A1: Correct position and direction	M1A1
	$(\mathbf{r}-(-\frac{1}{5}\mathbf{j}-\mathbf{k}))\times(\mathbf{i}-\frac{1}{11}\mathbf{j}+\frac{8}{11}\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	**dd**M1A1
	Case 2: Eliminates *y*		
	$x-5y-2z=3,\ 7x+5y-9z=8\ \Rightarrow 40y+15z=-13$		
	$y=t\Rightarrow z=-\frac{13}{5}-8t,\ y=-\frac{1}{5}-11t$	M1: Obtains *x*, *y* and *z* in terms of λ A1: Correct expressions	M1A1
	$Pos:-\frac{11}{5}\mathbf{i}-\frac{13}{5}\mathbf{k}\quad Dir:-\frac{11}{5}\mathbf{i}+\mathbf{j}-8\mathbf{k}$	M1: Uses their equations to obtain position and direction A1: Correct position and direction	M1A1
	$(\mathbf{r}-(-\frac{11}{5}\mathbf{i}-\frac{13}{5}\mathbf{k}))\times(-11\mathbf{i}+\mathbf{j}-8\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	ddM1A1
	Case 3: Eliminates *z*		
	$x-5y-2z=3,\ 7x+5y-9z=8\ \Rightarrow 8x-11z=11$		
	$z=t\Rightarrow x=\frac{11}{8}+\frac{11}{8}t,\ y=-\frac{13}{40}-\frac{1}{8}t$	M1: Obtains *x*, *y* and *z* in terms of λ A1: Correct expressions	M1A1
	$Pos:\frac{11}{8}\mathbf{i}-\frac{13}{40}\mathbf{j}\quad Dir:\frac{11}{8}\mathbf{i}-\frac{1}{8}\mathbf{j}+\mathbf{k}$	M1: Uses their equations to obtain position and direction A1: Correct position and direction	M1A1
	$(\mathbf{r}-(\frac{11}{8}\mathbf{i}-\frac{13}{40}\mathbf{j}))\times(\frac{11}{8}\mathbf{i}-\frac{1}{8}\mathbf{j}+\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	**dd**M1A1

	Alternative for part (c) by finding 2 points on the line		
	$x=0:(0,\ -\frac{1}{5},\ -1),\quad y=0:(-\frac{11}{5},\ 0,\ -\frac{13}{5}),\quad z=0:(\frac{11}{8},\ -\frac{13}{40},\ 0)$ M1: Attempts two points on the line A1: Two correct coordinates		M1A1
	Dir: $-\frac{1}{5}\mathbf{j}-\mathbf{k}-\left(-\frac{11}{5}\mathbf{i}-\frac{13}{5}\mathbf{k}\right)=\frac{11}{5}\mathbf{i}-\frac{1}{5}\mathbf{j}+\frac{8}{5}\mathbf{k}$	M1: Subtracts to obtain direction A1: Correct direction	M1A1
	$(\mathbf{r}-(-\frac{1}{5}\mathbf{j}-\mathbf{k}))\times(\frac{11}{5}\mathbf{i}-\frac{1}{5}\mathbf{j}+\frac{8}{5}\mathbf{k})=\mathbf{0}$	**dd**M1: (**r** – their point) × their direction "= 0" not required for this mark. **Dependent on both previous method marks.** A1: Correct equation (oe)	**dd**M1A1