Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Friday 21 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43149A

1. (*a*) Express $\dfrac{2}{(2r+1)(2r+3)}$ in partial fractions.

(2)

(*b*) Using your answer to (*a*), find, in terms of *n*,

$$\sum_{r=1}^{n} \frac{3}{(2r+1)(2r+3)}$$

Give your answer as a single fraction in its simplest form.

(3)

2.

$$z = 5\sqrt{3} - 5\text{i}$$

Find

(*a*) $|z|$,

(1)

(*b*) $\arg(z)$, in terms of π.

(2)

$$w = 2\left(\cos\frac{\pi}{4} + \text{i}\sin\frac{\pi}{4}\right)$$

Find

(*c*) $\left|\dfrac{w}{z}\right|$,

(1)

(*d*) $\arg\left|\dfrac{w}{z}\right|$, in terms of π.

(2)

3.

$$\frac{\text{d}^2y}{\text{d}x^2} + 4y - \sin x = 0$$

Given that $y = \dfrac{1}{2}$ and $\dfrac{\text{d}y}{\text{d}x} = \dfrac{1}{8}$ at $x = 0$,
find a series expansion for *y* in terms of *x*, up to and including the term in x^3.

(5)

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4. (*a*) Given that

$$z = r(\cos\theta + \mathrm{i}\sin\theta), \qquad r \in \mathbf{R}$$

prove, by induction, that $z^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta), \qquad n \in \mathbf{Z}^+$.

(5)

$$w = 3\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$$

(*b*) Find the exact value of w^5, giving your answer in the form $a + \mathrm{i}b$, where $a, b \in \mathbf{R}$.

(2)

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5. (*a*) Find the general solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 4x^2$$

(5)

(*b*) Find the particular solution for which $y = 5$ at $x = 1$, giving your answer in the form $y = \mathrm{f}(x)$.

(2)

(*c*) (i) Find the exact values of the coordinates of the turning points of the curve with equation $y = \mathrm{f}(x)$, making your method clear.

(ii) Sketch the curve with equation $y = \mathrm{f}(x)$, showing the coordinates of the turning points.

(5)

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6. (*a*) Use algebra to find the exact solutions of the equation

$$|2x^2 + 6x - 5| = 5 - 2x$$

(6)

(*b*) On the same diagram, sketch the curve with equation $y = |2x^2 + 6x - 5|$ and the line with equation $y = 5 - 2x$, showing the x-coordinates of the points where the line crosses the curve.

(3)

(*c*) Find the set of values of x for which

$$|2x^2 + 6x - 5| > 5 - 2x$$

(3)

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7. (*a*) Show that the transformation $y = xv$ transforms the equation

$$4x^2\frac{d^2y}{dx^2} - 8x\frac{dy}{dx} + (8 + 4x^2)y = x^4 \qquad \text{(I)}$$

into the equation

$$4\frac{d^2v}{dx^2} + 4v = x \qquad \text{(II)}$$

(6)

(*b*) Solve the differential equation (II) to find v as a function of x.

(6)

(*c*) Hence state the general solution of the differential equation (I).

(1)

8.

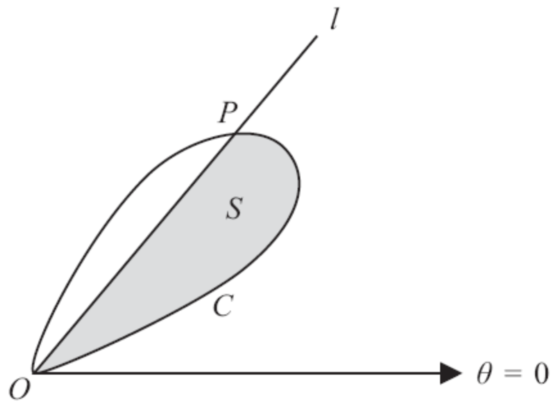

Figure 1

Figure 1 shows a curve C with polar equation $r = a\sin 2\theta$, $0 \leq \theta \leq \frac{\pi}{2}$, and a half-line l.

The half-line l meets C at the pole O and at the point P. The tangent to C at P is parallel to the initial line. The polar coordinates of P are (R, φ).

(*a*) Show that $\cos \varphi = \frac{1}{\sqrt{3}}$.

(6)

(*b*) Find the exact value of R.

(2)

The region S, shown shaded in Figure 1, is bounded by C and l.

(*c*) Use calculus to show that the exact area of S is

$$\frac{1}{36}a^2\left(9\arccos\left(\frac{1}{\sqrt{3}}\right)+\sqrt{2}\right)$$

(7)

TOTAL FOR PAPER: 75 MARKS

END