Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Subsidiary

Monday 30 January 2012 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Given that $z_1 = 1 - \text{i}$,

(a) find $\arg(z_1)$.

(2)

Given also that $z_2 = 3 + 4\text{i}$, find, in the form $a + \text{i}b$, $a, b \in \mathbb{R}$,

(b) $z_1 z_2$,

(2)

(c) $\dfrac{z_2}{z_1}$.

(3)

In part (b) and part (c) you must show all your working clearly.

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2. (a) Show that $\text{f}(x) = x^4 + x - 1$ has a real root α in the interval [0.5, 1.0].

(2)

(b) Starting with the interval [0.5, 1.0], use interval bisection twice to find an interval of width 0.125 which contains α.

(3)

(c) Taking 0.75 as a first approximation, apply the Newton Raphson process twice to $\text{f}(x)$ to obtain an approximate value of α. Give your answer to 3 decimal places.

(5)

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3. A parabola C has cartesian equation $y^2 = 16x$. The point $P(4t^2, 8t)$ is a general point on C.

(a) Write down the coordinates of the focus F and the equation of the directrix of C.

(3)

(b) Show that the equation of the normal to C at P is $y + tx = 8t + 4t^3$.

(5)

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4. A right angled triangle T has vertices $A(1, 1)$, $B(2, 1)$ and $C(2, 4)$. When T is transformed by the matrix $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, the image is T'.

(*a*) Find the coordinates of the vertices of T'.

(2)

(*b*) Describe fully the transformation represented by **P**.

(2)

The matrices $\mathbf{Q} = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ represent two transformations. When T is transformed by the matrix **QR**, the image is T''.

(*c*) Find **QR**.

(2)

(*d*) Find the determinant of **QR**.

(2)

(*e*) Using your answer to part (*d*), find the area of T''.

(3)

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5. The roots of the equation

$$z^3 - 8z^2 + 22z - 20 = 0$$

are z_1, z_2 and z_3 .

(*a*) Given that $z_1 = 3 + \text{i}$, find z_2 and z_3.

(4)

(*b*) Show, on a single Argand diagram, the points representing z_1, z_2 and z_3.

(2)

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6. (*a*) Prove by induction

$$\sum_{r=1}^{n} r^3 = \frac{1}{4} n^2(n + 1)^2.$$

(5)

(*b*) Using the result in part (*a*), show that

$$\sum_{r=1}^{n} (r^3 - 2) = \frac{1}{4} n(n^3 + 2n^2 + n - 8).$$

(3)

(*c*) Calculate the exact value of $\sum_{r=20}^{50} (r^3 - 2)$.

(3)

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7. A sequence can be described by the recurrence formula

$$u_{n+1} = 2u_n + 1, \qquad n \geq 1, \quad u_1 = 1.$$

(*a*) Find u_2 and u_3.

(2)

(*b*) Prove by induction that $u_n = 2^n - 1$.

(5)

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8. $$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}.$$

(*a*) Show that **A** is non-singular.

(2)

(*b*) Find **B** such that $\mathbf{BA}^2 = \mathbf{A}$.

(4)

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9. The rectangular hyperbola H has cartesian equation $xy = 9$.

The points $P\left(3p, \frac{3}{p}\right)$ and $Q\left(3q, \frac{3}{q}\right)$ lie on H, where $p \neq \pm q$.

(*a*) Show that the equation of the tangent at P is $x + p^2y = 6p$.

(4)

(*b*) Write down the equation of the tangent at Q.

(1)

The tangent at the point P and the tangent at the point Q intersect at R.

(*c*) Find, as single fractions in their simplest form, the coordinates of R in terms of p and q.

(4)

TOTAL FOR PAPER: 75 MARKS

END