Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Subsidiary

Tuesday 22 June 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H35387A

1. $$z = 2 - 3i$$

(*a*) Show that $z^2 = -5 - 12i$.

(2)

Find, showing your working,

(*b*) the value of $|z^2|$,

(2)

(*c*) the value of $\arg(z^2)$, giving your answer in radians to 2 decimal places.

(2)

(*d*) Show z and z^2 on a single Argand diagram.

(1)

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2. $$\mathbf{M} = \begin{pmatrix} 2a & 3 \\ 6 & a \end{pmatrix}, \text{ where a is a real constant.}$$

(*a*) Given that $a = 2$, find $\mathbf{M}^{-1}$.

(3)

(*b*) Find the values of a for which $\mathbf{M}$ is singular.

(2)

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3. $$f(x) = x^3 - \frac{7}{x} + 2, \ x > 0.$$

(*a*) Show that $f(x) = 0$ has a root α between 1.4 and 1.5.

(2)

(*b*) Starting with the interval [1.4, 1.5], use interval bisection twice to find an interval of width 0.025 that contains α.

(3)

(*c*) Taking 1.45 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = x^3 - \dfrac{7}{x} + 2$, $x > 0$ to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

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4. $$f(x) = x^3 + x^2 + 44x + 150.$$

Given that $f(x) = (x + 3)(x^2 + ax + b)$, where a and b are real constants,

(a) find the value of a and the value of b.

(2)

(b) Find the three roots of $f(x) = 0$.

(4)

(c) Find the sum of the three roots of $f(x) = 0$.

(1)

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5. The parabola C has equation $y^2 = 20x$.

(a) Verify that the point $P(5t^2, 10t)$ is a general point on C.

(1)

The point A on C has parameter $t = 4$.
The line l passes through A and also passes through the focus of C.

(b) Find the gradient of l.

(4)

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6. Write down the 2 × 2 matrix that represents

(a) an enlargement with centre (0, 0) and scale factor 8,

(1)

(b) a reflection in the x-axis.

(1)

Hence, or otherwise,

(c) find the matrix **T** that represents an enlargement with centre (0, 0) and scale factor 8, followed by a reflection in the x-axis.

(2)

$$\mathbf{A} = \begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} k & 1 \\ c & -6 \end{pmatrix}, \text{ where } k \text{ and } c \text{ are constants.}$$

(d) Find **AB**.

(3)

Given that **AB** represents the same transformation as **T**,

(e) find the value of k and the value of c.

(2)

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7. $$\mathrm{f}(n) = 2^n + 6^n.$$

(*a*) Show that $\mathrm{f}(k+1) = 6\mathrm{f}(k) - 4(2^k)$.

(3)

(*b*) Hence, or otherwise, prove by induction that, for $n \in \mathbb{Z}^+$, $\mathrm{f}(n)$ is divisible by 8.

(4)

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8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point A on H has x-coordinate $3c$.

(*a*) Write down the y-coordinate of A.

(1)

(*b*) Show that an equation of the normal to H at A is

$$3y = 27x - 80c.$$

(5)

The normal to H at A meets H again at the point B.

(*c*) Find, in terms of c, the coordinates of B.

(5)

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9. (*a*) Prove by induction that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1).$$

(6)

Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$,

(*b*) show that

$$\sum_{r=1}^{n}(r+2)(r+3) = \frac{1}{3}n(n^2 + an + b),$$

where a and b are integers to be found.

(5)

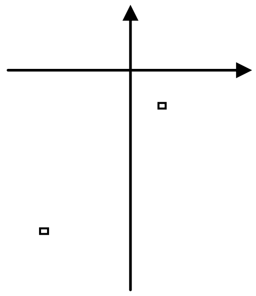
(*c*) Hence show that

$$\sum_{r=n+1}^{2n}(r+2)(r+3) = \frac{1}{3}n(7n^2 + 27n + 26).$$

(3)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	(a) $(2-3\text{i})(2-3\text{i}) = \ldots\ldots$ **Expand and use** $\text{i}^2 = -1$, getting completely correct expansion of 3 or 4 terms	M1
	Reaches $-5-12\text{i}$ after completely correct work (must see $4-9$) (*)	A1cso (2)
	(b) $\lvert z^2 \rvert = \sqrt{(-5)^2+(-12)^2} = 13$ **or** $\lvert z^2 \rvert = \sqrt{5^2+12^2} = 13$	M1 A1 (2)
	(c) $\tan\alpha = \frac{12}{5}$ (allow $-\frac{12}{5}$) or $\sin\alpha = \frac{12}{13}$ or $\cos\alpha = \frac{5}{13}$	M1
	$\arg(z^2) = -(\pi - 1.176\ldots) = -1.97$ (or 4.32) allow awrt	A1 (2)
	(d)  Both in correct quadrants. Approximate relative scale. No labels needed. Allow two diagrams if some indication of scale. Allow points or arrows	B1 (1)
		7 marks
2.	(a) $\mathbf{M} = \begin{pmatrix} 4 & 3 \\ 6 & 2 \end{pmatrix}$ Determinant: $(8-18) = -10$	B1
	$\mathbf{M}^{-1} = \frac{1}{-10}\begin{pmatrix} 2 & -3 \\ -6 & 4 \end{pmatrix}$ $\left[= \begin{pmatrix} -0.2 & 0.3 \\ 0.6 & -0.4 \end{pmatrix} \right]$	M1 A1 (3)
	(b) Setting $\Delta = 0$ and using $2a^2 \pm 18 = 0$ to obtain $a = $.	M1
	$a = \pm 3$	A1 cao (2)
		5 marks

Question Number	Scheme	Marks
3.	(a) $f(1.4) = ...$ and $f(1.5) = ...$ Evaluate both	M1
	$f(1.4) = -0.256$ (or $-\frac{32}{125}$), $f(1.5) = 0.708...$ (or $\frac{17}{24}$) **Change of sign, $\therefore$ root**	A1 (2)
	(b) $f(1.45) = 0.221...$ or 0.2 [$\therefore$ root is in [1.4, 1.45]]	M1
	$f(1.425) = -0.018...$ or -0.019 or -0.02	M1
	$\therefore$ root is in [1.425, 1.45]	A1cso (3) **5 marks**
	(c) $f'(x) = 3x^2 + 7x^{-2}$	M1 A1
	$f'(1.45) = 9.636...$ (Special case: $f'(x) = 3x^2 + 7x^{-2}$ +2 then $f'(1.45) = 11.636...$)	A1ft
	$x_1 = 1.45 - \frac{f(1.45)}{f'(1.45)} = 1.45 - \frac{0.221...}{9.636...} = 1.427$	M1 A1cao (5) **10 marks**
4.	(a) $a = -2$, $b = 50$	B1, B1 (2)
	(b) -3 is a root	B1
	Solving 3-term quadratic $x = \frac{2 \pm \sqrt{4-200}}{2}$ or $(x-1)^2 - 1 + 50 = 0$	M1
	$= 1 + 7i$, 1-7i	A1, A1ft (4)
	(c) $(-3) + (1+7i) + (1-7i) = -1$	B1ft (1) **7 marks**
5.	(a) $y^2 = (10t)^2 = 100t^2$ and $20x = 20 \times 5t^2 = 100t^2$	B1 (1)
	(b) Point *A* is (80, 40) (stated or seen on diagram). May be given in part (a)	B1
	Focus is (5, 0) (stated or seen on diagram) or (*a* , 0) with *a* = 5 May be given in part (a).	B1
	Gradient: $\frac{40-0}{80-5} = \frac{40}{75} \left(= \frac{8}{15}\right)$	M1 A1 (4) **5 marks**

Question Number	Scheme	Marks
6.	(a) $\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$	B1 (1)
	(b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	B1 (1)
	(c) $\mathbf{T} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & -8 \end{pmatrix}$	M1 A1 (2)
	(d) $\mathbf{AB} = \begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} k & 1 \\ c & -6 \end{pmatrix} = \begin{pmatrix} 6k+c & 0 \\ 4k+2c & -8 \end{pmatrix}$	M1 A1 A1 (3)
	(e) "$6k+c=8$" and "$4k+2c=0$" Form equations and solve simultaneously $k = 2$ and $c = -4$	M1 A1 (2) **9 marks**
7.	(a) LHS = $\text{f}(k+1) = 2^{k+1} + 6^{k+1}$ $= 2(2^k) + 6(6^k)$ $= 6(2^k + 6^k) - 4(2^k) = 6\text{f}(k) - 4(2^k)$ OR RHS = $= 6\text{f}(k) - 4(2^k) = 6(2^k + 6^k) - 4(2^k)$ $= 2(2^k) + 6(6^k)$ $= 2^{k+1} + 6^{k+1} = \text{f}(k+1)$ (*)	M1 A1 A1 (3)
	OR $\text{f}(k+1) - 6f(k) = 2^{k+1} + 6^{k+1} - 6(2^k + 6^k)$	M1
	$= (2-6)(2^k) = -4.2^k$, and so $\text{f}(k+1) = 6\text{f}(k) - 4(2^k)$	A1, A1 (3)
	(b) n = 1: $\text{f}(1) = 2^1 + 6^1 = 8$, **which is divisible** by 8	B1
	Either Assume f(*k*) divisible by 8 and try to use f(*k* + 1) = 6f(*k*) - 4(2^k) Show $4(2^k) = 4 \times 2(2^{k-1}) = 8(2^{k-1})$ or $8(\frac{1}{2}2^k)$ Or valid statement Deduction that result is implied for $n = k + 1$ and so is true for positive integers by induction (may include n = 1 true here) **Or** Assume f(*k*) divisible by 8 and try to use f (k + 1)– f (k) or f (k + 1) + f (k) including factorising $6^k = 2^k 3^k$ $= 2^3 2^{k-3}(1 + 5.3^k)$ or $= 2^3 2^{k-3}(3 + 7.3^k)$ o.e. Deduction that result is implied for $n = k + 1$ and so is true for positive integers by induction (must include explanation of why n = 2 is also true here)	M1 A1 A1cso (4) **7 marks**

Question Number	Scheme			Marks
8.	(a) $\frac{c}{3}$			B1 (1)
	(b) $y=\frac{c^2}{x}\Rightarrow\frac{dy}{dx}=-c^2x^{-2}$, **or** $y+x\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=-\frac{y}{x}$ **or** $x=c,\ y=-\frac{c}{t^2}$ so $\frac{dy}{dx}=-\frac{1}{t^2}$			B1
	and at A $\frac{dy}{dx}=-\frac{c^2}{(3c)^2}=-\frac{1}{9}$ so gradient of normal is 9			M1 A1
	Either $y-\frac{c}{3}=9(x-3c)$ **or** y=9x + k and use x=3c, y= $\frac{c}{3}$			M1
	$\Rightarrow$ $3y=27x-80c$ (*)			A1 (5)
	(c) $\frac{c^2}{x}=\frac{27x-80c}{3}$	$\frac{c^2}{y}=\frac{3y+80c}{27}$	$3\frac{c}{t}=27ct-80c$	M1
	$3c^2=27x^2-80cx$	$27c^2=3y^2+80cy$	$3c=27ct^2-80ct$	A1
	$(x-3c)(27x+c)=0$ so $x=$	$(y+27c)(3y-c)=0$ so $y=$	$(t-3)(27t+1)=0$ so $t=$	M1
	$x=-\frac{c}{27}$, $y=-27c$	$x=-\frac{c}{27}$, $y=-27c$	($t=-\frac{1}{27}$ and so) $x=-\frac{c}{27}$, $y=-27c$	A1 , A1 (5) **11 marks**

Question Number	Scheme	Marks
9.	(a) If $n = 1$, $\sum_{r=1}^{n} r^2 = 1$ and $\frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}\times 1\times 2\times 3 = 1$, so true for $n = 1$.	B1
	Assume result true for $n = k$	M1
	$\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$	M1
	$= \frac{1}{6}(k+1)(2k^2+7k+6)$ **or** $= \frac{1}{6}(k+2)(2k^2+5k+3)$ **or** $= \frac{1}{6}(2k+3)(k^2+3k+2)$	A1
	$= \frac{1}{6}(k+1)(k+2)(2k+3)$ $= \frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$ or equivalent	dM1
	True for $n = k + 1$ if true for $n = k$, (and true for $n = 1$) so true by induction for all n.	A1cso (6)
	(b) $\sum_{r=1}^{n}(r^2+5r+6) = \sum_{r=1}^{n} r^2 + 5\sum_{r=1}^{n} r +$ $(\sum_{r=1}^{n} 6)$	M1
	$\frac{1}{6}n(n+1)(2n+1) + \frac{5}{2}n(n+1),\quad + 6n$	A1, B1
	$= \frac{1}{6}n[(n+1)(2n+1) + 15(n+1) + 36]$	M1
	$= \frac{1}{6}n[2n^2+18n+52] = \frac{1}{3}n(n^2+9n+26)$ or $a = 9$, $b = 26$	A1 (5)
	(c) $\sum_{r=n+1}^{2n}(r+2)(r+3) = \frac{1}{3}2n(4n^2+18n+26) - \frac{1}{3}n(n^2+9n+26)$	M1 A1ft
	$\frac{1}{3}n(8n^2+36n+52-n^2-9n-26) = \frac{1}{3}n(7n^2+27n+26)$ (*)	A1cso (3) **14 marks**