Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Subsidiary Level

Thursday 11 June 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Orange or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1.

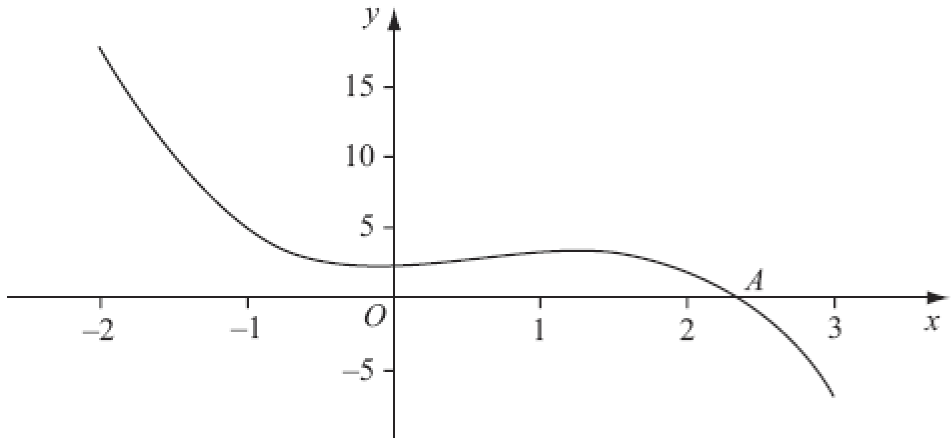

Figure 1

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x-axis at the point A where $x = \alpha$.

To find an approximation to α, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(*a*) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.
Give your answers to 3 decimal places where appropriate.

(3)

(*b*) Show that $\alpha = 2.359$ correct to 3 decimal places.

(3)

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2. (*a*) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$.

(2)

(*b*) Solve, for $0 \leq \theta < 360°$, the equation

$$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2.$$

(6)

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3. Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, \quad t \geq 0.$$

(*a*) Write down the number of rabbits that were introduced to the island.

(1)

(*b*) Find the number of years it would take for the number of rabbits to first exceed 1000.

(2)

(*c*) Find $\frac{dP}{dt}$.

(2)

(*d*) Find P when $\frac{dP}{dt} = 50$.

(3)

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4. (i) Differentiate with respect to x

(*a*) $x^2 \cos 3x$,

(3)

(*b*) $\frac{\ln(x^2 + 1)}{x^2 + 1}$.

(4)

(ii) A curve C has the equation

$$y = \sqrt{(4x+1)}, \quad x > -\tfrac{1}{4}, \quad y > 0.$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

5.

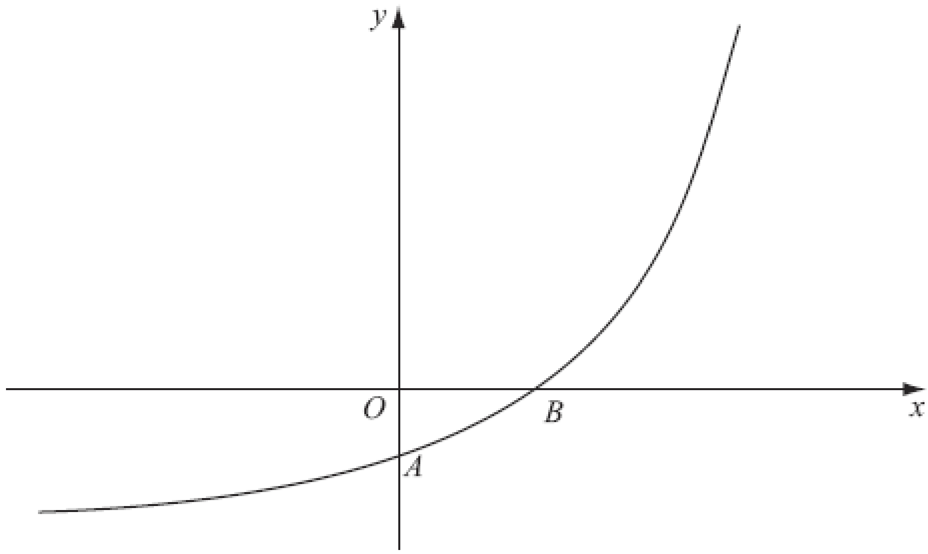

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0,\ 1 - k)$ and $B(\tfrac{1}{2}\ln k,\ 0)$, where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(*a*) $y = |\mathrm{f}(x)|$,

(3)

(*b*) $y = \mathrm{f}^{-1}(x)$.

(2)

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

Given that $\mathrm{f}(x) = \mathrm{e}^{2x} - k$,

(*c*) state the range of f,

(1)

(*d*) find $\mathrm{f}^{-1}(x)$,

(3)

(*e*) write down the domain of f^{-1}.

(1)

6. (*a*) Use the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$

(2)

The curves C_1 and C_2 have equations

$$C_1: y = 3\sin 2x$$

$$C_2: y = 4\sin^2 x - 2\cos 2x$$

(*b*) Show that the *x*-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

(3)

(*c*) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

(*d*) Hence find, for $0 \leq x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2,$$

giving your answers to 1 decimal place.

(4)

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7. The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R},\ x \neq -4,\ x \neq 2.$$

(*a*) Show that $f(x) = \dfrac{x-3}{x-2}$.

(5)

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R},\ x \neq \ln 2.$$

(*b*) Differentiate $g(x)$ to show that $g'(x) = \dfrac{e^x}{(e^x - 2)^2}$.

(3)

(*c*) Find the exact values of x for which $g'(x) = 1$

(4)

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8. (*a*) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$.

(1)

(*b*) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8 \cos x = 0.$$

giving your answers to 2 decimal places.

(5)

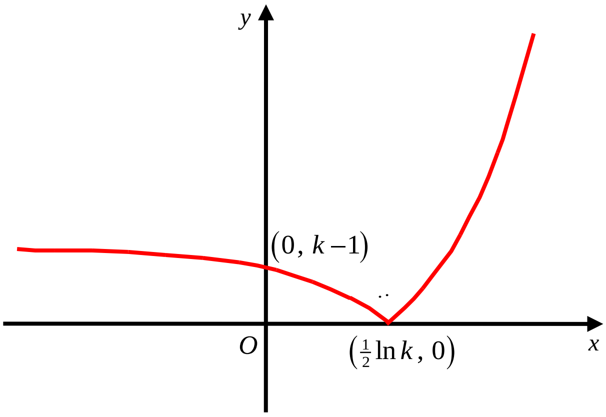
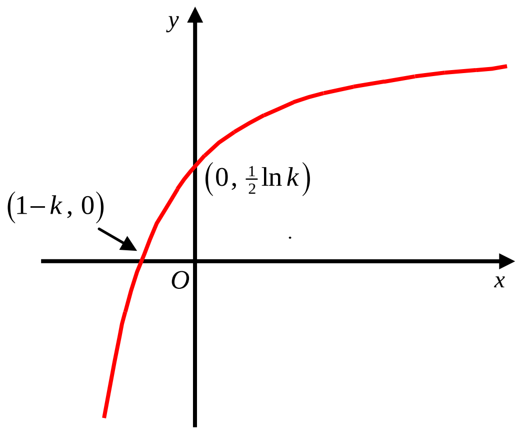
TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	Iterative formula: $x_{n+1} = \dfrac{2}{(x_n)^2} + 2\ ,\ x_0 = 2.5$	
	$x_1 = \dfrac{2}{(2.5)^2} + 2$	M1
	$x_1 = 2.32,\ \ x_2 = 2.371581451...$	A1
	$x_3 = 2.355593575...,\ \ x_4 = 2.360436923...$	A1 cso (3)
(b)	Let $\text{f}(x) = -x^3 + 2x^2 + 2 = 0$	
	$\text{f}(2.3585) = 0.00583577...$	M1
	$\text{f}(2.3595) = -0.00142286...$	M1
	Sign change (and $\text{f}(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)	A1 (3)
		(6 marks)
2. (a)	$\cos^2\theta + \sin^2\theta = 1\ \ (\div \cos^2\theta)$	
	$\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$	M1
	$1 + \tan^2\theta = \sec^2\theta$	
	$\tan^2\theta = \sec^2\theta - 1$ (as required)	A1 cso (2)
(b)	$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2,\ \ (\text{eqn } *)\ \ \ 0 \le \theta < 360^\circ$	
	$2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$	M1
	$2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$	
	$3\sec^2\theta + 4\sec\theta - 4 = 0$	M1
	$(\sec\theta + 2)(3\sec\theta - 2) = 0$	M1
	$\sec\theta = -2$ or $\sec\theta = \frac{2}{3}$	
	$\dfrac{1}{\cos\theta} = -2$ or $\dfrac{1}{\cos\theta} = \dfrac{2}{3}$	
	$\cos\theta = -\frac{1}{2}$; or $\cos\theta = \frac{3}{2}$	A1;
	$\alpha = 120^\circ$ or $\alpha =$ no solutions	
	$\theta_1 = 120^\circ$	A1
	$\theta_2 = 240^\circ$	B1 ft (6)
		(8 marks)

Question Number	Scheme	Marks
3. (a)	$P = 80\mathrm{e}^{\frac{t}{5}}$	
	$t = 0 \Rightarrow P = 80\mathrm{e}^{\frac{0}{5}} = 80(1) = \underline{80}$	B1 (1)
(b)	$P = 1000 \Rightarrow 1000 = 80\mathrm{e}^{\frac{t}{5}} \Rightarrow \frac{1000}{80} = \mathrm{e}^{\frac{t}{5}}$	M1
	$\therefore\ t = 5\ln\left(\frac{1000}{80}\right)$	
	$t = 12.6286...$	A1 (2)
(c)	$\frac{\mathrm{d}P}{\mathrm{d}t} = 16\mathrm{e}^{\frac{t}{5}}$	M1 A1 (2)
(d)	$50 = 16\mathrm{e}^{\frac{t}{5}}$	
	$\therefore\ t = 5\ln\left(\frac{50}{16}\right) \qquad \{= 5.69717...\}$	M1
	$P = 80\mathrm{e}^{\frac{1}{5}\left(5\ln\left(\frac{50}{16}\right)\right)}$ or $P = 80\mathrm{e}^{\frac{1}{5}(5.69717...)}$	M1
	$P = \frac{80(50)}{16} = \underline{250}$	A1 (3)
		(8 marks)

Question Number	Scheme	Marks
4. (i)(a)	$y = x^2 \cos 3x$	
	Apply product rule: $\begin{cases} u = x^2 & v = \cos 3x \\ \frac{\mathrm{d}u}{\mathrm{d}x} = 2x & \frac{\mathrm{d}v}{\mathrm{d}x} = -3\sin 3x \end{cases}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x\cos 3x - 3x^2 \sin 3x$	A1 A1 (3)
(i)(b)	$y = \frac{\ln(x^2+1)}{x^2+1}$	
	$u = \ln(x^2+1) \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2x}{x^2+1}$	M1 A1
	Apply quotient rule: $\begin{cases} u = \ln(x^2+1) & v = x^2+1 \\ \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2x}{x^2+1} & \frac{\mathrm{d}v}{\mathrm{d}x} = 2x \end{cases}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(\frac{2x}{x^2+1}\right)(x^2+1) - 2x\ln(x^2+1)}{(x^2+1)^2}$	A1
(ii)	$y = \sqrt{4x+1},\ x > -\frac{1}{4}$	
	At P, $y = \sqrt{4(2)+1} = \underline{\sqrt{9}} = \underline{3}$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \underline{\frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{(4x+1)^{\frac{1}{2}}}$	A1
	At P, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{(4(2)+1)^{\frac{1}{2}}}$	M1
	Hence m(**T**) $= \frac{2}{3}$	
	Either **T**: $y - 3 = \frac{2}{3}(x-2)$;	M1
	T: $3y - 9 = 2(x-2)$;	
	T: $3y - 9 = 2x - 4$	
	T: $\underline{2x - 3y + 5 = 0}$	A1 (6)
		(13 marks)

Question Number	Scheme		Marks
5. (a)		Curve retains shape when $x > \frac{1}{2}\ln k$	B1
		Curve reflects through the x-axis when $x < \frac{1}{2}\ln k$	B1
		$(0,\ k-1)$ and $(\frac{1}{2}\ln k,\ 0)$ marked in the correct positions.	B1 (3)
(b)		Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)	B1
		$(1-k,\ 0)$ and $(0,\ \frac{1}{2}\ln k)$	B1 (2)
(c)	Range of f: $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k,\ \infty)}$		B1 (1)
(d)	$y = e^{2x} - k \Rightarrow y + k = e^{2x}$		M1
	$\Rightarrow \ln(y+k) = 2x$		
	$\Rightarrow \frac{1}{2}\ln(y+k) = x$		M1
	Hence $f^{-1}(x) = \underline{\frac{1}{2}\ln(x+k)}$		<u>A1</u> cao (3)
	$f^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k,\ \infty)}$		B1ft (1)
			(10 marks)

Question Number	Scheme	Marks
6. (a)	$A=B \Rightarrow \cos(A+A) = \cos 2A = \underline{\cos A\cos A - \sin A \sin A}$	M1
	$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives	
	$\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required)	A1 (2)
(b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$	M1
	$3\sin 2x = 4\left(\frac{1-\cos 2x}{2}\right) - 2\cos 2x$	M1
	$3\sin 2x = 2(1-\cos 2x) - 2\cos 2x$	
	$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$	
	$3\sin 2x + 4\cos 2x = 2$	A1 (3)
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$	
	$3\sin 2x + 4\cos 2x = R\cos 2x \cos\alpha + R\sin 2x \sin\alpha$	
	Equate $\sin 2x$: $3 = R\sin\alpha$ Equate $\cos 2x$: $4 = R\cos\alpha$	
	$R = \sqrt{3^2 + 4^2}; = \sqrt{25} = 5$	B1
	$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...°$	M1 A1
	Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$	A1 (3)
(d)	$3\sin 2x + 4\cos 2x = 2$	
	$5\cos(2x - 36.87) = 2$	
	$\cos(2x - 36.87) = \frac{2}{5}$	M1
	$(2x - 36.87) = 66.42182...°$	A1
	$(2x - 36.87) = 360 - 66.42182...°$	
	Hence, $x = 51.64591...°, 165.22409...°$	A1 A1 (4)
		(12 marks)

Question Number	Scheme	Marks
7. (a)	$\mathrm{f}(x) = 1 - \dfrac{2}{(x+4)} + \dfrac{x-8}{(x-2)(x+4)} \quad x \in \mathbb{R},\ x \neq -4,\ x \neq 2.$	
	$\mathrm{f}(x) = \dfrac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$	M1 A1
	$= \dfrac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$	
	$= \dfrac{x^2 + x - 12}{[(x+4)(x-2)]}$	A1
	$= \dfrac{(x+4)(x-3)}{[(x+4)(x-2)]}$	M1
	$= \dfrac{(x-3)}{(x-2)}$	A1 cso (5)
(b)	$\mathrm{g}(x) = \dfrac{e^x - 3}{e^x - 2} \quad x \in \mathbb{R},\ x \neq \ln 2.$	
	Apply quotient rule: $\left\{ \begin{array}{ll} u = e^x - 3 & v = e^x - 2 \\ \dfrac{du}{dx} = e^x & \dfrac{dv}{dx} = e^x \end{array} \right\}$	
	$\mathrm{g}'(x) = \dfrac{e^x(e^x - 2) - e^x(e^x - 3)}{(e^x - 2)^2}$	M1 A1
	$= \dfrac{e^x}{(e^x - 2)^2}$	A1 cso (3)
(c)	$\mathrm{g}'(x) = 1 \Rightarrow \dfrac{e^x}{(e^x - 2)^2} = 1$	
	$e^x = (e^x - 2)^2$	M1
	$e^x = e^{2x} - 2e^x - 2e^x + 4$	
	$\underline{e^{2x} - 5e^x + 4} = 0$	A1
	$(e^x - 4)(e^x - 1) = 0$	M1
	$e^x = 4$ or $e^x = 1$	
	$x = \ln 4$ or $x = 0$	A1 (4)
		(12 marks)

Question Number	Scheme	Marks
8. (a)	$\sin 2x = \underline{2\sin x\cos x}$	B1 (1)
(b)	$\operatorname{cosec} x - 8\cos x = 0\,, \quad 0 < x < \pi$	
	$\dfrac{1}{\sin x} - 8\cos x = 0$	M1
	$\dfrac{1}{\sin x} = 8\cos x$	
	$1 = 8\sin x\cos x$	
	$1 = 4(2\sin x\cos x)$	
	$1 = 4\sin 2x$	M1
	$\underline{\sin 2x = \tfrac{1}{4}}$	<u>A1</u>
	Radians $2x = \{0.25268..., 2.88891...\}$	
	Degrees $2x = \{14.4775..., 165.5225...\}$	
	Radians $x = \{0.12634..., 1.44445...\}$	A1
	Degrees $x = \{7.23875..., 82.76124...\}$	A1 cao(5)
		(6 marks)