C3 January 2007

1. (a) By writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(5)

(b) Given that $\sin\theta = \frac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$.

(2)

2.

$$f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}, \quad x \neq -2.$$

(a) Show that $f(x) = \frac{x^2 + x + 1}{(x+2)^2}$, $x \neq -2$.

(4)

(b) Show that $x^2 + x + 1 > 0$ for all values of x.

(3)

(c) Show that $f(x) > 0$ for all values of x, $x \neq -2$.

(1)

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3. The curve C has equation

$$x = 2\sin y.$$

(a) Show that the point $P\left(\sqrt{2}, \frac{\pi}{4}\right)$ lies on C.

(1)

(b) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ at P.

(4)

(c) Find an equation of the normal to C at P. Give your answer in the form $y = mx + c$, where m and c are exact constants.

(4)

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4. (i) The curve C has equation

$$y = \frac{x}{9 + x^2}.$$

Use calculus to find the coordinates of the turning points of C.

(6)

(ii) Given that

$$y = (1 + e^{2x})^{\frac{3}{2}},$$

find the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}\ln 3$.

(5)

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5. **Figure 1**

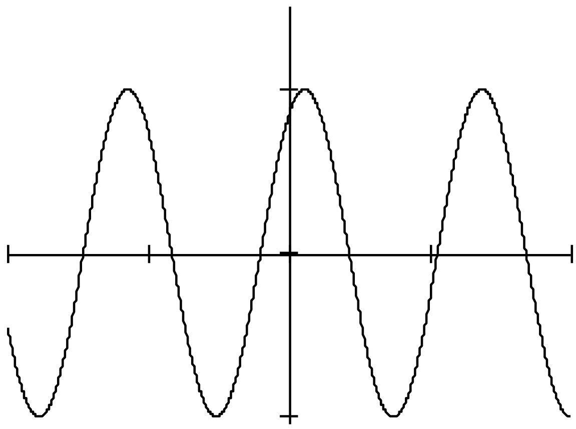

Figure 1 shows an oscilloscope screen.

The curve shown on the screen satisfies the equation

$$y = \sqrt{3}\cos x + \sin x.$$

(a) Express the equation of the curve in the form $y = R\sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

(b) Find the values of x, $0 \leqslant x < 2\pi$, for which $y = 1$.

(4)

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6. The function f is defined by

$$f : x \mapsto \ln(4-2x), \quad x < 2 \quad \text{and} \quad x \in \mathbb{R}.$$

(a) Show that the inverse function of f is defined by

$$f^{-1} : x \mapsto 2 - \frac{1}{2}e^{x}$$

and write down the domain of f^{-1}.

(4)

(b) Write down the range of f^{-1}.

(1)

(c) In the space provided on page 16, sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes.

(4)

The graph of $y = x + 2$ crosses the graph of $y = f^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\frac{1}{2}e^{x_n}, \quad x_0 = -0.3$$

is used to find an approximate value for k.

(d) Calculate the values of x_1 and x_2, giving your answers to 4 decimal places.

(2)

(e) Find the value of k to 3 decimal places.

(2)

7.

$$f(x) = x^4 - 4x - 8.$$

(a) Show that there is a root of $f(x) = 0$ in the interval [–2, –1].

(3)

(b) Find the coordinates of the turning point on the graph of $y = f(x)$.

(3)

(c) Given that $f(x) = (x-2)(x^3 + ax^2 + bx + c)$, find the values of the constants, a, b and c.

(3)

(d) In the space provided on page 21, sketch the graph of $y = f(x)$.

(3)

(e) Hence sketch the graph of $y = |f(x)|$.

(1)

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8. (i) Prove that

$$\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x.$$

(3)

(ii) Given that

$$y = \arccos x, \quad -1 \leqslant x \leqslant 1 \text{ and } 0 \leqslant y \leqslant \pi,$$

(a) express $\arcsin x$ in terms of y.

(2)

(b) Hence evaluate $\arccos x + \arcsin x$. Give your answer in terms of π.

(1)

TOTAL FOR PAPER: 75 MARKS

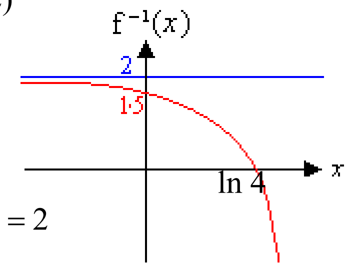
edexcel

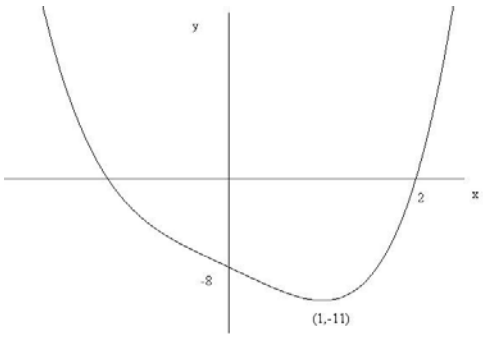
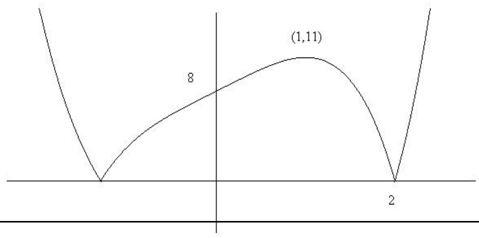
January 2007
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $\sin 3\theta = \sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$	B1
	$= 2\sin\theta\cos^2\theta + (1-2\sin^2\theta)\sin\theta$	B1 B1
	$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$	M1
	$= 3\sin\theta - 4\sin^3\theta$ * cso	A1 **(5)**
	(b) $\sin 3\theta = 3\times\frac{\sqrt{3}}{4} - 4\left(\frac{\sqrt{3}}{4}\right)^3 = \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \frac{9\sqrt{3}}{16}$ or exact equivalent	M1 A1 **(2)**
		[7]
2.	(a) $f(x) = \frac{(x+2)^2, -3(x+2)+3}{(x+2)^2}$	M1 A1, A1
	$= \frac{x^2+4x+4-3x-6+3}{(x+2)^2} = \frac{x^2+x+1}{(x+2)^2}$ * cso	A1 **(4)**
	(b) $x^2+x+1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4},\ > 0$ for all values of x.	M1 A1, A1 **(3)**
	(c) $f(x) = \frac{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}{(x+2)^2}$	
	Numerator is positive from (b)	
	$x \neq -2 \Rightarrow (x+2)^2 > 0$ (Denominator is positive)	
	Hence $f(x) > 0$	B1 **(1)**
		[8]
	Alternative to (b)	
	$\frac{d}{dx}(x^2+x+1) = 2x+1 = 0 \Rightarrow x = -\frac{1}{2} \Rightarrow x^2+x+1 = \frac{3}{4}$	M1 A1
	A parabola with positive coefficient of x^2 has a minimum $\Rightarrow x^2+x+1 > 0$	A1 **(3)**
	Accept equivalent arguments	

Question Number	Scheme	Marks
3.	(a) $y=\frac{\pi}{4} \Rightarrow x=2\sin\frac{\pi}{4}=2\times\frac{1}{\sqrt{2}}=\sqrt{2} \Rightarrow P\in C$	B1 **(1)**
	Accept equivalent (reversed) arguments. In any method it must be clear that $\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ or exact equivalent is used.	
	(b) $\frac{dx}{dy}=2\cos y$ *or* $1=2\cos y\frac{dy}{dx}$	M1 A1
	$\frac{dy}{dx}=\frac{1}{2\cos y}$ May be awarded after substitution	M1
	$y=\frac{\pi}{4} \Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{2}}$ * cso	A1 **(4)**
	(c) $m'=-\sqrt{2}$	B1
	$y-\frac{\pi}{4}=-\sqrt{2}\left(x-\sqrt{2}\right)$	M1 A1
	$y=-\sqrt{2}x+2+\frac{\pi}{4}$	A1 **(4)**
		[9]
4.	(i) $\frac{dy}{dx}=\frac{\left(9+x^2\right)-x\left(2x\right)}{\left(9+x^2\right)^2}$ $\left(=\frac{9-x^2}{\left(9+x^2\right)^2}\right)$	M1 A1
	$\frac{dy}{dx}=0 \Rightarrow 9-x^2=0 \Rightarrow x=\pm 3$	M1 A1
	$\left(3,\frac{1}{6}\right), \left(-3,-\frac{1}{6}\right)$ Final two A marks depend on second M only	A1, A1 **(6)**
	(ii) $\frac{dy}{dx}=\frac{3}{2}\left(1+e^{2x}\right)^{\frac{1}{2}}\times 2e^{2x}$	M1 A1 A1
	$x=\frac{1}{2}\ln 3 \Rightarrow \frac{dy}{dx}=\frac{3}{2}\left(1+e^{\ln 3}\right)^{\frac{1}{2}}\times 2e^{\ln 3}=3\times 4^{\frac{1}{2}}\times 3=18$	M1 A1 **(5)**
		[11]

Question Number	Scheme		Marks	
5.	(a) $R^2 = (\sqrt{3})^2 + 1^2 \Rightarrow R = 2$		M1 A1	
	$\tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$	accept awrt 1.05	M1 A1	**(4)**
	(b) $\sin(x + \text{ their } \alpha) = \frac{1}{2}$		M1	
	$x + \text{ their } \alpha = \frac{\pi}{6} \left(\frac{5\pi}{6}, \frac{13\pi}{6}\right)$		A1	
	$x = \frac{\pi}{2}, \frac{11\pi}{6}$	accept awrt 1.57, 5.76	M1 A1	**(4)**
				[8]
	The use of degrees loses only one mark in this question. Penalise the first time it occurs in an answer and then ignore.			

Question Number	Scheme	Marks
6.	(a) $y = \ln(4-2x)$	
	$e^y = 4-2x$ leading to $x = 2 - \frac{1}{2}e^y$ Changing subject and removing ln	M1 A1
	$y = 2 - \frac{1}{2}e^x \Rightarrow f^{-1} \mapsto 2 - \frac{1}{2}e^x$ * cso	A1
	Domain of f^{-1} is $\mathbb{R}$	B1 **(4)**
	(b) Range of f^{-1} is $f^{-1}(x) < 2$ (and $f^{-1}(x) \in \mathbb{R}$)	B1 **(1)**
	(c)  $y = 2$	
	Shape	B1
	1.5	B1
	ln 4	B1
		B1 **(4)**
	(d) $x_1 \approx -0.3704, \quad x_2 \approx -0.3452$ cao	B1, B1 **(2)**
	If more than 4 dp given in this part a maximum on one mark is lost. Penalise on the first occasion.	
	(e) $x_3 = -0.354\,030\,19 \ldots$	
	$x_4 = -0.350\,926\,88 \ldots$	
	$x_5 = -0.352\,017\,61 \ldots$	
	$x_6 = -0.351\,633\,86 \ldots$ Calculating to at least x_6 to at least four dp	M1
	$k \approx -0.352$ cao	A1 **(2)** **[13]**
	Alternative to (e)	
	$k \approx -0.352$ Found in any way	
	Let $g(x) = x + \frac{1}{2}e^x$	
	$g(-0.3515) \approx +0.0003, \quad g(-0.3525) \approx -0.001$	M1
	Change of sign (and continuity) $\Rightarrow k \in (-0.3525, -0.3515)$	
	$\Rightarrow k = -0.352$ (to 3 dp)	A1 **(2)**

Question Number	Scheme	Marks
7.	(a) $f(-2) = 16 + 8 - 8\ (=16) > 0$	B1
	$f(-1) = 1 + 4 - 8\ (= -3) < 0$	B1
	Change of sign (and continuity) $\Rightarrow$ root in interval $(-2, -1)$	B1ft **(3)**
	ft their calculation as long as there is a sign change	
	(b) $\frac{dy}{dx} = 4x^3 - 4 = 0 \quad \Rightarrow \quad x = 1$	M1 A1
	Turning point is $(1, -11)$	A1 **(3)**
	(c) $a = 2, b = 4, c = 4$	B1 B1 B1 **(3)**
	(d) 	
	Shape	B1
	ft their turning point in correct quadrant only	B1 ft
	2 and −8	B1 **(3)**
	(e) 	
	Shape	B1 **(1)** **[13]**

Question Number	Scheme	Marks
8.	(i) $\sec^2 x - \operatorname{cosec}^2 x = (1+\tan^2 x) - (1+\cot^2 x)$	M1 A1
	$= \tan^2 x - \cot^2 x$ ∗ cso	A1 **(3)**
	(ii)(a) $y = \arccos x \Rightarrow x = \cos y$	B1
	$x = \sin\left(\frac{\pi}{2} - y\right) \Rightarrow \arcsin x = \frac{\pi}{2} - y$	B1 **(2)**
	Accept $\arcsin x = \arcsin \cos y$	
	(b) $\arccos x + \arcsin x = y + \frac{\pi}{2} - y = \frac{\pi}{2}$	B1 **(1)**
		[6]
	Alternatives for (i)	
	$\sec^2 x - \tan^2 x = 1 = \operatorname{cosec}^2 x - \cot^2 x$	M1 A1
	Rearranging $\sec^2 x - \operatorname{cosec}^2 x = \tan^2 x - \cot^2 x$ ∗ cso	A1 **(3)**
	$\left(\text{LHS} = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}\right)$	
	RHS $= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x} = \frac{\sin^4 x - \cos^4 x}{\cos^2 x \sin^2 x} = \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{\cos^2 x \sin^2 x}$	M1
	$= \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}$	A1
	= LHS ∗ or equivalent	A1 **(3)**