Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Friday 5 June 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Orange or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H34263A

1. Use calculus to find the value of

$$\int_1^4 (2x + 3\sqrt{x}) \ \mathrm{d}x .$$

(5)

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2. (*a*) Find the first 3 terms, in ascending powers of *x*, of the binomial expansion of

$$(2 + kx)^7$$

where *k* is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 is 6 times the coefficient of *x*,

(*b*) find the value of *k*.

(2)

3.

$$f(x) = (3x - 2)(x - k) - 8$$

where *k* is a constant.

(*a*) Write down the value of f(*k*).

(1)

When f(*x*) is divided by $(x - 2)$ the remainder is 4.

(*b*) Find the value of *k*.

(2)

(*c*) Factorise f(*x*) completely.

(3)

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4. (*a*) Complete the table below, giving values of $\sqrt{(2^x + 1)}$ to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
$\sqrt{(2^x + 1)}$	1.414	1.554	1.732	1.957			3

(2)

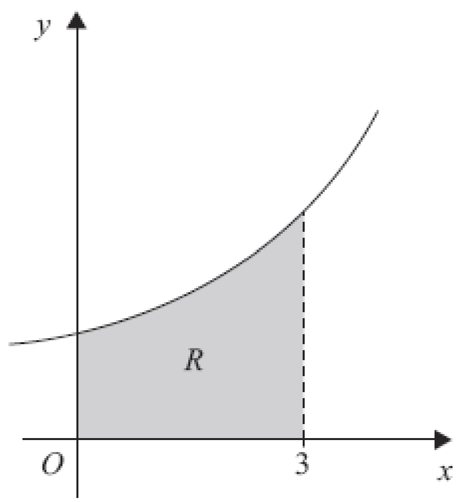

Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y = \sqrt{(2^x + 1)}$, the x-axis and the lines $x = 0$ and $x = 3$

(*b*) Use the trapezium rule, with all the values from your table, to find an approximation for the area of R.

(4)

(*c*) By reference to the curve in Figure 1 state, giving a reason, whether your approximation in part (*b*) is an overestimate or an underestimate for the area of R.

(2)

5. The third term of a geometric sequence is 324 and the sixth term is 96.

(*a*) Show that the common ratio of the sequence is $\frac{2}{3}$.

(2)

(*b*) Find the first term of the sequence.

(2)

(*c*) Find the sum of the first 15 terms of the sequence.

(3)

(*d*) Find the sum to infinity of the sequence.

(2)

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6. The circle C has equation

$$x^2 + y^2 - 6x + 4y = 12$$

(*a*) Find the centre and the radius of C.

(5)

The point $P(-1, 1)$ and the point $Q(7, -5)$ both lie on C.

(*b*) Show that PQ is a diameter of C.

(2)

The point R lies on the positive y-axis and the angle $PRQ = 90°$.

(*c*) Find the coordinates of R.

(4)

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7. (i) Solve, for $-180° \leq \theta < 180°$,

$$(1 + \tan\theta)(5\sin\theta - 2) = 0.$$

(4)

(ii) Solve, for $0 \leq x < 360°$,

$$4\sin x = 3\tan x.$$

(6)

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8. (*a*) Find the value of y such that

$$\log_2 y = -3.$$

(2)

(*b*) Find the values of x such that

$$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x.$$

(5)

9.

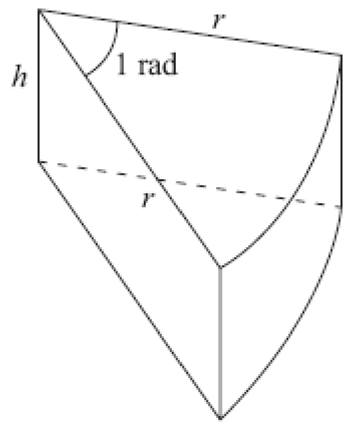

Figure 2

Figure 2 shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height h cm. The cross section is a sector of a circle. The sector has radius r cm and angle 1 radian.

The volume of the box is 300 cm^3.

(*a*) Show that the surface area of the box, S cm^2, is given by

$$S = r^2 + \frac{1800}{r}.$$

(5)

(*b*) Use calculus to find the value of r for which S is stationary.

(4)

(*c*) Prove that this value of r gives a minimum value of S.

(2)

(*d*) Find, to the nearest cm^2, this minimum value of S.

(2)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$\int\left(2x+3x^{\frac{1}{2}}\right)\,dx=\frac{2x^2}{2}+\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$	M1 A1 A1
	$\int_1^4\left(2x+3x^{\frac{1}{2}}\right)\,dx=\left[x^2+2x^{\frac{3}{2}}\right]_1^4=(16+2\times 8)-(1+2)$	M1
	$= 29$ (29 + C scores A0)	A1 (5)
		(5 marks)
2. (a)	$(7\times\ldots\times x)$ or $(21\times\ldots\times x^2)$ The 7 or 21 can be in 'unsimplified' form.	M1
	$(2+kx)^7=2^7+2^6\times 7\times kx+2^5\times\binom{7}{2}k^2x^2$	
	$= 128;\quad +448kx,\quad +672k^2x^2$	B1; A1, A1
	(If $672kx^2$ follows $672(kx)^2$, ignore subsequent working and allow A1)	(4)
(b)	$6\times 448k=672k^2$	M1
	$k = 4$ (Ignore k = 0, if seen)	A1 (2)
		(6 marks)
3. (a)	$f(k)=-8$	B1 (1)
(b)	$f(2)=4\Rightarrow\quad 4=(6-2)(2-k)-8$	M1
	So $k = -1$	A1 (2)
(c)	$f(x)=3x^2-(2+3k)x+(2k-8)\quad =3x^2+x-10$	M1
	$= (3x - 5)(x + 2)$	M1 A1 (3)
		(6 marks)
4. (a)	$x = 2$ gives 2.236 (allow AWRT) Accept √5	B1
	$x = 2.5$ gives 2.580 (allow AWRT) Accept 2.58	B1 (2)
(b)	$\left(\frac{1}{2}\times\frac{1}{2}\right)$, $[(1.414+3)+2(1.554+1.732+1.957+2.236+2.580)]$	B1,[M1A1ft]
	$= 6.133$ (AWRT 6.13, even following minor slips)	A1 (4)
(c)	Overestimate	B1
	'Since the trapezia lie above the curve', or an equivalent explanation, or sketch of (one or more) trapezia above the curve on a diagram (or on the given diagram, in which case there should be reference to this). (Note that there must be some reference to a trapezium or trapezia in the explanation or diagram).	B1 (2)
		(8 marks)

Question Number	Scheme	Marks
5. (a)	$324r^3 = 96$ or $r^3 = \frac{96}{324}$ or $r^3 = \frac{8}{27}$	M1
	$r = \frac{2}{3}$ (*)	A1cso (2)
(b)	$a\left(\frac{2}{3}\right)^2 = 324$ or $a\left(\frac{2}{3}\right)^5 = 96$ $a = \ldots,$ 729	M1, A1 (2)
(c)	$S_{15} = \frac{729\left(1-\left[\frac{2}{3}\right]^{15}\right)}{1-\frac{2}{3}}, \quad = 2182.00\ldots$ (awrt 2180)	M1A1ft, (3)
(d)	$S_\infty = \frac{729}{1-\frac{2}{3}}, \quad = 2187$	M1, A1 (2)
		[9]
		(9 marks)
6. (a)	$(x-3)^2 - 9 + (y+2)^2 - 4 = 12$ Centre is (3, –2)	M1 A1, A1
	$(x-3)^2 + (y+2)^2 = 12 + "9" + "4"$ $r = \sqrt{12+"9"+"4"} = 5$ (or $\sqrt{25}$ **)**	M1 A1 (5)
(b)	$PQ = \sqrt{(7--1)^2 + (-5-1)^2}$ or $\sqrt{8^2+6^2}$	M1
	$= 10 = 2\times$radius, $\therefore$diam. (N.B. For A1, need a comment or conclusion)	A1 (2)
(c)	*R* must lie on the circle (angle in a semicircle theorem)… often <u>implied</u> by <u>a diagram with *R* on the circle</u> or by subsequent working)	B1
	$x = 0 \Rightarrow \quad y^2 + 4y - 12 = 0$	M1
	$(y - 2)(y + 6) = 0$ $y = \ldots\ldots$ (M is dependent on previous M)	
	$y = -6$ or 2 (Ignore $y = -6$ if seen, and 'coordinates' are not required))	M1 A1 (4)
		(11 marks)
7. (i)	$\tan\theta = -1 \Rightarrow \quad \theta = -45, \quad 135$	B1, B1ft
	$\sin\theta = \frac{2}{5} \Rightarrow \quad \theta = 23.6, \quad 156.4$ (AWRT: 24, 156)	B1, B1ft (4)
(ii)	$4\sin x = \frac{3\sin x}{\cos x}$	M1
	$4\sin x\cos x = 3\sin x \quad \Rightarrow \quad \sin x(4\cos x - 3) = 0$	M1
	Other possibilities (after squaring): $\sin^2 x(16\sin^2 x - 7) = 0$, $(16\cos^2 x - 9)(\cos^2 x - 1) = 0$	
	$x = 0$, 180 <u>seen</u>	B1, B1
	$x = 41.4$, 318.6 (awrt: 41, 319)	B1, B1ft (6)
		(10 marks)

Question Number	Scheme	Marks
8. (a)	$\log_2 y = -3 \Rightarrow y = 2^{-3}$	M1
	$y = \frac{1}{8}$ or 0.125	A1 (2)
(b)	$32 = 2^5$ or $16 = 2^4$ or $512 = 2^9$	M1
	[or $\log_2 32 = 5\log_2 2$ or $\log_2 16 = 4\log_2 2$ or $\log_2 512 = 9\log_2 2$]	
	[or $\log_2 32 = \frac{\log_{10} 32}{\log_{10} 2}$ or $\log_2 16 = \frac{\log_{10} 16}{\log_{10} 2}$ or $\log_2 512 = \frac{\log_{10} 512}{\log_{10} 2}$]	
	$\log_2 32 + \log_2 16 = 9$	A1
	$(\log x)^2 = ...$ or $(\log x)(\log x) = ...$ (May not be seen explicitly, so M1 may be implied by later work, and the base may be 10 rather than 2)	M1
	$\log_2 x = 3 \Rightarrow x = 2^3 = 8$	A1
	$\log_2 x = -3 \Rightarrow x = 2^{-3} = \frac{1}{8}$	A1ft (5)
		(7 marks)
9. (a)	(Arc length =) $r\theta = r \times 1 = r$. Can be awarded by implication from later work, e.g. $3rh$ or $(2rh + rh)$ in the S formula. (Requires use of $\theta = 1$).	B1
	(Sector area =) $\frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times 1 = \frac{r^2}{2}$. Can be awarded by implication from later work, e.g. the correct volume formula. (Requires use of $\theta = 1$).	B1
	Surface area = 2 sectors + 2 rectangles + curved face	
	$(= r^2 + 3rh)$ (See notes below for what is allowed here)	M1
	Volume = 300 = $\frac{1}{2}r^2 h$	B1
	Sub for h: $S = r^2 + 3 \times \frac{600}{r} = r^2 + \frac{1800}{r}$ (*)	A1cso (5)
(b)	$\frac{dS}{dr} = 2r - \frac{1800}{r^2}$ or $2r - 1800r^{-2}$ or $2r + -1800r^{-2}$	M1A1
	$\frac{dS}{dr} = 0 \Rightarrow r^3 = ...,$ $r = \sqrt[3]{900}$, or AWRT 9.7 (NOT -9.7 or ± 9.7)	M1, A1 (4)
(c)	$\frac{d^2S}{dr^2} = ...$ and consider sign, $\frac{d^2S}{dr^2} = 2 + \frac{3600}{r^3} > 0$ so point is a minimum	M1, A1ft (2)
(d)	$S_{min} = (9.65...)^2 + \frac{1800}{9.65...}$	
	(Using their value of r, however found, in the <u>given</u> S formula)	M1
	$= 279.65...$ (AWRT: 280) (Dependent on full marks in part (b))	A1 (2)
		(13 marks)