$\S$A AEA Trial 2001 Final ~~1st~~ Draft | Marks

		Marks
1. (a) G.P. $a=1$ $r=t$, $S_n = \dfrac{t^{n+1}-1}{t-1}$	Spot GP	M1 A1
$S = 1+2t+\dots+nt^{n-1} = \dfrac{d}{dt}(S_n) = \dfrac{(t-1)(n+1)t^n - [t^{n+1}-1]\times 1}{(t-1)^2}$	$\frac{d}{dt}S_n$ Quot rule	M1 M1
$= \left(nt^{n+1} \cancel{+t^{n+1}} - (n+1)t^n \cancel{-t^{n+1}} + 1\right)/(t-1)^2$		
$= \dfrac{nt^{n+1}-(n+1)t^n+1}{(t-1)^2}$		A1 (5)
(b) $t=3$, $n=2001$ $\Rightarrow$ Sum $= \dfrac{2001\times 3^{2002} - 2002\times 3^{2001}+1}{2^2} = \dfrac{4001\times 3^{2001}+1}{4}$		B1 (1)
(c) $1+t+\dots+t^n \to n+1$, $1+2t+\dots+nt^{n-1} \to \dfrac{n(n+1)}{2}$		B1 (1) (7)
2. (a) $I_s = \int_0^{\pi/2} e^{2x}\, d(-\cos x)$ $\boxed{S \equiv I_s \quad C \equiv I_c}$	Attempt parts	M1
$= \left[-\cos x\, e^{2x}\right]_0^{\pi/2} + 2\int_0^{\pi/2} e^{2x}\cos x\, dx$		A1 (ignore limits)
$\therefore I_s = (-0)-(-1) + 2I_c$ or $I_s = 1+2I_c$ (*)		A1 c.s.o. (3)
(b) $I_c = \int_0^{\pi/2} e^{2x}\, d(\sin x)$		M1
$= \left[\sin x\, e^{2x}\right]_0^{\pi/2} - 2\int_0^{\pi/2} e^{2x}\sin x\, dx$		A1
i.e. $I_c = e^{\pi} - 0 - 2I_s$ i.e. $I_c = e^{\pi} - 2I_s$		A1
Sub: $I_s = 1 + 2\left[e^{\pi} - 2I_s\right]$		M1
	collect I_s	M1
$5I_s = 1+2e^{\pi}$ $\therefore I_s = \frac{1}{5}(1+2e^{\pi})$		A1 (6) (9)
3. $3\cos 4\theta + \cos^2\theta = 0.5$ $[\cos 2A = 2\cos^2 A - 1]$	$\cos^2\theta \to \cos 2\theta$	M1
$6\cos 4\theta + 2\cos^2\theta = 1$	$\cos 4\theta \to \cos 2\theta$	M1
$12\cos^2 2\theta - 6 + \cos 2\theta = 0$	Correct quadratic	A1
$(4\cos 2\theta + 3)(3\cos 2\theta - 2) = 0$	Attempt to solve	M1
$\Rightarrow \cos 2\theta = \frac{2}{3}$ or $-\frac{3}{4}$		A1, A1
$\cos 2\theta = \frac{2}{3} \Rightarrow 2\theta = 48.19, 311.81, 408.19, 671.81$	one sol^n	M1
$\therefore \theta = 24.1, 155.9, 204.1, 335.9$	comp. sol^n	M1
	+360° sol^n	M1
$\cos 2\theta = -\frac{3}{4} \Rightarrow 2\theta = 138.59, 221.41, 498.59, 581.41$	$\div 2$ (at correct stage)	M1
$\therefore \theta = 69.3, 110.7, 249.3, 290.7$	8 sol^{ns} (−1 eeoo)	A2/1/0
		(12)

§A AEA Trial 2001 Final / 2nd Draft — Marks

4 (a) $\ln(a+b) \neq \ln(a) + \ln(b)$ (line 2) — B1 (1)

(b) $y = \ln\left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right)$ — $\sec^2 + \operatorname{cosec}^2 \to \frac{1}{s^2c^2}$ — M1

$= \ln\left(\frac{1}{\sin^2 x \cos^2 x}\right)$ — use of $\sin 2x$ — M1

$= -2\ln(\sin x \cos x)$

$= -2\ln\frac{1}{2} - 2\ln(\sin 2x)$ — correct diff^n of ln — M1

correct manipulation of trig or ln — M1

$\therefore \frac{dy}{dx} = -\frac{2}{\sin 2x} \times 2\cos 2x = \underline{\underline{-4\cot 2x}}$ — A1 cso (5)

(c) Require: $\frac{p'+q'}{p+q} = \frac{p'}{p} + \frac{q'}{q}$ — each side — M1, M1

forming eqn — M1

$= \frac{p'q + q'p}{pq}$ — A1

i.e. $\frac{\frac{d}{dx}(p+q)}{p+q} = \frac{\frac{d}{dx}(pq)}{pq}$ i.e. $\frac{d}{dx}(\ln[p+q]) = \frac{d}{dx}[\ln(pq)]$ — M1 A1

Integrating — M1

i.e. $pq = A(p+q)$ — A1

$\therefore\ p(q-A) = Aq$

i.e. $\underline{\underline{p = \frac{Aq}{q-A}}}$, where A is a constant — (8) (14)

5 (a) By symmetry consider RHS.

$\frac{25}{4} - x^2 = 2kx - kx^2$ — M1

i.e. $(k-1)x^2 - 2kx + \frac{25}{4} = 0$ — Quadratic in x — M1

Need equal roots $\therefore$ $4k^2 = \cancel{4} \times \frac{25}{\cancel{4}} \times (k-1)$ — M1

A1

i.e. $4k^2 - 25k + 25 = 0$

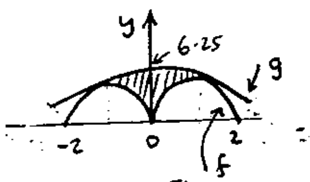
$(4k-5)(k-5) = 0$ — M1

i.e. $\underline{k = 5 \text{ or } 5/4}$ — A1

(b) $x = \text{"}\frac{-b}{2a}\text{"} = \frac{2k}{2(k-1)} = \frac{k}{k-1}$ — M1

$\because x < 2$ we need $\underline{k = 5}$ i.e. $\left(x = \underline{\underline{5/4}}\right)$ — M1 (8)

(c)



symmetric — B1

touch x 2 — B1

touch at $\left(\pm\frac{5}{4}, \pm\frac{75}{16}\right)$ — B1, B1 (4)

(d) Area $= 2\times\int_0^{5/4}\left(6.25 - x^2 - [10x - 5x^2]\right)dx$ — $\int g - f$ — M1

$2\times$ + Limits — M1

$\int$ — M1 A1

$= 2\times\left[6.25x - 5x^2 + \frac{4}{3}x^3\right]_0^{5/4}$

$= 2\times\left[\frac{25}{4}\times\frac{5}{4} - \frac{5\times 25}{16} + \frac{4}{3}\times\frac{125}{64}\right] = 2\times\frac{125}{48} = \frac{125}{24}$ or $5\frac{5}{24}$ — M1 A1 (6) (18)

6 (a) $\sum_1^n e^r$ G.P. $a=e,\ r=e$ $\therefore$ sum $= \dfrac{e(e^n-1)}{e-1}$ — M1, A1 (2)

(b) $\sum_{r=1}^{n} \ln\left(\frac{r+2}{r}\right) = \sum_1^n \ln(r+2) - \ln(r)$ — M1

$= \cancel{\ln 3} - \ln 1 + \cancel{\ln 4} - \ln 2 + \cancel{\ln 5} - \cancel{\ln 3} \dots + \ln(n+1) - \cancel{\ln(n-1)} + \ln(n+2) - \cancel{\ln(n)}$ — M1 List; M1 Cancelling

$= \ln(n+2) + \ln(n+1), - \ln 2 - \ln 1$ — A1, A1 correct terms left

$= \ln\left[\dfrac{(n+2)(n+1)}{2}\right]$ — A1 (6)

(c) $\sum_{r=1}^{n} \dfrac{1}{r(r+2)} = \frac{1}{2}\sum_1^n\left[\frac{1}{r} - \frac{1}{r+2}\right]$ — M1 A1 (inc. ½)

$= \frac{1}{2}\left[\cancel{\frac{1}{n}} - \frac{1}{n+2} + \cancel{\frac{1}{n-1}} - \frac{1}{n+1} + \cancel{\frac{1}{n-2}} - \cancel{\frac{1}{n}} \dots \cancel{\frac{1}{3}} - \cancel{\frac{1}{5}} + \frac{1}{2} - \cancel{\frac{1}{4}} + \frac{1}{1} - \cancel{\frac{1}{3}}\right]$ — M1 List; M1 Cancelling

$= \frac{1}{2}\left[\frac{3}{2} - \frac{1}{n+2} - \frac{1}{n+1}\right]$ — A1 (5)

(d) $\sum_0^n \binom{n}{r}\tan^{2r}\theta$ cf $\sum_0^n \binom{n}{r} a^r b^{n-r}$ $a = \tan^2\theta,\ b=1$ — M1 A1

$\therefore \sum_0^n \binom{n}{r}\tan^{2r}\theta = (1+\tan^2\theta)^n$ $\left[\text{or } (\sec^2\theta)^n\right]$ — M1 A1

$\therefore \sum_1^n \binom{n}{r}\tan^{2r}\theta = \sec^{2n}\theta - 1$ o.e. — A1 (5)

(e) (c) is convergent, $S_\infty = \frac{3}{4}$ — B1, B1 (2)

(20)

7 (a) Let $z = x + iy$ — 2 eqns M1

$\Rightarrow x^2 - y^2 = 5$ — A1

$2xy = -12 \Rightarrow xy = -6$ or $x = -\frac{6}{y}$ — Solve → quad in x^2 or y^2 M1

$\therefore \frac{36}{y^2} - y^2 = 5 \Rightarrow y^4 + 5y^2 - 36 = 0$

$(y^2+9)(y^2-4) = 0$ — x A1

$\therefore y^2 = 4 \Rightarrow y = \pm 2$ so $z = -3+2i$ (or $3-2i$) — y A1 (5)

(b) $x^2 + (1-4i)x - (5-i) = 0$ — M1 Quadratic

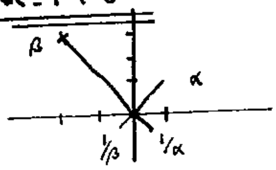
$\therefore x = \dfrac{-(1-4i) \pm \sqrt{1-16-8i+20-4i}}{2}$ — M1, Use of formula

$x = \dfrac{-(1-4i) \pm \sqrt{5-12i}}{2}$ — A1

i.e. — M1 Use of (a)

$x = (-1+4i-3+2i)/_2$ or $\frac{1}{2}(-1+4i+3-2i)$ — A1 ✓ α, β correct

$\therefore \beta = -2+3i$ and $\alpha = 1+i$ — A1, A1 (7)



$\therefore$ — α, β B1, B1

(c) $\frac{1}{\beta} = \frac{1}{13}(-2-3i)$; $\frac{1}{\alpha} = \frac{1}{2}(1-i)$ — $\frac{1}{\alpha}, \frac{1}{\beta}$ plots M1; A1, A1 (5)

(d) $\alpha^4 = -4$ — B1

$\alpha^{2001} = (\alpha^4)^{500} \times \alpha$ — M1

$= 2^{1000}(1+i)$ — A1 (3)

(20)

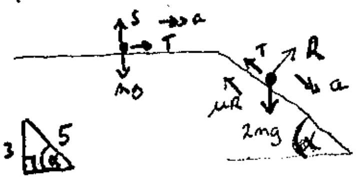
(8) (a)

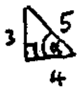
$mu = \frac{1}{4}mu + 2mv$ — M1

$\Rightarrow \quad v = \underline{\underline{\frac{3u}{8}}}$ — A1 (2)

(b) Rel. speed is $\frac{u}{8}$, $\therefore$ time $= \underline{\underline{\frac{8l}{u}}}$ — M1 A1 (2)

(c) $mu \to mw + 2mw \quad \Rightarrow \quad w = \underline{\underline{\frac{u}{3}}}$ — M1 A1 (2)





$\underline{F=ma}$ on A $\quad ma = T$ — B1

on B $\quad R = 2mg\cos\alpha$ — B1

$\searrow \; 2mg\sin\alpha - T - \mu R = 2ma$ — M1 A1

Solving $\quad 2mg(\sin\alpha - \mu\cos\alpha) = 3ma$ — M1

B will accelerate down slope if $\sin\alpha > \mu\cos\alpha$

$\mu < \tan\alpha = \frac{3}{4}$ — M1

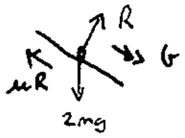
or

then acceleration $\quad \underline{\underline{a = \frac{2g}{15}(3-4\mu)}} \quad (\underline{\mu < \tfrac{3}{4}})$ — M1 A1

$\underline{\mu = \frac{3}{4}}$ and acceleration is zero so particles carry on moving with const. vel. — B1

$\underline{\mu > 3/4}$ B slows up but A keeps moving so string becomes slack — B1

Assuming A does not catch up with B then B will decelerate and stop — B1

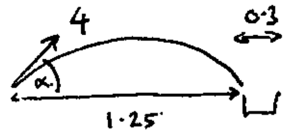


$2mg\sin\alpha - \mu R = 2mb$ — M1 A1

$\therefore \quad b = g(\sin\alpha - \mu\cos\alpha)$

$= \underline{\underline{\frac{g}{5}(3 - 4\mu)}}$ or deceleration $\underline{\underline{\frac{g}{5}(4\mu-3)}}$ — A1 (14) (20)

9. (a)



$\rightarrow\ x = 4\cos\alpha\, t$ B1

$\uparrow\ y = 0 = 4\sin\alpha t - 4.9t^2$ B1

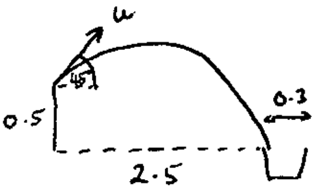
$y = 0 \Rightarrow \quad t = (0)$ or $\dfrac{4\sin\alpha}{4.9}$; $\Rightarrow \quad x = \dfrac{16\sin\alpha\cos\alpha}{4.9}$ $t =$ $x(\alpha)$ M1, M1

$\therefore \quad 1.25 < \dfrac{8\sin 2\alpha}{4.9} < 1.55$ Inequalities for $f(\alpha)$ M1 A1

$\Rightarrow \quad 0.7656\ldots < \sin 2\alpha < 0.949\ldots$ M1

$\Rightarrow \quad \underline{\underline{25.0 < \alpha < 35.8}}$; $\underline{\underline{54.2 < \alpha < 65.0}}$ A1, A1 (9)

(b)



$\rightarrow \quad x = u\cos 45\, t = \dfrac{u}{\sqrt{2}}t$ B1

$\downarrow \quad y = -0.5 = \dfrac{u}{\sqrt{2}}t - 4.9t^2$ M1 A1

sub for x: $\quad -0.5 = x - \dfrac{9.8}{u^2}x^2$ equ in x M1

ie. $9.8x^2 - u^2x - 0.5u^2 = 0 \quad \Rightarrow x = \dfrac{u^2 \pm \sqrt{u^4 + 19.6u^2}}{19.6}$ (take +) M1

$\therefore \quad 2.5 \times 19.6 < u^2 + \sqrt{u^4 + 19.6u^2} < 2.8 \times 19.6$ M1

$(2.5 \times 19.6 - u^2)^2 < u^4 + 19.6u^2$ | $u^4 + 19.6u^2 < (2.8 \times 19.6 - u^2)^2$

$\Rightarrow \quad \dfrac{2.5^2 \times 19.6^2}{6 \times 19.6} < u^2$ | $\Rightarrow \quad u^2 < \dfrac{2.8^2 \times 19.6^2}{6.6 \times 19.6}$ M1

| $\Rightarrow \quad u < 4.825\ldots$ A1

$\Rightarrow \quad 4.52\ldots < u$ A1

ie. $\quad \underline{\underline{4.5 < u < 4.8}}$ (9)

(c) Air resistance has to be considered. B1

Energy will be lost contracting air resistance so paper ~~will~~ might fall short. B1 (2)

or size of paperball

§B AEA Trial 2001 STATISTICS Final Draft | Marks

10 (a) X = no. of defectives in a sample of 10 $\therefore X \sim B(10, p)$ — B1, B1

$P(\text{Accept delivery}) = P(X \le 1) + P(X = 2) \times P(X = 0)$ — M1 A1

$= q^{10} + 10q^9p + 45q^8p^2 \times q^{10}$

$= \underline{q^9(q + 10p + 45p^2q^9)}$ — A1 cso. (5)

(b) N = no. of components sampled — B1

n:	10	20
$P(N=n)$:	$P(X \le 1)$	$P(X \ge 2)$

M1

$\therefore E(N) = 10 \times (q^{10} + 10q^9p) + 20[1 - (q^{10} + 10q^9p)]$ — M1 A1

$= \underline{20 - 10(q^{10} + 10q^9p)}$ or $\underline{20 - 10q^9(q + 10p)}$ (o.e.) — A1 (5)

(c) Y = no. of defectives in a sample of 15 $Y \sim B(15, p)$

$P(\text{Accept delivery}) = P(Y \le 2) = q^{15} + 15q^{14}p + 105q^{13}p^2$ — M1, M1 A1

$(= q^{13}(q^2 + 15pq + 105p^2))$ (3)

Probability of rejecting delivery

	Manager	Assistant
$p=0.05$	0.04145	0.0362
$p=0.10$	0.19636	0.18406

M1 A1

Costs

	MANAGER Sampling Rejection	ASSISTANT Sampling Rejection
$p=0.05$	$(20 - 9.1386) \times 5 + 1000 \times 0.04145 = \underline{£95.76}$	$15 \times 5 + 1000 \times 0.0362 = \underline{£111.20}$
$p=0.10$	$(20 - 7.361) \times 5 + 500 \times 0.19636 = \underline{£161.38}$	$15 \times 5 + 500 \times 0.18406 = \underline{£167.03}$

M1 (Sample + Rejection)
M1 Use of E(N)
A1/1 any two A1 all four A
B1 ✓ (7) 20

In general manager's scheme is best

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11 (a) X = no. of hesitations in 10 mins $X \sim P(5)$ $P(X \ge 3) = 1 - P(X \le 2) = 1 - 0.1247$ — M1

$= \underline{0.8753}$ — A1 (2)

(b) $P(X \ge 5 \mid X \ge 2) = \dfrac{P(X \ge 5)}{P(X \ge 2)}, = \dfrac{1 - P(X \le 4)}{1 - P(X \le 1)} = \dfrac{1 - 0.4405}{1 - 0.0404}, = \dfrac{0.5595}{0.9596} = \underline{0.5831}$ — M1A1, M1A1, A1 (5)

(c) Y = no. of minutes with no hesitation. $Y \sim B(10, e^{-0.5}) = B(10, 0.6065\ldots)$ — Bin M1, p B1

$P(Y \le 7) = 1 - [P(8) + P(9) + P(10)]; = 1 - [45(0.6\ldots)^8(0.39\ldots)^2 + 10(0.6\ldots)^9(0.3\ldots)^1 + (0.6\ldots)^{10}]$ — M1; M1 A1

$= 1 - [0.1276\ldots + \qquad = \underline{0.8219\ldots}$ — A1 (6)

(d) Different random variables or some of the other 3 minutes could contain more than 1 hesitation — B1 (1)

(e) Let M = no. of hesitations in 1st min. $M \sim Po(\lambda)$ — M1 A1

$0.5 = e^{-\lambda} \Rightarrow \lambda = \underline{\ln 2}$

Let N = no. of hesitations in next 9 mins $N \sim Po(3)$ — B1

Cases:

M:	0	1	2	≥ 3
N:	≥ 3	≥ 2	≥ 1	≥ 0

Cases M1

$\therefore \text{Prob} = \frac{1}{2}[1 - 0.4232] + \frac{1}{2}\ln 2[1 - 0.1991] + \frac{1}{2}\frac{(\ln 2)^2}{2!}[1 - 0.0498] + [1 - \frac{1}{2}(1 + \ln 2 + \frac{(\ln 2)^2}{2!})]$ — M1

$= 0.2884 + 0.2776 + 0.11413 + 0.0333 = \underline{0.7134}$ — A1 (6). 20

Answers

12

(a) Let the numbers of type A, B and C rugs made be x_1, x_2, x_3

Maximise $P = 5x_1 + 6x_2 + 4x_3$ (B1)

Subject to $x_1 + 4x_2 + 3x_3 + r = 900$ (M1 A2)

$3x_1 + 2x_2 + 3x_3 + s = 450$

$x_1 + 2x_2 + x_3 + t = 240$ (4)

b.v.	x_1	x_2	x_3	r	s	t	value	
r	1	4	3	1	0	0	900	(B1)
s	3	2	3	0	1	0	450	
t	1	2	1	0	0	1	240	
P	−5	−6	−4	0	0	0	0	(M1 A1)
r	−1	0	1	1	0	−2	420	$R_1 - 4R_3$
s	2	0	2	0	1	−1	200	$R_2 - 2R_3$
x_2	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	120	$R_3 \div 2$
P	−2	0	−1	0	0	3	720	$R_4 + 6R_3$
r	0	0	2	1	$\frac{1}{2}$	$-2\frac{1}{2}$	520	$R_1 + R_2$
x_1	1	0	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	100	$R_2 \div 2$
x_2	0	1	$-\frac{1}{2}$	0	$-\frac{1}{4}$	$\frac{3}{4}$	70	$R_3 - \frac{1}{2}R_2$
P	0	0	1	0	1	2	920	$R_4 + 2R_2$

(M1 A1)

(6)

Now optimal Profit = £920 (M1)

number of type A = 100, type B = 70 type C = 0. (A1)

(2)

(b) Do not increase amount of type I. r = 520 kg already. So 520 kg of type I is not being used. (B1)

(c) Add x kg to type II and $40 - x$ to type III. (B1)

Final column values in tableau 1 R_2 and R_3 are now $450 + x$ and $280 - x$

In either case $\frac{280 - x}{2} < \frac{450 + x}{2}$ so 1st pivot element unchanged.

(The columns containing the pivot elements will not change.) (M1)

In the second tableau. The 1st row will never be selected since it would give a negative result. The final column values in rows 2 and 3

Answers

8

become $160 + 2x$ and $140 - \frac{1}{2}x$ (A1)

Row 2 will be selected if

$$(160 + 2x) \div 2 < (140 - \tfrac{1}{2}x) \div \tfrac{1}{2}$$

ie $$80 + x < 280 - x$$

ie $$2x < 200$$ (A1)

$$x < 100$$

Since x is < 100 row 2 will continue to supply the 2nd (and first) pivot element. (4)

(d) In the final tableau the final column values are.

$R_1 = 420 + 3x$ $R_2 = 80 + x$ $R_3 = 100 - x$ (M1)

$R_4 = 1000 - x$

we wish to maximise profit, ie, $1000 - x$ $\therefore x = 0$ (A1)

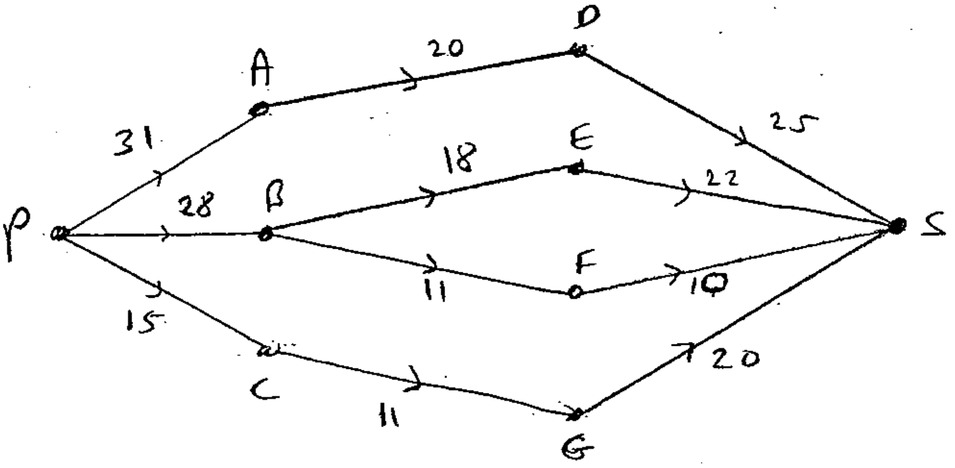
$\therefore$ profit $= £1000$

number of type A $= 100$, type B $= 80$, type C $= 0$ (B1)

(3)

Answers

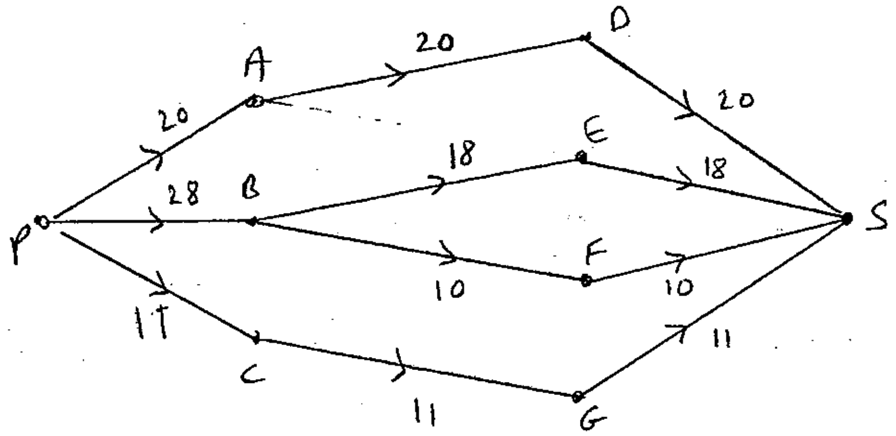
13 (a)



M1 A1 A1

(3)

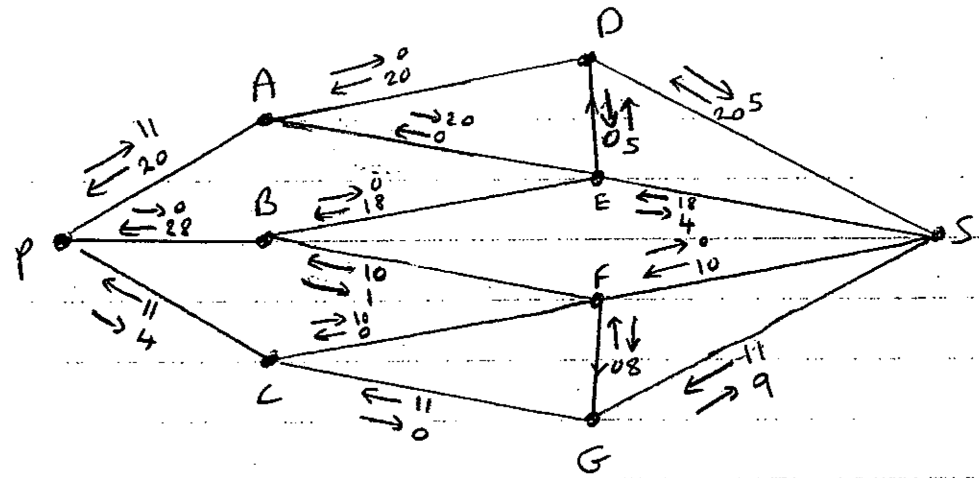
(b) Min cut AD, PB, CG M1 A1



M1 A1

(4)

(c)



M1 A1 A1

Increasing flow eg.

PAEDS – 5 M1

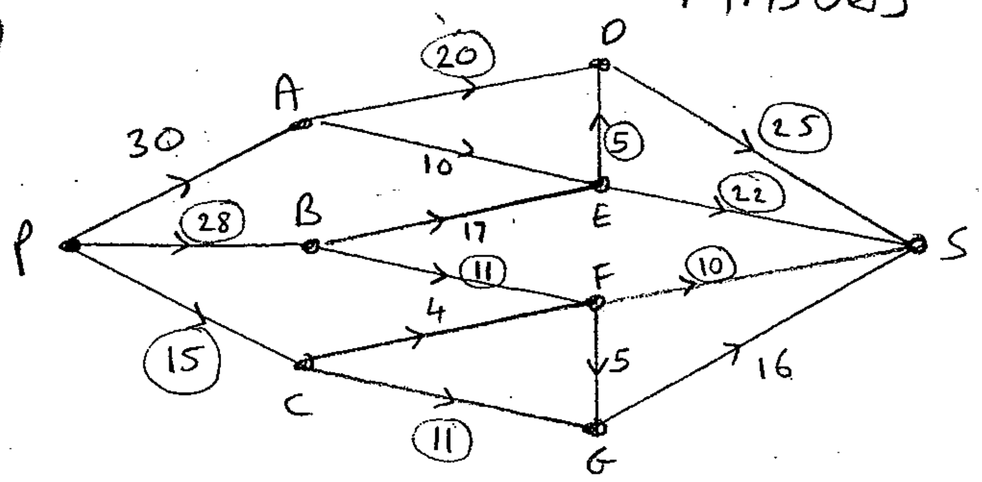
PAES – 4 A1

PCFGS – 4

rerouting PAEBFGS – 1 M1 A1

Max flow = 73 = min cut PG, BF, ES, DS M1 A1 A1

Answers



D M1

A1

A1

(13)