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British Mathematical Olympiad

Round 1 : Wednesday, 30 November 2005

Time allowed $3\frac{1}{2}$ hours.

Instructions

- *Full written solutions - not just answers - are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.*
- *One* complete *solution will gain more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all the problems.*
- *Each question carries 10 marks. However, earlier questions tend to be easier. In general you are advised to concentrate on these problems first.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Start each question on a fresh sheet of paper. Write on one side of the paper only. On each sheet of working write the number of the question in the top* left *hand corner and your name, initials and school in the top* right *hand corner.*
- *Complete the cover sheet provided and attach it to the front of your script, followed by your solutions in question number order.*
- *Staple all the pages neatly together in the top* left *hand corner.*

Do not turn over until **told to do so.**

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2005/6 British Mathematical Olympiad
Round 1

1. Let n be an integer greater than 6. Prove that if $n-1$ and $n+1$ are both prime, then $n^2(n^2+16)$ is divisible by 720. Is the converse true?

2. Adrian teaches a class of six pairs of twins. He wishes to set up teams for a quiz, but wants to avoid putting any pair of twins into the same team. Subject to this condition:

   i) In how many ways can he split them into two teams of six?

   ii) In how many ways can he split them into three teams of four?

3. In the cyclic quadrilateral $ABCD$, the diagonal AC bisects the angle DAB. The side AD is extended beyond D to a point E. Show that $CE = CA$ if and only if $DE = AB$.

4. The equilateral triangle ABC has sides of integer length N. The triangle is completely divided (by drawing lines parallel to the sides of the triangle) into equilateral triangular cells of side length 1.

   A continuous route is chosen, starting inside the cell with vertex A and always crossing from one cell to another through an edge shared by the two cells. No cell is visited more than once. Find, with proof, the greatest number of cells which can be visited.

5. Let G be a convex quadrilateral. Show that there is a point X in the plane of G with the property that every straight line through X divides G into two regions of equal area if and only if G is a parallelogram.

6. Let T be a set of 2005 coplanar points with no three collinear. Show that, for any of the 2005 points, the number of triangles it lies strictly within, whose vertices are points in T, is even.