

## Upper And Lower Bounds

(Usually I would say that you should round answers to 3 sig figs etc, but just this once I would like you to write answers showing all digits on your calculator display. Thanks.)

1. Write down the lower bound and upper bounds for the following:

(a) 110 (correct to the nearest integer).

109.5, 110.5

(b) 110 (correct to the nearest ten).

105, 115

(c) 110 (correct to the nearest five).

107.5, 112.5

(d) 123.8 (correct to 1dp).

123.75, 123.85

(e) 4500 (correct to 3 sf).

4495, 4505

2. If  $x = 3.6$  and  $y = 4.1$ , both correct to 1 decimal place, find the upper bound and lower bounds for

(a)  $x + y$ .

7.6, 7.8

(b)  $xy$ .

14.3775, 15.1475

(c)  $\frac{1}{x} + \frac{1}{y}$ .

0.514936458161, 0.528603721092

(d)  $\frac{1}{x} - \frac{1}{y}$ .

0.027059022493, 0.040726285423

3. A sprinter is running the 100 metres dash. The length of the track is 100 m (correct to the nearest metre). Her time to finish the race is 12.1 seconds (correct to 1 dp). Find the upper bound for her average speed.

8.320248962656

4. A bowl is made out a solid hemisphere of radius 6 cm (correct to nearest cm) with another hemisphere of radius 5.1 cm (correct to 1 dp) carved out. If the density of the material is  $2 \text{ g/cm}^3$ , find the upper bound for the mass of the bowl in grams.

610.8821678918

5. A rectangular swimming pool is 25 metres by 10 metres (both correct to the nearest metre). A path is constructed around the edge of the pool of width 1 metre (correct to nearest metre). [i.e. rectangle around a rectangle.]. Find the lower bound for the area of the path.

6. A solid cone has radius 12.5 cm (correct to 3 sf) and perpendicular height 20 cm (correct to nearest cm). Find the lower bound for its total surface area.

7. A solid metal cylinder has height 2 metres and radius 1 metre (both correct to the nearest metre). It is melted down and cast into 10 spheres. Find the upper bound for the radius of each individual sphere.

$r_{\max} = \frac{3}{4}$

8. If  $y = 2^x$  and  $x = 2.7$  (correct to 1dp), find the upper bound for  $y$ .

6.72717132203

9. If  $y = \frac{1}{x+1}$  and  $x = 7$  (correct to nearest integer), find the upper bound for  $y$ .

10. If  $y = \frac{1}{x+1}$  and  $y = 7$  (correct to nearest integer), find the upper bound for  $x$ .

11. If  $y = 3^{-x}$  and  $x = 0.8$  (correct to 1dp), find the lower bound for  $y$ .

2.544210651641

12. If  $y = 0.6^x$  and  $x = 1.1$  (correct to 1dp), find the upper bound for  $y$ .

0.584869282782

13. If  $y = x^2 - 2x + 1$  and  $x = 1$  (correct to the nearest integer), find the lower bound for  $y$ .  0 (exactly)

14. If  $y = 2 \sin x$  and  $x = 39$  (correct to nearest integer), find the upper bound for  $y$ .  1.272156440556

15. If  $y = 3 \cos x$  and  $x = 39$  (correct to nearest integer), find the upper bound for  $y$ .  2.347824470557

16. If  $y = \tan x$  and  $x = 100$  (correct to nearest 25 degrees), find the upper bound for  $y$ .

Unbounded (plus infinity in dreadful terms).