

Simultaneous Equations

Patrons are reminded to try to use the substitution method where possible. Look for an x or y to isolate from one equation. Then substitute this into the *other* equation. For example solve

$$\begin{aligned} 3x + y &= 7 \\ 2x + 5y &= 2 \end{aligned}$$

From the first we can see that $y = 7 - 3x$. Substituting this into the second we find $2x + 5(7 - 3x) = 2$ which solves to $x = \frac{33}{13}$. We then place this value into $y = 7 - 3x$ to discover $y = -\frac{8}{13}$. So $(x, y) = (\frac{33}{13}, -\frac{8}{13})$

Present answers in the form $(x, y) = (-1, \frac{1}{2})$. (Don't forget the brackets!)

1.	$x + y = 3$ $x - y = 2$	$(x, y) = (\frac{5}{2}, \frac{1}{2})$	12.	$4x - y = 2$ $3x + 4y = 1$	$(x, y) = (\frac{9}{19}, -\frac{2}{19})$
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2.	$x - y = 4$ $x + y = 7$	$(x, y) = (\frac{11}{2}, \frac{3}{2})$	13.	$5x + 4y = 1$ $x - 3y = 0$	$(x, y) = (\frac{3}{19}, \frac{1}{19})$
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3.	$2x + y = 3$ $x - 2y = 2$	$(x, y) = (\frac{8}{5}, -\frac{1}{5})$	14.	$x + y = -7$ $2x + 3y = 4$	$(x, y) = (-25, 18)$
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4.	$x + 2y = 4$ $3x - 2y = -7$	$(x, y) = (-\frac{3}{4}, \frac{19}{8})$	15.	$2x + y = 2$ $x - 6y = 1$	$(x, y) = (1, 0)$
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5.	$5x - y = 4$ $4x - 5y = 0$	$(x, y) = (\frac{20}{21}, \frac{16}{21})$	16.	$3x + 2y = 1$ $y - 2x = -3$	$(x, y) = (1, -1)$
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6.	$2x - 3y = 5$ $3x + 2y = 2$	$(x, y) = (\frac{16}{13}, -\frac{11}{13})$	17.	$a - b = 2$ $3a - 2b = -4$	$(a, b) = (-8, -10)$
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7.	$4x - 2y = -9$ $3x + 5y = 3$	$(x, y) = (-\frac{3}{2}, \frac{3}{2})$	18.	$4x + 7y = 10$ $3x - y = -2$	$(x, y) = (-\frac{4}{25}, \frac{38}{25})$
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8.	$x - 2y = 2$ $3x + 2y = 1$	$(x, y) = (\frac{3}{4}, -\frac{5}{8})$	19.	$2x + 3y = 1$ $3x - 4y = 2$	$(x, y) = (\frac{10}{17}, -\frac{1}{17})$
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9.	$3x - y = 3$ $2x + 3y = -1$	$(x, y) = (\frac{8}{11}, -\frac{9}{11})$	20.	$5x + y = 7$ $4x - \frac{1}{2}y = 2$	$(x, y) = (\frac{11}{13}, \frac{36}{13})$
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10.	$y - 2x = 5$ $5x - 7y = 2$	$(x, y) = (-\frac{37}{9}, -\frac{29}{9})$	21.	$x + 5y = 0$ $3x + 4y = -1$	$(x, y) = (-\frac{5}{11}, \frac{1}{11})$
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11.	$x + 3y = 4$ $5x - 2y = 6$	$(x, y) = (\frac{26}{17}, \frac{14}{17})$	22.	$\frac{x + y}{2} - \frac{x - y}{3} = 1$ $x - 2y = 2$	$(x, y) = (\frac{22}{7}, \frac{4}{7})$
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$$23. \frac{x+2y}{3} - \frac{x-3y}{7} = x \cdot \quad (x, y) = \left(\frac{23}{37}, \frac{17}{37}\right)$$

$$x+3y=2$$

$$24. \begin{cases} x+ay=0 \\ 2x+3y=-1 \end{cases} \cdot \quad (x, y) = \left(\frac{-a}{3-2a}, \frac{1}{2a-3}\right)$$

$$25. \begin{cases} kx+y=4 \\ 2x-3y=2 \end{cases} \cdot \quad (x, y) = \left(\frac{14}{2+3k}, \frac{8-2k}{2+3k}\right)$$

$$26. \begin{cases} ax+4y=6 \\ bx-y=5 \end{cases} \cdot \quad (x, y) = \left(\frac{26}{a+4b}, \frac{6b-5a}{a+4b}\right)$$

$$27. \begin{cases} kx+y=1 \\ 5x-ky=m \end{cases} \cdot \quad (x, y) = \left(\frac{m+k}{5+k^2}, \frac{5-km}{5+k^2}\right)$$

$$28. \begin{cases} x+y=1 \\ ax+by=1 \end{cases} \cdot \quad (x, y) = \left(\frac{1-b}{a-b}, \frac{a-1}{a-b}\right)$$

$$29. \begin{cases} x+ay=3 \\ ax+by=4 \end{cases} \cdot \quad (x, y) = \left(\frac{3b-4a}{b-a^2}, \frac{4-3a}{b-a^2}\right)$$

$$30. \begin{cases} 4x+y=3 \\ ax+by=c \end{cases} \cdot \quad (x, y) = \left(\frac{c-3b}{a-4b}, \frac{3a-4c}{a-4b}\right)$$

$$31. \frac{x+by}{2} - \frac{x-y}{3} = 1 \cdot \quad (x, y) = \left(\frac{6a+4-6b}{3a+2-b}, \frac{4}{3a+2-b}\right)$$

$$32. \frac{ax+1}{2} + \frac{by+2}{3} = 1 \cdot \quad (x, y) = \left(\frac{3a+2b}{20b-9a^2}, \frac{3a+10}{9a^2-20b}\right)$$

$$\frac{5x+1}{3} + \frac{ay+1}{2} = 1$$

$$33. \begin{cases} 23x+21y=1 \\ 21x+23y=-1 \end{cases} \cdot \quad (x, y) = \left(\frac{1}{2}, -\frac{1}{2}\right)$$

Now solve the following simultaneous equations in three unknowns.

1.

$$\begin{aligned}x + y + z &= 1 \\2x + 3y + z &= 6 \\x - y + 2z &= -5\end{aligned}$$

$$x = 1, y = 2, z = -2$$

2.

$$\begin{aligned}x + y + z &= 1 \\x - y + 2z &= 2 \\2x + 3y + 3z &= 3\end{aligned}$$

$$x = 0, y = 0, z = 1$$

3.

$$\begin{aligned}2a + b + 3c &= -7 \\a - b + 2c &= -4 \\3a + 2b - c &= 11\end{aligned}$$

$$a = 3, b = -1, c = -4$$

4.

$$\begin{aligned}2x + 3y - z &= 2 \\4x - y + 2z &= 5 \\2x + y - 3z &= -4\end{aligned}$$

$$x = \frac{1}{2}, y = 1, z = 2$$

5.

$$\begin{aligned}x - y + z &= 1 \\2x + 2y + 3z &= 1 \\x - y - 4z &= 2\end{aligned}$$

$$x = 1, y = -\frac{1}{5}, z = -\frac{1}{5}$$

6.

$$\begin{aligned}a + 2b + c &= 3 \\2a + b + c &= 1 \\a - b + 2c &= 0\end{aligned}$$

$$a = -\frac{2}{3}, b = \frac{4}{3}, c = 1$$

7.

$$\begin{aligned}p + q + r &= -1 \\2p + q + 2r &= -1 \\p + 3q &= 1\end{aligned}$$

$$p = 4, q = -1, r = -4$$

8.

$$\begin{aligned}x + y - z &= 1 \\x - 2y + 3z &= 0 \\x - y + 2z &= -1\end{aligned}$$

$$x = 2, y = -5, z = -4$$

9.

$$\begin{aligned}4x - 5y + 2z &= -2 \\5x + 7y + 3z &= 3 \\2x + 3y + z &= 1\end{aligned}$$

$$x = -\frac{8}{11}, y = \frac{4}{11}, z = \frac{15}{11}$$