

Prime Factorisation, HCFs And LCMs

HCF = Highest Common Factor.

LCM = Lowest Common Multiple.

Patrons are reminded that it is *always* preferable to factories a number into its prime factor form.

For example

$3960 \Rightarrow \text{BAD}$,

$2^3 \times 3^2 \times 5 \times 11 \Rightarrow \text{GOOD}$.

1. Express the following numbers as a product of primes

(a) 60.

$$2^2 \times 3 \times 5$$

(b) 81.

$$3^4$$

(c) 53.

$$53$$

(d) 102.

$$2 \times 3 \times 17$$

(e) 72.

$$2^3 \times 3^2$$

(f) 1 485.

$$3^3 \times 5 \times 11$$

(g) 605.

$$5 \times 11^2$$

(h) 11 375.

$$5^3 \times 7 \times 13$$

(i) 230.

$$2 \times 5 \times 23$$

(j) 8 856.

$$2^3 \times 3^3 \times 41$$

(k) 91 500.

$$2^2 \times 3 \times 5^3 \times 61$$

(l) 59 290.

$$2 \times 5 \times 7^2 \times 11^2$$

2. Find the smallest integer that 11340 needs to be multiplied by to make a perfect square.

3. Find the smallest integer that 38808 needs to be multiplied by to make a perfect cube.

4. Find the HCF and LCM of 25 and 30.

$$\text{HCF} = 5, \text{LCM} = 2 \times 3 \times 5^2 = 150$$

5. Find the HCF and LCM of 100 and 110.

$$\text{HCF} = 2 \times 5 = 10, \text{LCM} = 2^2 \times 5^2 \times 11 = 1100$$

6. Find the HCF and LCM of 72 and 90.

$$\text{HCF} = 2 \times 3^2 = 18, \text{LCM} = 2^3 \times 3^2 \times 5 = 360$$

7. Find the HCF and LCM of $2^3 \times 3 \times 7$ and $3^2 \times 5 \times 7$.

$$\text{HCF} = 3 \times 7 = 21, \text{LCM} = 2^3 \times 3^2 \times 5 \times 7 = 2520$$

8. Find the HCF and LCM of $2^2 \times 5^3 \times 11^2$ and $3 \times 5^4 \times 7^2$.

$$\text{HCF} = 5^3 = 125, \text{LCM} = 2^2 \times 3 \times 5^4 \times 7^2 \times 11^2 = 44467500$$

9. (Thanks to JM for this gem.) The LCM of two numbers is 84. If one of the numbers is 28, find all possible values of the other number.

$$3, 6, 12, 21, 42, 84$$

10. (Thanks, again, to JM for this gem.) Find the smallest integer greater than 1 with the property that division by each of 3, 4 and 5 yields a remainder of 1.

11. (Maclaurin) How many positive integers leave a remainder of 31 when divided into 2011?