New GCSE Topics

In order to cater for the new grade 9, some new topics were added to the Edexcel IGCSE syllabus. The following is a selection of questions on these topics.

Completing The Square

1. Complete the square on $x^2 + 6x - 1$. $(x+3)^2 - 10$
2. Complete the square on $x^2 - 10x + 3$. $(x-5)^2 - 22$
3. Complete the square on $x^2 + 5x$. $(x+\frac{5}{2})^2 - \frac{25}{4}$
4. Complete the square on $x^2 + 2ax - 1$. $(x+a)^2 - a^2 - 1$
5. Complete the square on $2x^2 + 16x + 3$. $2(x+4)^2 - 29$
6. Complete the square on $3x^2 + 6x - 13$. $3(x+1)^2 - 16$
7. Complete the square on $-x^2 + 8x - 3$. $-(x-4)^2 + 13$
8. By completing the square, solve the equation $x^2 + 4x - 1 = 0$. $x = -2 \pm \sqrt{5}$
9. By completing the square, solve the equation $y^2 - y - 5 = 0$. $\frac{1 \pm \sqrt{21}}{2}$
10. By completing the square, find the vertex (i.e. the maximum or minimum) of the curve $y = x^2 - 8x + 5$. $(4, -11)$
11. By completing the square, find the vertical line of symmetry of the curve $y = x^2 - 2x + 1$. $x = 1$

Arithmetic Sequences

1. The nth term of a sequence is given by $3n + 5$. Write down the first five terms. 8, 11, 14, 17, 20
2. The nth term of a sequence is given by $n^2 - n$. Write down the first five terms. 0, 2, 6, 12, 20
3. Find an expression for the nth term of the following:
   (a) $4, 7, 10, 13, 16, \ldots$. $3n + 1$
   (b) $100, 95, 90, 85, 80, \ldots$. $-5n + 105$
   (c) $-11, -4, 3, 10, 17, \ldots$. $3n - 18$
4. Find the sum of $3 + 5 + 7 + 9 + 11 + \ldots$ (200 terms). 40400
5. Find the sum of $10 + 15 + 20 + 25 + 30 + \ldots$ (1000 terms). 2507500
6. Find the sum of $5 + 7 + 9 + 11 + 13 + \cdots + 553$. (Hint: find how many terms there are.) 76725

Perpendicular Lines

1. Find the equation of the line perpendicular to $y = \frac{1}{3}x - 1$ which passes through $(3, -2)$. Give your answer in the form $y = mx + c$. $y = -3x + 7$
2. Find the equation of the line perpendicular to $y = 2x + 3$ which passes through $(-1, 4)$. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers. $x + 2y - 7 = 0$

Surds

1. Simplify $\sqrt{8}$. $2\sqrt{2}$
2. Simplify $\sqrt{45}$. $3\sqrt{5}$
3. Simplify $\sqrt{200}$. $10\sqrt{2}$
4. Simplify $\dfrac{4}{\sqrt{2}}$. $2\sqrt{2}$
5. Simplify $\dfrac{100}{\sqrt{5}}$. $20\sqrt{5}$
6. Simplify $\dfrac{9}{\sqrt{27}}$. $\sqrt{3}$
7. Simplify $\dfrac{1}{1+\sqrt{2}}$. $\sqrt{2}-1$
8. Simplify $\dfrac{1-\sqrt{3}}{\sqrt{3}+1}$. $\sqrt{3}-2$

Expanding Brackets

1. Expand and simplify $x(x-2)(x+5)$. x^3+3x^2-10x
2. Expand and simplify $(y-4)(y-1)(y+2)$. y^3-3y^2-6y+8
3. Expand and simplify $(x-1)^3$. x^3-3x^2+3x-1
4. Expand and simplify $(2x+1)^3$. $8x^3+12x^2+6x+1$
5. Expand and simplify $(1-x^2)^3$. $1-3x^2+3x^4-x^6$
6. Expand and simplify $(x+1)^3-x(2x+3)(x-1)$. $-x^3+2x^2+6x+1$

Proof

1. Prove that the sum of five consecutive positive integers is always divisible by 5.

   Boils down to $5n+10=5(n+2)$. So a multiple of 5.

2. Prove (for positive integer n) that $(n+1)^2-2n-1$ always represents a square number.

   Boils down to n^2. So a square.

3. Prove (for positive integer n) that $(n-1)^2+4n+1$ always represents a square number.

   Boils down to $(n+1)^2$. So a square.

4. Prove (for positive integer n) that $3n^2-3n+1+(n-1)^3$ always represents a cube number.

   Boils down to n^3. So a cube.

5. Prove that the angle subtended in a semicircle is always 90°.

   Draw radii. Two isosceles triangles. Then consider angle sum of overall triangle.

Trig Graphs & Graphical Transformations

1. Sketch the following graphs:

   (a) $y = \sin x$. — Sine wave

   (b) $y = \cos x$. — Cosine wave

   (c) $y = \tan x$. — Tan 'wave'

   (d) $y = 2\cos x$. — Cos wave. Up to 2, down to -2

   (e) $y = \sin(x + 90)$. — Cos wave

   (f) $y = \cos x + 1$. — Cos wave up one unit

   (g) $y = \sin(2x)$. — Sine wave squished so wavelength 180 instead of 360

   (h) $y = 5\sin 3x$. — Sine wave up to 5 down to -5, wavelength 120

   (i) $y = \tan(x - 90)$. — Tan wave right 90, so asymptote on y-axis

2. By sketching the graph of $y = \sin x$ and $y = \frac{1}{2}$, find all solutions to $\sin x = \frac{1}{2}$ in the range $0 < x < 720$. — $x = 30, 150, 390, 510$