Harder Problems

1. (a) Solve the simultaneous equations equations $\begin{array}{r} y = 2x^2 + x + 1 \\ y + 17 = 13x \end{array}$. $\boxed{(3, 22)}$

   (b) What does your answer say about the geometric relationship between the two in the xy–plane? $\boxed{\text{Tangent}}$

2. Solve $\frac{5}{x-1} + \frac{9}{x+1} = 8$. $\boxed{x = 2 \text{ or } x = -\frac{1}{4}}$

3. Make x the subject of $\frac{ax-b}{cy+dx} = e$. $\boxed{x = \frac{cey+b}{a-de}}$

4. Find the range of the function $f(x) = 7 + 4x - x^2$ if the domain is all the real numbers. [Hint: Sketch] $\boxed{f(x) \leqslant 11}$

5. Given $f(x) = \frac{ax}{b-x}$ find

   (a) $f^{-1}(x)$. $\boxed{f^{-1}(x) = \frac{bx}{a+x}}$

   (b) $ff(x)$. $\boxed{ff(x) = \frac{a^2x}{b^2-bx-ax}}$

   (c) $fff(x)$. $\boxed{fff(x) = \frac{a^3x}{b^3-b^2x-a^2x-abx}}$

6. Given $g(x) = \frac{3x-a}{7-kx}$ find

   (a) $g^{-1}(x)$. $\boxed{g^{-1}(x) = \frac{7x+a}{3+kx}}$

   (b) $gg(x)$. $\boxed{gg(x) = \frac{9x-10a+akx}{49-10kx+ak}}$

   (c) $ggg(x)$. $\boxed{\frac{27x-79a+13akx-a^2k}{343-79kx+17ak-ak^2x}}$

7. The normal to $y = 3x^2 - x + 2$ has gradient 4. Find where on the curve the normal exists. $\boxed{(\frac{1}{8}, \frac{123}{64})}$

8. Find the equation of the tangent to $y = x^2 + 2x - 1$ when $x = p$ in the form $ax + by + c = 0$. $\boxed{(2p+2)x + y + 1 + 2p^2 = 0}$

9. Find the equation of the normal to $y = \frac{1}{2x^2}$ when $x = k$ in the form $ax + by + c = 0$. $\boxed{k^3x - 2k^2y + 1 - k^4}$

10. The tangent to $y = x^2$ when $x = p$ crosses the x–axis at P and the y–axis at Q. If the origin is O, find the area of triangle OPQ. $\boxed{\frac{p^3}{4}}$

11. Differentiate the following with respect to x:

    (a) $(x+2)(x-1)(x+3)$. $\boxed{3x^2 + 8x + 1}$

    (b) $(x+a)(x-a)(x+b)$. $\boxed{3x^2 + 2bx - a^2}$

    (c) $3\sqrt{x} - \frac{1}{2x}$. $\boxed{\frac{3}{2\sqrt{x}} + \frac{1}{2x^2}}$

    (d) $\frac{3x^3-7x^2+2x-5}{x^2}$. $\boxed{3 - \frac{2}{x^2} + \frac{10}{x^3}}$

    (e) $\frac{(\sqrt{x}+x^2)(x-3)}{\sqrt{x}}$. $\boxed{1 + \frac{5}{2}x^{\frac{3}{2}} - \frac{9}{2}\sqrt{x}}$

12. Find the point of intersection of the tangent to $y = x^3 - 3x + 1$ when $x = 1$ and the normal when $x = -1$. $\boxed{(-1, -1)}$

13. Find the point of intersection of the tangent to $y = (2x-1)(x+3)$ when $x = 1$ and the normal when $x = 2$. $\boxed{(\frac{131}{59}, \frac{884}{59})}$