Gradient, Mid-Point & Length

Gradient between (x_1, y_1) and (x_2, y_2) is

$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Also the length between (x_1, y_1) and (x_2, y_2) is (by Pythagoras)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(\text{change in } x)^2 + (\text{change in } y)^2}.$$

Also the midpoint between (x_1, y_1) and (x_2, y_2) is

$$\text{mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\text{average of the } x\text{'s, average of the } y\text{'s}\right)$$

Gradient

1. Find the gradient between the points $(\frac{1}{2}, -2)$ and $(4, -1)$. $\boxed{\frac{2}{7}}$
2. Find the gradient between the points $(-\frac{1}{2}, \frac{2}{3})$ and $(1, 2)$. $\boxed{\frac{8}{9}}$
3. The gradient between the points $(-2, p)$ and $(3, 1)$ is 2. Find p. $\boxed{p = -9}$
4. The gradient between the points $(-1, 2)$ and $(p, -1)$ is $-\frac{1}{2}$. Find p. $\boxed{p = 5}$

Mid-Point

1. Find the mid-point between the points $(3, 6)$ and $(-1, 10)$. $\boxed{(1, 8)}$
2. Find the mid-point between the points $(-1, \frac{3}{2})$ and $(4, 2)$. $\boxed{(\frac{3}{2}, \frac{7}{4})}$
3. The mid-point between the points $(3, q)$ and $(p, 11)$ is $(7, -1)$. Find p & q. $\boxed{p = 11, q = -13}$
4. The mid-point between the points $(-1, q)$ and $(p, \frac{2}{3})$ is $(2, -1)$. Find p & q. $\boxed{p = 5, q = -\frac{8}{3}}$

Length

1. Find the distance between the points $(1, 3)$ and $(6, -9)$. $\boxed{13}$
2. Find the distance between the points $(-2, 30)$ and $(4, 22)$. $\boxed{10}$
3. The distance between the points $(4, 5)$ and $(7, p)$ is 5. Find the possible values of p.
   $\boxed{p = 1 \text{ or } p = 9}$
4. The distance between the points $(2, 1)$ and $(1, p)$ is $\sqrt{5}$. Find the possible values of p.
   $\boxed{p = -1 \text{ or } p = 3}$
5. The distance between the points $(-1, 3)$ and $(p, 2)$ is $\sqrt{10}$. Find the possible values of p.
   $\boxed{p = -4 \text{ or } p = 2}$
6. The distance between the points $(7, 5)$ and $(p + 1, 7)$ is $\sqrt{8}$. Find the possible values of p.
   $\boxed{p = 4 \text{ or } p = 8}$
7. The distance between the points $(2p, 7)$ and $(3, 8)$ is $\sqrt{2}$. Find the possible values of p.
   $\boxed{p = 1 \text{ or } p = 2}$

8. The distance between the points $(5, -2)$ and $(2p + 1, 0)$ is $\sqrt{5}$. Find the possible values of p.

$p = \frac{5}{2}$ or $p = \frac{3}{2}$

9. The distance between the points $(p, 1)$ and $(2, p + 1)$ is $\sqrt{2}$. Find the possible values of p.

$p = 1$ (repeated)

10. The distance between the points $(p, 3)$ and $(5, p + 5)$ is 5. Find the possible values of p.

$p = 1$ or $p = 2$

11. The distance between the points $(2p, 1)$ and $(0, p - \frac{1}{2})$ is $\sqrt{2}$. Find the possible values of p.

$p = \frac{1}{10}$ or $p = \frac{1}{2}$