Functions

1. If f$(x) = 2x + 3$ find (fully simplified):

(a) $f(1)$, $\boxed{5}$

(b) $f(\frac{1}{2})$, $\boxed{4}$

(c) $f(x+1)$, $\boxed{2x+5}$

(d) $f(1-3x)$. $\boxed{5-6x}$

2. If $g(x) = 2x^2 - x + 1$ find (fully simplified):

(a) $g(5)$, $\boxed{46}$

(b) $g(-2)$, $\boxed{11}$

(c) $g(\frac{2}{3})$, $\boxed{\frac{11}{9}}$

(d) $g(x+1)$, $\boxed{2x^2+3x+2}$

(e) $g(x^2-1)$, $\boxed{2x^4-5x^2+4}$

(f) $g(\sqrt{x})$. $\boxed{2x-\sqrt{x}+1}$

3. If $h(x) = \frac{2}{3+x}$ find (fully simplified):

(a) $h(7)$, $\boxed{\frac{1}{5}}$

(b) $h(-\frac{1}{2})$, $\boxed{\frac{4}{5}}$

(c) $h(x-3)$, $\boxed{\frac{2}{x}}$

(d) $h(\frac{1}{x})$, $\boxed{\frac{2x}{3x+1}}$

(e) $h(\frac{7}{x-8})$. $\boxed{\frac{2x-16}{3x-17}}$

(f) $h(\frac{ax}{4-bx})$. $\boxed{\frac{8-2bx}{12-3bx+ax}}$

4. If $l(x) = \frac{1+2x}{2-3x}$ find (fully simplified):

(a) $l(-3)$, $\boxed{-\frac{5}{11}}$

(b) $l(\frac{1}{4})$, $\boxed{\frac{6}{5}}$

(c) $l(x+1)$, $\boxed{-\frac{2x+3}{3x+1}}$

(d) $l(\frac{1}{x})$, $\boxed{\frac{x+2}{2x-3}}$

(e) $l(\frac{x}{x+1})$, $\boxed{\frac{3x+1}{2-x}}$

(f) $l(\frac{x-1}{x-2})$. $\boxed{\frac{4-3x}{1+x}}$

5. If $f(x) = 2x - 3$, $g(x) = x^2 + x$ and $h(x) = \frac{1}{x+1}$ solve the following:

(a) $f(x) = 3$, $\boxed{x=3}$

(b) $f(x+3) = 8$, $\boxed{x=\frac{5}{2}}$

(c) $fff(x) = 2x - 1$, $\boxed{x=\frac{10}{3}}$

(d) $hf(x) = 7$, $\boxed{x=\frac{15}{14}}$

(e) $gf(x) = 0$, $\boxed{x=1 \text{ or } x=\frac{3}{2}}$

(f) $fh(x) = -1$, $\boxed{x=0}$

(g) $fg(x) = -3$, $\boxed{x=0 \text{ or } x=-1}$

6. If $f(x) = x + 1$, $g(x) = x^2$ and $h(x) = \frac{1}{x}$ solve the following:

(a) $f(x) = 2$, $\boxed{x=1}$

(b) $f(x+3) = -\frac{1}{2}$, $\boxed{x=-\frac{9}{2}}$

(c) $ff(x) = 2x - 1$, $\boxed{x=3}$

(d) $fff(x) = 2x + 6$, $\boxed{x=-3}$

(e) $hf(x) = x$, $\boxed{x=\frac{-1\pm\sqrt{5}}{2}}$

(f) $gf(x) = 1$, $\boxed{x=0 \text{ or } x=-2}$

(g) $fh(x) = 0$, $\boxed{x=-1}$

(h) $fg(x) = 2$, $\boxed{x=1 \text{ or } x=-1}$

7. If $f(x) = 2x + 1$, $g(x) = x^2 + x$ and $h(x) = \frac{1}{x}$, find fully simplified expressions for:

(a) $fg(x)$. $\boxed{2x^2+2x+1}$

(b) $gf(x)$. $\boxed{4x^2+6x+2}$

(c) $hf(x)$. $\boxed{\frac{1}{2x+1}}$

(d) $hfg(x)$. $\boxed{\frac{1}{2x^2+2x+1}}$

(e) $ff(x)$. $\boxed{4x+3}$

(f) $fff(x)$. $\boxed{8x+7}$

(g) $gg(x)$. $\boxed{x^4+2x^3+2x^2+x}$

(h) $hh(x)$. $\boxed{x}$

8. Find the natural domain of the following functions:

(a) $f(x) = -x + 7$. $\boxed{x \epsilon \mathbb{R}}$

(b) $f(x) = x^2 - 2x + 1$. $\boxed{x \epsilon \mathbb{R}}$

(c) $f(x) = \frac{1}{x}$. $\boxed{x \neq 0}$

(d) $f(x) = \sqrt{x - 6}$. $\boxed{x \geqslant 6}$

(e) $f(x) = \sqrt{2x + 1}$. $\boxed{x \geqslant -\frac{1}{2}}$

(f) $f(x) = \sqrt{1 - 4x}$. $\boxed{x \leqslant \frac{1}{4}}$

(g) $f(x) = \frac{x+1}{x-1}$. $\boxed{x \neq 1}$

(h) $f(x) = \frac{ax+b}{cx+d}$. $\boxed{x \neq -\frac{d}{c}}$

(i) $f(x) = \frac{\sqrt{7-3x}}{x+3}$. $\boxed{x \leqslant \frac{7}{3} \text{ and } x \neq -3}$

9. Find the inverses of the following functions:

(a) $f(x) = 2x - 1$. $\boxed{f^{-1}(x) = \frac{x+1}{2}}$

(b) $f(x) = \frac{1}{x}$. $\boxed{f^{-1}(x) = \frac{1}{x}}$

(c) $f(x) = ax + b$. $\boxed{f^{-1}(x) = \frac{x-b}{a}}$

(d) $f(x) = \sqrt{2 - x}$. $\boxed{f^{-1}(x) = 2 - x^2}$

(e) $f(x) = \frac{1}{1+x}$. $\boxed{f^{-1}(x) = \frac{1}{x} - 1 = \frac{1-x}{x}}$

(f) $f(x) = \frac{1+x}{1-x}$. $\boxed{f^{-1}(x) = \frac{x-1}{x+1}}$

(g) $f(x) = \frac{2-3x}{1+5x}$. $\boxed{f^{-1}(x) = \frac{2-x}{3+5x}}$

(h) $f(x) = 1 + \frac{3x}{1-x}$. $\boxed{f^{-1}(x) = \frac{x-1}{x+2}}$

(i) $f(x) = 5 - \frac{3}{x-2}$. $\boxed{f^{-1}(x) = \frac{2x-13}{x-5} = 2 + \frac{3}{5-x}}$

(j) $f(x) = a + \frac{1+x}{1-ax}$. $\boxed{f^{-1}(x) = \frac{x-a-1}{1+ax-a^2}}$

(k) $f(x) = \frac{ax+b}{cx+d}$. $\boxed{f^{-1}(x) = \frac{b-dx}{cx-a} = \frac{dx-b}{a-cx}}$

(l) $f(x) = \sqrt{\frac{ax+x+c}{x-c}}$. $\boxed{f^{-1}(x) = \frac{c(1+x^2)}{x^2-a-1}}$

10. The following functions are self-inverse (i.e. $f(x) \equiv f^{-1}(x)$ and $ff(x) = x$). Find conditions on the constants for this to be true:

(a) $f(x) = x + a$. $\boxed{a = 0}$

(b) $f(x) = \frac{x+a}{x-3}$. □