# Single Pure – Implicit Functions

Patrons are reminded not to smoke in lessons. They are also reminded that the theory behind implicit functions is the chain rule. If you need to differentiate $f(y)$ with respect to $x$, then you *don't* obtain $f'(y)$. However with the chain rule we find

$$\frac{d}{dx}(f(y)) = \frac{d}{dy}(f(y)) \times \frac{dy}{dx} = f'(y)\frac{dy}{dx}$$

by sneaking in a couple of helpful '$dy$s'. So you differentiate the $f(y)$ as you would expect, but then multiply by a $\frac{dy}{dx}$ straight away.

1. Find (fully simplified) expressions for $\frac{dy}{dx}$ for the following implicitly defined functions:
   (a) $x^2 + y^2 = r^2$.
   (b) $x^3 + e^y = 1$.
   (c) $\sin 2x + \cos 3y = xy$.
   (d) $e^{\sin x} - 2x^2y^3 = 1$.
   (e) $x^n + y^n = xy^2$.
2. Find the equations of the tangents or normals to the following at the required points of the following implicitly defined functions:
   (a) Tangent at $(3, -4)$ on $x^2 + y^2 = 5$.
3. Find the stationary points of the following implicitly defined functions:
   (a) $x^2 + 3xy + y^2 = 4$.
   (b) $x^2 - 6xy - y^2 = 10$.
4. Find the gradient of the curve $4x^2 + 2xy + y^2 = 12$ at the point $(1, 2)$.
5. The equation of a curve is $x^2 + 3xy + 4y^2 = 58$. Find the equation of the normal at the point $(2, 3)$ on the curve, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.
6. Find the equation of the normal to the curve $x^3 + 4x^2y + y^3 = 6$ at the point $(1, 1)$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.
7. Find the equation of the normal to the curve $x^3 + 2x^2y = y^3 + 15$ at the point $(2, 1)$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.