

Single Mechanics - Vector Motion 1

Patrons are reminded that for bodies travelling at *constant velocity*, $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$, where \mathbf{r}_0 is the position of the body at the 'start' (i.e. when one's clock is started).

- At noon Sachin starts at position $(3\mathbf{i} + 4\mathbf{j})$. He travels at a constant velocity of $(-\mathbf{i} + 2\mathbf{j}) \text{ ms}^{-1}$. Harry starts at a position of $(-10\mathbf{i} + 3\mathbf{j})$. He travels at a constant velocity of $(\mathbf{i} + \mathbf{j}) \text{ ms}^{-1}$.
 - What is Sachin's speed?
 - On what bearing is Harry travelling?
 - Find an expression for Sachin's position after t seconds.
 - Do Sachin and Harry meet? If they do meet, give the time and position that they meet.
- At noon Frasier starts at position $(-\mathbf{i} + 3\mathbf{j})$. He travels at a constant velocity of $(2\mathbf{i} - 4\mathbf{j}) \text{ ms}^{-1}$. Niles starts at a position of $(21\mathbf{i} - 11\mathbf{j})$. He travels at a constant velocity of $(-\mathbf{i} - 3\mathbf{j}) \text{ ms}^{-1}$.
 - What is Frasier's speed?
 - On what bearing is Niles travelling?
 - Find an expression for Frasier's position after t seconds.
 - Do Frasier and Niles meet? If they do meet, give the time and position that they meet.
 - A church lies at position $(-30\mathbf{i} + 3\mathbf{j})$. At what time is Niles due south of the church?
- At noon Tom starts at position $(7\mathbf{i} - 89\mathbf{j})$. He travels at a constant velocity of $(3\mathbf{i} + 5\mathbf{j}) \text{ ms}^{-1}$. Theo starts at a position of $(-11\mathbf{i} + 19\mathbf{j})$. He travels at a constant velocity of $(4\mathbf{i} - \mathbf{j}) \text{ ms}^{-1}$.
 - What is Tom's speed?
 - On what bearing is Theo travelling?
 - Find an expression for Theo's position after t seconds.
 - Do Tom and Theo meet? If they do meet, give the time and position that they meet.
 - A school lies at position $(50\mathbf{i} - \mathbf{j})$. At what time is Tom due north of the school?
- At noon Pietro starts at position $(26\mathbf{i} + \mathbf{j})$. He travels at a constant velocity of $(-2\mathbf{i} + 7\mathbf{j}) \text{ ms}^{-1}$. Adriana starts at a position of $(11\mathbf{i} + 9\mathbf{j})$. She travels at a constant velocity of $(\mathbf{i} + 5\mathbf{j}) \text{ ms}^{-1}$.
 - What is Adriana's speed?
 - On what bearing is Pietro travelling?
 - Find an expression for Adriana's position after t seconds.
 - Do Pietro and Adriana meet? If they do meet, give the time and position that they meet.
 - A convent lies at position $(-\mathbf{i} + 70\mathbf{j})$. At what time is Adriana due east of the church?
- At noon Ed starts at position $(6\mathbf{i} - 6\mathbf{j})$. He travels at a constant velocity of $(\mathbf{i} + \mathbf{j}) \text{ ms}^{-1}$. Robin starts at a position of $(9\mathbf{i} - 8\mathbf{j})$. He travels at a constant velocity of $(3\mathbf{j} - 2\mathbf{i}) \text{ ms}^{-1}$.
 - What is Ed's speed?
 - On what bearing is Robin travelling?
 - Find an expression for Ed's position after t seconds.

- (d) Do Ed and Robin meet? If they do meet, give the time and position that they meet.
- (e) When $t = 7$, what is the distance between Ed and Robin?
6. At noon Charlotte starts at position $(3\mathbf{i} + a\mathbf{j})$. She travels at a constant velocity of $(\mathbf{i} - \mathbf{j}) \text{ ms}^{-1}$. Simon starts at a position of $(2\mathbf{i} - 15\mathbf{j})$. He travels at a constant velocity of $(3\mathbf{i} + \mathbf{j}) \text{ ms}^{-1}$.
- (a) What is Charlotte's speed?
- (b) On what bearing is Simon travelling?
- (c) Find an expression for Simon's position after t seconds.
- (d) Given that Charlotte and Simon do meet, find the value of a .
- (e) When $t = 10$, what is the distance between Charlotte and Simon?
7. (Harder) At noon Cara starts at position $(14\mathbf{i} + 2\mathbf{j})$. She travels at a constant velocity of $(-\mathbf{i} + 6\mathbf{j}) \text{ ms}^{-1}$. Tom starts at a position of $(-2\mathbf{i} - 5\mathbf{j})$. He travels at a constant velocity of $(3\mathbf{i} + 5\mathbf{j}) \text{ ms}^{-1}$.
- (a) At what time are Cara and Tom closest to each other? $t = \frac{57}{17}$
- (b) At this time what is the distance between them? $d = \frac{44\sqrt{17}}{17}$
8. (Harder) At noon Chris starts at position $(-\mathbf{i} - 13\mathbf{j})$. He travels at a constant velocity of $(4\mathbf{i} + \mathbf{j}) \text{ ms}^{-1}$. Tim starts at a position of $(-2\mathbf{i} + 9\mathbf{j})$. He travels at a constant velocity of $(3\mathbf{i} - \mathbf{j}) \text{ ms}^{-1}$.
- (a) At what time are Chris and Tim closest to each other?
- (b) At this time what is the distance between them?
9. (Harder) At noon Ross starts at position $(-14\mathbf{i} + 7\mathbf{j})$. He travels at a constant velocity of $(\mathbf{i} - 4\mathbf{j}) \text{ ms}^{-1}$. Rachel starts at a position of $(5\mathbf{i} + 10\mathbf{j})$. She travels at a constant velocity of $(-2\mathbf{j}) \text{ ms}^{-1}$.
- (a) At what time are Ross and Rachel closest to each other?
- (b) At this time what is the distance between them?
10. (Harder) At noon Sarah starts at position $(a\mathbf{i} + b\mathbf{j})$. She travels at a constant velocity of $(x\mathbf{i} + y\mathbf{j}) \text{ ms}^{-1}$. Jemima starts at a position of $(c\mathbf{i} + d\mathbf{j})$. She travels at a constant velocity of $(\theta\mathbf{i} + \phi\mathbf{j}) \text{ ms}^{-1}$. Find an expression for the time at which they are closest. $t = \frac{(x-\theta)(c-a) + (y-\phi)(d-b)}{(x-\theta)^2 + (y-\phi)^2}$
11. (Harder) At noon Maris starts at position $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$. She travels at a constant velocity of $(-2\mathbf{i} + \mathbf{j} - \mathbf{k}) \text{ ms}^{-1}$. Lilith starts at a position of $(-2\mathbf{i} - 4\mathbf{j} + 10\mathbf{k})$. She travels at a constant velocity of $(-2\mathbf{i} - \mathbf{j} - 4\mathbf{k}) \text{ ms}^{-1}$.
- (a) At what time are Maris and Lilith closest to each other?
- (b) At this time what is the distance between them?