

## Isomorphisms Introduction Worksheet

- Determine which of the following groups of order 4 are isomorphic:
  - $\{3, 6, 9, 12\}$  under multiplication modulo 15.
  - $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$  under matrix multiplication.
  - $\left\{ \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ -1 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ i & i \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ -i & -i \end{pmatrix} \right\}$  under matrix multiplication.
  - $\{p(x) = x, q(x) = \frac{2}{2-x}, r(x) = \frac{x-2}{x-1}, s(x) = \frac{2(x-1)}{x}\}$  under composition of functions.
  - $\{a(x) = x, b(x) = 1 - \frac{1}{2x}, c(x) = \frac{1-x}{1-2x}, d(x) = \frac{1}{2-2x}\}$  under composition of functions.
- Give examples of geometric groups which are isomorphic to:
  - $\{1, 3, 7, 9\}$  under multiplication modulo 10.
  - $\{i(x) = x, a(x) = \frac{1}{x}, b(x) = -x, c(x) = -\frac{1}{x}\}$  under composition of functions.
  - $\{1, -1, \frac{1}{2}(1+i\sqrt{3}), \frac{1}{2}(1-i\sqrt{3}), \frac{1}{2}(-1+i\sqrt{3}), \frac{1}{2}(-1-i\sqrt{3})\}$  under multiplication of complex numbers.
  - $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} \right\}$  where  $\omega^3 = 1$ , under matrix multiplication.
- Prove that all groups with three elements are isomorphic.
- Prove that the group of symmetries of the rhombus is isomorphic to the group of symmetries of a rectangle. How many different one-to-one correspondences are there?