FP2 Transformations Of The Complex Plane

1. Shamelessly stolen from Parsonson, who is far better than I am at coming up with questions that work out nicely. Find the image of the circle $|z| = 1$ under the following transformations.

   (a) $w = \dfrac{2}{i - 2z}$. $\boxed{|w - 2i| = 2|w| \quad \Rightarrow \quad u^2 + (v - \tfrac{2}{3})^2 = \tfrac{16}{9}}$

   (b) $w = \dfrac{2 + z}{i - z}$. $\boxed{|w + 2i| = |w + 1| \quad \Rightarrow \quad 2u - 4v = 3}$

   (c) $w = \dfrac{1 + z}{1 - iz}$. $\boxed{|w - 1| = |w - i| \quad \Rightarrow \quad v = u}$

   (d) $w = \dfrac{2z - i}{1 + z}$. $\boxed{|w + i| = |w - 2| \quad \Rightarrow \quad 4u + 2v = 3}$

   (e) $w = \dfrac{iz + 1 - i}{(1 + i)z - 1}$. $\boxed{|w + 1 - i| = \sqrt{2}|w - \tfrac{1+i}{2}| \quad \Rightarrow \quad (u - 2)^2 + v^2 = 5}$

   (f) $w = \dfrac{(1 + i)z - 1}{(1 - i)z + i}$. $\boxed{|w - i| = \sqrt{2}|w - i| \quad \Rightarrow \quad \text{Point } i}$

2. Find the cartesian equation of the image of the following loci under the given transformations.

   (a) $|z| = 3$ under $w = z + 1 + i$. $\boxed{|w - 1 - i| = 3 \quad \Rightarrow \quad (u - 1)^2 + (v - 1)^2 = 9}$

   (b) $|z - 2i| = |z|$ under $w = 3z - i$. $\boxed{|w - 5i| = |w + i| \quad \Rightarrow \quad v = 2}$

   (c) $|z + 1 + i| = 4$ under $w = i - 2z$. $\boxed{|w - 2 - 3i| = 8 \quad \Rightarrow \quad (u - 2)^2 + (v - 3)^2 = 64}$

   (d) $|z + 2 + i| = |z - 4i|$ under $w = \dfrac{z + i}{3}$. $\boxed{|w + \tfrac{2}{3}| = |w - \tfrac{5}{3}i| \quad \Rightarrow \quad 12u + 30v = 21}$

   (e) $|z - 3| = |z|$ under $w = \dfrac{1}{z - 3}$. $\boxed{\tfrac{1}{3} = |w + \tfrac{1}{3}| \quad \Rightarrow \quad (u + \tfrac{1}{3})^2 + v^2 = \tfrac{1}{9}}$

   (f) $|z - 3i| = |z + 2|$ under $w = iz + 4$. $\boxed{|w - 1| = |w - 4 + 2i| \quad \Rightarrow \quad 6u - 4v = 19}$

   (g) $|z - i| = 1$ under $w = \dfrac{z - i}{z + i}$. $\boxed{2|w| = |w - 1| \quad \Rightarrow \quad (u + \tfrac{1}{3})^2 + v^2 = \tfrac{4}{9}}$

   (h) $|z - i| = 2$ under $w = \dfrac{iz}{2 + iz}$. $\boxed{|w + 1| = 2|w - 1| \quad \Rightarrow \quad (u - \tfrac{5}{3})^2 + v^2 = \tfrac{16}{9}}$

   THE REST HAVEN'T REALLY WORKED

   (i) $|z - 4i| = 2$ under $w = \dfrac{z + 3i}{z + 2}$. $\boxed{\sqrt{5}|w - \tfrac{14+7i}{10}| = |w - 1| \quad \Rightarrow}$

   (j) $2|z| = |z - 2i|$ under $w = \dfrac{z}{z + 1}$. $\boxed{2|w| = \sqrt{5}|w - \tfrac{4+2i}{5}| \quad \Rightarrow}$

   (k) $|z + i| = |z - 2 + 3i|$ under $w = \dfrac{1}{z}$. $\boxed{|w - i| = \sqrt{13}|w - \tfrac{2+3i}{13}| \quad \Rightarrow}$

   (l) $|2z + 3| = |z - 3i|$ under $w = \dfrac{1}{z - 2}$. $\boxed{7|w + \tfrac{2}{7}| = \sqrt{13}|w + \tfrac{2+3i}{13}| \quad \Rightarrow}$