C4 Vector Scalar Product

1. Find the value of a that satisfies $\begin{pmatrix} 2 \\ -1 \\ a \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix} = 0$. $\boxed{a = \frac{7}{5}}$

2. Find the value of a that satisfies $\begin{pmatrix} a \\ a \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix} = 0$. $\boxed{a = -28}$

3. Find the value of a that makes the lines $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ a \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 7 \\ 8 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$ perpendicular. $\boxed{a = -\frac{1}{5}}$

4. Find the (acute) angle between the lines $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$. $\boxed{68.6^\circ}$

5. Find the (acute) angle between the lines $\mathbf{r} = \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}$. $\boxed{13.5^\circ}$

6. Find the (acute) angle between the line $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -1 \\ 5 \end{pmatrix}$ and the line joining $(2, -2, 1)$ and $(-1, 3, 3)$. $\boxed{75.5^\circ}$

7. Find the (acute) angle between the line $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ and the line joining $(3, -4, 0)$ and $(0, 2, 5)$. $\boxed{68.0^\circ}$

8. Find the (acute) angle between the line joining $(0, 0, 2)$ and $(3, 2, 1)$ and the line joining $(-1, -1, 3)$ and $(0, -2, 7)$. $\boxed{79.1^\circ}$

9. Find the (acute) angle between the line joining $(1, -1, 0)$ and $(3, 3, 4)$ and the line joining $(2, 0, 5)$ and $(1, -3, 4)$. $\boxed{25.2^\circ}$

10. If $A = (1, 2, 6)$, $B = (-1, 3, -1)$ and $O = (0, 0, 0)$ find
    (a) $A\hat{O}B$, $\boxed{92.7^\circ}$
    (b) $O\hat{A}B$, $\boxed{26.8^\circ}$
    (c) $A\hat{B}O$. $\boxed{60.5^\circ}$

11. If $A = (3, 0, 1)$, $B = (1, -1, 2)$ and $C = (0, 0, 4)$ find $A\hat{B}C$. $\boxed{120^\circ}$

12. If $A = (3, 0, 1)$, $B = (1, 4, 2)$ and $C = (-1, 0, 5)$ find $A\hat{C}B$. $\boxed{48.96^\circ}$

13. If $D = (-1, 2, 1)$, $E = (-1, -5, 0)$ and $F = (3, -3, -2)$ find $E\hat{D}F$. $\boxed{40.5^\circ}$

14. If $R = (0, 1, -1)$, $S = (2, 0, -1)$ and $T = (4, 1, 2)$ find $R\hat{S}T$. $\boxed{1.938 \text{ radians}}$

15. If $P = (-1,-1,-1)$, $Q = (1,3,2)$ and $R = (0,5,1)$ find $Q\hat{R}P$. $\boxed{55.0^\circ}$

16. (a) Find the point on the line $\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$ closest to the origin. $\boxed{\left(-\frac{1}{7}, -\frac{3}{7}, -\frac{2}{7}\right)}$

(b) Hence find the exact shortest distance from the origin to the line. $\boxed{\frac{\sqrt{14}}{7}}$

17. (a) Find the point on the line $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ closest to the origin. $\boxed{\left(\frac{15}{14}, \frac{6}{7}, -\frac{3}{14}\right)}$

(b) Hence find the exact shortest distance from the origin to the line. $\boxed{\frac{3\sqrt{42}}{14}}$

18. Find the point on the line joining $(3,2,-1)$ and $(-1,-1,0)$ closest to the origin. $\boxed{\left(\frac{1}{13}, -\frac{5}{26}, -\frac{7}{26}\right)}$

19. Find the exact shortest distance from the origin to the line joining $(0,-3,4)$ and $(1,2,3)$.

$\boxed{\frac{\sqrt{942}}{9}}$

20. (a) Find the point on the line $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$ closest to the point $(1,0,-3)$. $\boxed{\left(\frac{7}{3}, \frac{2}{3}, -\frac{8}{3}\right)}$

(b) Hence find the exact shortest distance from the point $(1,0,-3)$ to the line. $\boxed{\frac{\sqrt{21}}{3}}$

21. (a) Find the point on the line $\mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ closest to the point $(3,-3,2)$.

$\boxed{\left(\frac{19}{9}, -\frac{28}{9}, \frac{32}{9}\right)}$

(b) Hence find the exact shortest distance from the point $(3,-3,2)$ to the line. $\boxed{\frac{\sqrt{29}}{3}}$

22. (a) Find the point on the line $\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$ closest to the point $(0,1,-3)$.

$\boxed{\left(\frac{23}{9}, \frac{7}{9}, \frac{4}{9}\right)}$

(b) Hence find the exact shortest distance from the point $(0,1,-3)$ to the line. $\boxed{\frac{\sqrt{166}}{3}}$

23. (a) Find the point on the line $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ closest to the point $(3,0,1)$.

$\boxed{\left(\frac{28}{9}, -\frac{10}{9}, -\frac{13}{9}\right)}$

(b) Hence find the exact shortest distance from the point $(3,0,1)$ to the line. $\boxed{\frac{\sqrt{65}}{3}}$

24. Find the point on the line $\mathbf{r} = \lambda \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ closest to the point (p,q,r). $\boxed{\left(\frac{a(ap+bq+cr)}{a^2+b^2+c^2}, \frac{b(ap+bq+cr)}{a^2+b^2+c^2}, \frac{c(ap+bq+cr)}{a^2+b^2+c^2}\right)}$