C4 Trigonometric Integration

Evaluate the following integrals by means of the given trigonometric substitution.

1. $\displaystyle\int \frac{1}{1+x^2}\,dx$ with $x = \tan\theta$. $\boxed{\tan^{-1}x + c}$

2. $\displaystyle\int_0^2 \frac{2}{4+x^2}\,dx$ with $x = 2\tan\theta$. $\boxed{\frac{\pi}{4}}$

3. $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$ with $x = \sin\theta$. $\boxed{\sin^{-1}x + c}$

4. $\displaystyle\int_0^3 \frac{1}{\sqrt{9-x^2}}\,dx$ with $x = 3\sin\theta$. $\boxed{\frac{\pi}{2}}$

5. $\displaystyle\int \sqrt{1-x^2}\,dx$ with $x = \sin\theta$. $\boxed{\frac{1}{2}\left(x\sqrt{1-x^2} + \sin^{-1}x\right) + c}$

6. $\displaystyle\int_0^1 \sqrt{4-x^2}\,dx$ with $x = 2\sin\theta$. $\boxed{\frac{3\sqrt{3}+2\pi}{6}}$

7. $\displaystyle\int \frac{1}{(x+1)^2+4}\,dx$ with $x+1 = 2\tan\theta$. $\boxed{\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + c}$

8. $\displaystyle\int_{-5}^1 \frac{1}{(x+2)^2+9}\,dx$ with $x+2 = 3\tan\theta$. $\boxed{\frac{\pi}{6}}$

9. $\displaystyle\int \frac{\sqrt{x^2-1}}{x}\,dx$ with $x = \sec\theta$. $\boxed{\sqrt{x^2-1} + \tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) + c}$

10. $\displaystyle\int \frac{5}{4+8x^2}\,dx$ with $\sqrt{2}x = \tan\theta$. $\boxed{\frac{5\sqrt{2}}{8}\tan^{-1}(\sqrt{2}x) + c}$

11. $\displaystyle\int \frac{2}{\sqrt{3-x^2}}\,dx$ with $x = \sqrt{3}\sin\theta$. $\boxed{2\sin^{-1}\left(\frac{\sqrt{3}x}{3}\right) + c}$

12. $\displaystyle\int \frac{3}{5+2x^2}\,dx$ with $\sqrt{2}x = \sqrt{5}\tan\theta$. $\boxed{\frac{3\sqrt{10}}{10}\tan^{-1}\left(\frac{\sqrt{10}x}{5}\right) + c}$

13. $\displaystyle\int \frac{5}{7+3x^2}\,dx$ with $\sqrt{3}x = \sqrt{7}\tan\theta$. $\boxed{\frac{5\sqrt{21}}{21}\tan^{-1}\left(\frac{\sqrt{21}x}{7}\right) + c}$

14. $\displaystyle\int \sqrt{4-25x^2}\,dx$ with $5x = 2\sin\theta$. $\boxed{\frac{\sqrt{4-25x^2}}{2}x + \frac{2}{5}\sin^{-1}\left(\frac{5x}{2}\right) + c}$