

C3 Test

You *may* use a calculator. Slovenly work will be penalized.

1. By writing $\sin 105$ as $\sin(60 + 45)$, expand the latter into the form $\frac{\sqrt{k+\sqrt{2}}}{4}$ where k is to be determined. $k = 6$

2. Differentiate the following with respect to x . You must simplify your answers:

(a) $x^3(x-1)^7$.

$x^2(10x-3)(x-1)^6$

(b) $\frac{1}{\sqrt{2-3x}}$.

$\frac{3}{2(2-3x)^{3/2}}$

3. Solve in the range $0 < x < 360$ the following (where answers are not exact, give them to one decimal place):

(a) $2 \sin 2x = \cos x$.

$x = 14.5$ or $x = 90$ or $x = 165.5$ or $x = 270$

(b) $\tan 2x = 4 \tan x$.

$x = 35.5$ or $x = 144.7$ or $x = 180$ or $x = 215.3$ or $x = 324.7$

4. Find the volume formed when $y = 2x + 1$ is rotated round the x -axis between $x = 0$ and $x = 2$.

$\frac{62\pi}{3}$

5. Solve $5 \cot \theta + 2 \operatorname{cosec}^2 \theta = 5$ in the range $0 < \theta < 2\pi$. Give answers to two decimal places.

$\theta = 1.11$ or $\theta = 4.25$ or $\theta = 2.82$ or $\theta = 5.96$

6. (a) Show that the equation $x^3 - x - 2 = 0$ has a root between 1 and 2.

- (b) Show that the equation can be rearranged into the form $x = g(x)$ where

$$g(x) = \sqrt[3]{x+2}.$$

- (c) Using $x_0 = 1.5$, iterate the above formula to find the solution to 3dp. $x = 1.521$ (to 3dp)

7. (a) Write $3 \cos x + 5 \sin x$ in the form $R \cos(x - \alpha)$, where R and α are to be determined.

$\sqrt{34} \cos(x - 59.04)$

- (b) Hence solve $3 \cos x + 5 \sin x = -1$ in the range $-180 < x < 360$. Give answers to 1dp.

$x = -40.8$ or $x = 158.9$ or $x = 319.2$

- (c) What is the maximum value of $3 \cos x + 5 \sin x$, and for which value of x does this maximum occur at? $\sqrt{34}$ when $x = 59.04$