Reverse Chain Rule a.k.a. Inspection a.k.a. Spotting The Answer

Recall that integration is (by definition) the inverse operation to differentiation. Therefore if you think you've found an integral you should be able to differentiate your answer and get back to where you started.

For this sheet the starting point here is the following:

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \qquad \frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(\ln x) = \frac{1}{x},$$
$$\frac{d}{dx}(\sin x) = \cos x, \qquad \frac{d}{dx}(\cos x) = -\sin x, \qquad \frac{d}{dx}(\tan x) = \sec^2 x,$$
$$\frac{d}{dx}(\sinh x) = \cosh x, \qquad \frac{d}{dx}(\cosh x) = \sinh x, \qquad \frac{d}{dx}(\tanh x) = \text{sech}^2 x.$$

Combining the above with the chain rule we find the following (here $f \equiv f(x)$ and $f' \equiv f'(x)$):

$$\frac{d}{dx}(af^n) = anf'f^{n-1}, \qquad \frac{d}{dx}(e^f) = f'e^f, \qquad \frac{d}{dx}(\ln f) = \frac{f'}{f},$$
$$\frac{d}{dx}(\sin f) = f'\cos f, \qquad \frac{d}{dx}(\cos f) = -f'\sin f, \qquad \frac{d}{dx}(\tan f) = f'\sec^2 f,$$
$$\frac{d}{dx}(\sinh f) = f'\cosh f, \qquad \frac{d}{dx}(\cosh f) = f'\sinh f, \qquad \frac{d}{dx}(\tanh f) = f'\text{sech}^2 f.$$

Note that in *every one* of the above there is an $f'(x)$ term. Therefore if you see an integral of something where a function is 'wrapped up' within another function (e.g. e^{x^2+1}) then you should look for something which looks like (just about) the derivative of the internal function (here $2x$); you then know that this will be a reverse chain rule integral.