

## LM0 Beginning of Term Problems

1. What is the value of the following?

$$\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{\dots}}}}}}}$$

2.  $N$  is a four-digit positive integer, not ending in zero, and  $R(N)$  is the four-digit integer obtained by reversing the digits of  $N$ . For example,  $R(3475) = 5743$ .

Determine all such integers  $N$  for which  $R(N) = 4N + 3$ .

3. The Dwarfs in the Land-under-the-Mountain have just adopted a completely decimal currency system based on the Pippin, with gold coins to the value of 1 Pippin, 10 Pippins, 100 Pippins and 1000 Pippins.

In how many ways is it possible for a Dwarf to pay, in exact coinage, a bill of 1997 Pippins?

4. Let  $ABCD$  be a convex quadrilateral. The midpoints of  $AB$ ,  $BC$ ,  $CD$  and  $DA$  are  $P$ ,  $Q$ ,  $R$  and  $S$ , respectively. Given that the quadrilateral  $PQRS$  has area 1, prove that the area of the quadrilateral  $ABCD$  is 2.

5. An old map of Treasure Island reveals the whereabouts of a hidden treasure. There are three prominent landmarks called  $A$ ,  $B$ , and  $C$  on the island. The directions on the map are as follows: "First, walk in a straight line from  $A$  to  $B$ , then turn  $90^\circ$  to the left and walk the same distance again to locate point  $D$ . Next, walk in a straight line from  $A$  to  $C$ , then turn  $90^\circ$  to the right and walk the same distance again to locate point  $E$ . The treasure is located at the midpoint of the straight line joining  $D$  and  $E$ ." The owner of the map has sailed to the island and found two of the landmarks. Unfortunately, landmark  $A$  has been wiped out by time and it is not certain which of the two remaining landmarks is  $B$ . Can you help find the location of the treasure?