

## Definite Integration

1. Sketch the graph of  $y = 4x^2 - 9$ , and find the area bounded by this curve and the  $x$ -axis.
2. Find the areas bounded by  $y = 4(x^2 - 4)(3 - x)$  and the  $x$ -axis (i) for which  $y > 0$ ; (ii) for which  $y < 0$ . Write down the value of

$$\int_{-2}^3 4(x^2 - 4)(3 - x) dx.$$

3. Evaluate  $\int_1^3 (4 + 3x - x^3) dx$ . What can you deduce from your results about the graph of  $y = 4 + 3x - x^3$  between  $x = 1$  and  $x = 3$ ?

4. Evaluate  $\int_0^2 (x^2 + 1) dx$ . Explain, with the aid of a sketch, how you could deduce the value of

$$\int_{-2}^2 (x^2 + 1) dx.$$

Show, preferably by means of another sketch, that

$$\int_{-2}^2 x(x^2 + 1) dx.$$

5. Sketch the curve  $y = x(x - 2)$ . Find an expression for

$$F(t) \equiv \int_{-1}^t x(x - 2) dx$$

and sketch the curve  $y = F(t)$  for  $-1 \leq x \leq 3$ .

Find the maximum and minimum values of  $F(t)$ , and the values of  $t$  for which these occur. How do these relate to the first graph?

6. (a) Use the result

$$\int_a^b f(x) dx = F(b) - F(a),$$

where  $F'(x) \equiv f(x)$ , to show that

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

and interpret this in terms of areas when  $a < b < c$ .

- (b) Show that putting  $a = b$  in (a) suggests that

$$\int_a^a f(x) dx = 0,$$

and explain how this fits with the interpretation of a definite integral as an area.

- (c) Show that putting  $c = a$  in (a) suggests that

$$\int_b^a f(x) dx = - \int_a^b f(x) dx.$$

[The results in (b) and (c) are used as the definitions of  $\int_a^b f(x) dx$  when  $a \geq b$ .]

7. Calculate the coordinates of the points where the curve  $y = x^2$  crosses the line  $y = 3x + 10$  and show them in a sketch of a line and the curve. On your diagram shade the area which satisfies both  $y \geq x^2$  and  $y \leq 3x + 10$ . Calculate this area.
8. Sketch the curves  $y = x^2$  and  $y = 8 - x^2$ , and show that they intersect at the points  $(2, 4)$  and  $(-2, 4)$ . Calculate the area enclosed by the two curves.
9. Find the area enclosed by the curves  $y = x^2$  and  $y = x^3$ .
10. Sketch the curves
- $$y = \frac{4}{x^2}, y = x(x + 3) \text{ and } y = x - \frac{x^2}{4}.$$
- for  $0 \leq x \leq 2$ . You *should* find they cross at  $(0, 0)$ ,  $(1, 4)$  and  $(2, 1)$  and that the three curves enclose an area between these points. Find the enclosed area.
11. Show that if  $n \geq 1$ , the area enclosed by the curves  $y = x^n$  and  $y = x^{1/n}$  is  $\frac{n-1}{n+1}$ .
12. Sketch the curve  $y = (x + 2)^2$ , and shade the region defined by the inequalities  $x \geq 0$ ,  $y \geq (x + 2)^2$ ,  $y \leq 9$ . Calculate the area of this region.
13. Draw a rough sketch of the curve  $y^2 = 8x + 8$ . Calculate the area enclosed by the curve and the line  $x = 1$ .