

Completing The Square Worksheet

1. Complete the square on the following:

(a) $x^2 + 4x - 1$.

(b) $x^2 - 3x + 5$.

(c) $h^2 + h$.

(d) $2t^2 + 4t + 7$.

(e) $-a^2 + 6a - 10$.

(f) $-3\theta^2 + 12\theta + 1$.

2. Find the coordinate of the turning point of the following curves and determine their natures

(a) $y = (x - 1)^2$.

(b) $y = -(x + 3)^2 - 7$.

(c) $y = 2(x - 4)^2 + 8$.

(d) $y = 4 - x^2$.

(e) $y = 3 + (2 - x)^2$.

(f) $y = x^2 + 3x - 7$.

(g) $y = 2x^2 + 8x + 3$.

3. Find the equation of the new curve after the following transformations:

(a) $y = x^2$ after a translation $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

(b) $y = x^2 + 4x - 3$ after a translation $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$.

(c) $y = 2x^2 + 8x + 1$ after a translation $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

(d) $y = -x^2 + 2x - 1$ after a reflection in the y -axis.

(e) $y = 3x^2 + 6x + 2$ after a reflection in the x -axis.

(f) $y = -2x^2 - x + \frac{1}{2}$ after a translation $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and *then* a reflection in the x -axis.

4. By completing the square solve the following equations:

(a) $0 = x^2 + x - 6$.

(b) $0 = x^2 + 4x - 1$.

(c) $2x^2 + x = x^2 - 5x + 7$.

5. What is the maximum value of $\frac{1}{(x - 3)^2 + 4}$ and for what value of x does it occur?

6. What is the minimum value of $\frac{5}{-(x - 3)^2 - 2}$ and for what value of x does it occur?

7. What is the maximum value of $\frac{5}{3x^2 + 12x + 90}$ and for what value of x does it occur?

8. If $x = 3$ is a root of the quadratic equation $f(x) = 0$ and $(1, -4)$ is the vertex of $y = f(x)$, what must the other root be? Also guess an equation for $f(x)$ and verify your answer.

9. If $x = a$ is a root of the quadratic equation $f(x) = 0$ and (b, c) is the vertex of $y = f(x)$, what must the other root be?

10. If $x = a$ and $x = b$ are roots of a quadratic equation, what must the equation of the line of symmetry be?
11. What is the minimum value of $5[(x - 1)(x - 2)(x - 3)]^2 + 7$ and for which value(s) of x does it occur?
12. What is the maximum value of $5 - 3[1 - \sin \theta]^2$ and for which value(s) of θ does it occur?
13. What is the minimum and maximum value of $\frac{5}{3 + 2 \sin \theta}$?
14. What is the minimum and maximum value of $\frac{1}{10 - 2 \cos \theta}$?
15. By completing the square on the equation $ax^2 + bx + c = 0$ and solving the resulting equation for x , derive the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$