OCR
RECOGNISING ACHIEVEMENT

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education**
**Advanced General Certificate of Education**

# MATHEMATICS 4731

Mechanics 4

**Specimen Paper**

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

**TIME** 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use 9.8 m $s^{-2}$.
- You are permitted to use a graphic calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

**This question paper consists of 3 printed pages and 1 blank page.**

**1** A circular flywheel of radius 0.2 m is rotating freely about a fixed axis through its centre and perpendicular to its plane. The moment of inertia of the flywheel about the axis is $0.37 \text{ kg m}^2$. When the angular speed of the flywheel is $8 \text{ rad s}^{-1}$ a particle of mass 0.75 kg, initially at rest, sticks to a point on the circumference of the flywheel. Find

**(i)** the angular speed of the flywheel immediately after the particle has stuck to it, [4]

**(ii)** the loss of energy that results when the particle sticks to the flywheel. [2]

**2** A uniform solid sphere, of mass 4 kg and radius 0.1 m, is rotating freely about a fixed axis with angular speed $20 \text{ rad s}^{-1}$. The axis is a diameter of the sphere. A couple, having constant moment 0.36 N m about the axis and acting in the direction of rotation, is then applied for 6 seconds. For this time interval, find

**(i)** the angular acceleration of the sphere, [3]

**(ii)** the angle through which the sphere turns, [2]

**(iii)** the work done by the couple. [2]

**3** The region bounded by the *x*-axis, the *y*-axis, and the curve $y = 4 - x^2$ for $0 \leqslant x \leqslant 2$, is occupied by a uniform lamina of mass 35 kg. The unit of length is the metre. Show that the moment of inertia of the lamina about the *y*-axis is $28 \text{ kg m}^2$. [8]

**4** A straight rod *AB* of length *a* has variable density, and at a distance *x* from *A* its mass per unit length is $k\left(1+\frac{x^2}{a^2}\right)$, where *k* is a constant.

**(i)** Find the distance of the centre of mass of the rod from *A*. [6]

You are given that the moment of inertia of the rod about a perpendicular axis through *A* is $\frac{8}{15}ka^3$.

**(ii)** Show that the period of oscillation of the rod as a compound pendulum, when freely pivoted at the other end *B*, is $2\pi\sqrt{\frac{22a}{35g}}$. [5]

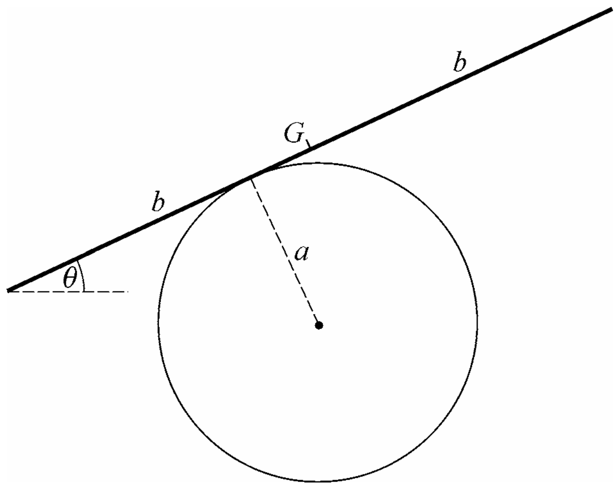
**5** A uniform rod *AB*, of mass *m* and length $2a$, is free to rotate in a vertical plane about a fixed horizontal axis through *A*. The rod is released from rest with *AB* horizontal. Air resistance may be neglected. For the instant when the rod has rotated through an angle $\frac{1}{6}\pi$,

**(i)** show that the angular acceleration of the rod is $\frac{(3\sqrt{3})g}{8a}$, [2]

**(ii)** find the angular speed of the rod, [3]

**(iii)** show that the force acting on the rod at *A* has magnitude $\frac{\sqrt{103}}{8}mg$. [7]

**6**




A cylinder with radius $a$ is fixed with its axis horizontal. A uniform rod, of mass $m$ and length $2b$, moves in a vertical plane perpendicular to the axis of the cylinder, maintaining contact with the cylinder and not slipping (see diagram). When the rod is horizontal, its mid-point $G$ is in contact with the cylinder. You are given that, when the rod makes an angle $\theta$ with the horizontal, the height of $G$ above the axis of the cylinder is $a(\theta \sin\theta + \cos\theta)$.

**(i)** By considering the potential energy of the rod, show that $\theta = 0$ is a position of stable equilibrium. [6]

**(ii)** You are also given that, when $\theta$ is small, the kinetic energy of the rod is approximately $\frac{1}{6}mb^2\dot{\theta}^2$.

Show that the approximate period of small oscillations about the position $\theta = 0$ is $\dfrac{2\pi b}{\sqrt{(3ga)}}$. [7]

**7** An unidentified object $U$ is flying horizontally due east at a constant speed of $220 \text{ m s}^{-1}$. An aircraft is 15 000 m from $U$ and is at the same height as $U$. The bearing of $U$ from the aircraft is $310°$.

**(i)** Assume that the aircraft flies in a straight line at a constant speed of $160 \text{ m s}^{-1}$.

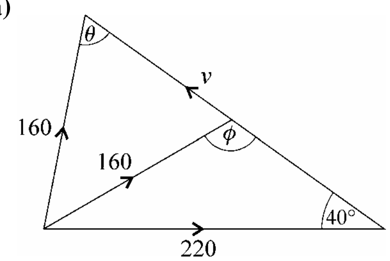
**(a)** Find the bearings of the two possible directions in which the aircraft can fly to intercept $U$. [6]

**(b)** Given that the interception occurs in the shorter of the two possible times, find the time taken to make the interception. [5]

**(ii)** Assuming instead that the aircraft flies in a straight line at a constant speed of $130 \text{ m s}^{-1}$, show that the nearest the aircraft can come to $U$ is approximately 988 m. [4]

| | | | |
|---|---|---|---|
| **1** | **(i)** MI with particle is $0.37+0.75\times0.2^2=0.4$ | M1 | For $0.75\times0.2^2$ |
| | | A1 | For correct MI, stated or implied |
| | $0.4\omega=0.37\times8$ | M1 | For relevant use of cons. of ang. mom. |
| | Hence angular speed is $7.4\text{ rad s}^{-1}$ | A1 **4** | For correct value 7.4 |
| | **(ii)** K.E. loss $\frac{1}{2}\times0.37\times8^2-\frac{1}{2}\times0.4\times7.4^2=0.888\text{ J}$ | M1 | For an correct relevant use of $\frac{1}{2}I\omega^2$ |
| | | A1√ **2** | For correct value for the KE loss |
| | | **6** | |
| **2** | **(i)** $I=\frac{2}{5}\times4\times0.1^2=0.016$ | B1 | For correct use of $\frac{2}{5}mr^2$ |
| | $0.36=0.016\alpha$ | M1 | For use of $C=I\alpha$ to find $\alpha$ |
| | Hence angular acceleration is $22.5\text{ rad s}^{-2}$ | A1 **3** | For correct value 22.5 |
| | **(ii)** $\theta=20\times6+\frac{1}{2}\times22.5\times6^2$ | M1 | For use of $\theta=\omega_0t+\frac{1}{2}\alpha t^2$ to find $\theta$ |
| | Angle turned through is 525 radians | A1√ **2** | For correct answer 525 |
| | **(iii)** Work done $=0.36\times525=189\text{ J}$ | M1 | For use of $C\theta$, or increase in $\frac{1}{2}I\omega^2$ |
| | | A1√ **2** | For correct answer 189 |
| | | **7** | |
| **3** | *EITHER*: Area is $\int_0^2(4-x^2)\,dx=\left[4x-\frac{1}{3}x^3\right]_0^2=\frac{16}{3}$ | M1 | For evaluation of $\int_0^2 y\,dx$ |
| | | A1 | For correct value $\frac{16}{3}$ |
| | Hence $\frac{16}{3}\rho=35\Rightarrow\rho=\frac{105}{16}$ | B1√ | For correct density |
| | $I=\int_0^2\rho x^2y\,dx=\frac{105}{16}\int_0^2x^2(4-x^2)\,dx$ | M1 | For use of $\int x^2y\,dx$ |
| | | A1√ | For correct expression for $I$ |
| | $=\frac{105}{16}\left[\frac{4}{3}x^3-\frac{1}{5}x^5\right]_0^2=\frac{105}{16}\times\frac{64}{15}=28$ | A1 | For correct indefinite integral $\frac{4}{3}x^3-\frac{1}{5}x^5$ |
| | | A1√ | For correct numerical expression $\frac{64}{15}\rho$ |
| | | A1 | For obtaining given answer 28 correctly |
| | *OR*: Area is $\int_0^4(4-y)^{\frac{1}{2}}\,dy=\left[-\frac{2}{3}(4-y)^{\frac{3}{2}}\right]_0^4=\frac{16}{3}$ | M1 | For evaluation of $\int_0^4 x\,dy$ |
| | | A1 | For correct value $\frac{16}{3}$ |
| | Hence $\frac{16}{3}\rho=35\Rightarrow\rho=\frac{105}{16}$ | B1√ | For correct density |
| | $I=\frac{1}{3}\rho\int_0^4x^3\,dy=\frac{35}{16}\int_0^4(4-y)^{\frac{3}{2}}\,dy$ | M1 | For use of $\frac{1}{3}\int x^3\,dy$ |
| | | A1√ | For correct expression for $I$ |
| | $=\frac{35}{16}\left[-\frac{2}{5}(4-y)^{\frac{5}{2}}\right]_0^4=\frac{35}{16}\times\frac{64}{5}=28$ | A1 | For correct indefinite integral $-\frac{2}{5}(4-y)^{\frac{5}{2}}$ |
| | | A1√ | For correct numerical expression $\frac{1}{3}\rho\times\frac{64}{5}$ |
| | | A1 **8** | For obtaining given answer 28 correctly |
| | | **8** | |

| | | | |
|---|---|---|---|
| **4** **(i)** | Moment @ $A = \int_0^a kx\left(1+\frac{x^2}{a^2}\right)dx = k\left[\frac{x^2}{2}+\frac{x^4}{4a^2}\right]_0^a$ | M1 | For attempted integration of $\rho x$ with limits |
| | $= \frac{3}{4}ka^2$ | A1 | For correct MI $\frac{3}{4}ka^2$ |
| | Mass of rod is $\int_0^a k\left(1+\frac{x^2}{a^2}\right)dx = k\left[x+\frac{x^3}{3a^2}\right]_0^a$ | M1 | For attempted integration of $\rho$ with limits |
| | $= \frac{4}{3}ka$ | A1 | For correct mass $\frac{4}{3}ka$ |
| | Hence $\frac{4}{3}ka\bar{x} = \frac{3}{4}ka^2 \Rightarrow \bar{x} = \frac{9}{16}a$ | M1 | For moments equation for $\bar{x}$ |
| | | A1 **6** | For correct answer $\frac{9}{16}a$ |
| **(ii)** | $I_G = I_A - m(\bar{x})^2 = \frac{8}{15}ka^3 - \frac{4}{3}ka\left(\frac{9}{16}a\right)^2 = \frac{107}{960}ka^3$ | B1 | For stating correct relation $I_G = I_A - m(\bar{x})^2$ |
| | $I_B = I_G + m(a-\bar{x})^2 = \frac{107}{960}ka^3 + \frac{4}{3}ka\left(\frac{7}{16}a\right)^2 = \frac{11}{30}ka^3$ | M1 | For correct use of \|\| axes to find $I_B$ |
| | | A1 | For correct value $\frac{11}{30}ka^3$, or equivalent |
| | Period is $2\pi\sqrt{\frac{\frac{11}{30}ka^3}{(\frac{4}{3}ka)g(\frac{7}{16}a)}} = 2\pi\sqrt{\frac{22a}{35g}}$ | M1 | For correct use of $2\pi\sqrt{\frac{I}{mgh}}$ |
| | | A1 **5** | For showing given answer correctly |
| | | **11** | |
| **5** **(i)** | $mga\cos\frac{1}{6}\pi = \frac{4}{3}ma^2\alpha$ | M1 | For use of $C = I_A\alpha$ |
| | Hence $\alpha = \frac{(3\sqrt{3})g}{8a}$ | A1 **2** | For obtaining given answer correctly |
| **(ii)** | $\frac{1}{2}\times\frac{4}{3}ma^2\times\omega^2 = mga\sin\frac{1}{6}\pi$ | M1 | For relevant use of conservation of energy |
| | | A1 | For correct equation |
| | Hence $\omega = \sqrt{\left(\frac{3g}{4a}\right)}$ | A1 **3** | For correct answer |
| **(iii)** | Res \|\| rod: $R - mg\sin\frac{1}{6}\pi = ma\omega^2$ | M1 | For Newton II equation with 3 terms |
| | Hence $R = \frac{1}{2}mg + \frac{3}{4}mg = \frac{5}{4}mg$ | A1√ | For correct component |
| | Res ⊥ rod: $mg\cos\frac{1}{6}\pi - S = ma\alpha$ | M1 | For Newton II equation with 3 terms |
| | | A1 | For correct equation |
| | Hence $S = \left(\frac{1}{2}\sqrt{3}\right)mg - \left(\frac{3}{8}\sqrt{3}\right)mg = \left(\frac{1}{8}\sqrt{3}\right)mg$ | A1 | For correct component |
| | Magnitude is $\sqrt{(R^2+S^2)} = \frac{1}{8}mg\sqrt{(10^2+3)}$ | M1 | For correct method for resultant |
| | $= \frac{\sqrt{103}}{8}mg$ | A1 **7** | For obtaining given answer correctly |
| | | **12** | |

| | | | |
|---|---|---|---|
| **6** **(i)** $V = mga(\theta\sin\theta + \cos\theta)$, so | | | |
| $\frac{\mathrm{d}V}{\mathrm{d}\theta} = mga(\theta\cos\theta + \sin\theta - \sin\theta) = mga\theta\cos\theta$ | M1 | | For differentiation using product rule |
| | A1 | | For correct derivative |
| Hence equilibrium at $\theta = 0$, since $\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0$ | A1 | | For showing the given result correctly |
| $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = mga(\cos\theta - \theta\sin\theta)$ | M1 | | For differentiating again using product rule |
| | A1 | | For correct second derivative |
| When $\theta = 0$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = mga > 0$, so equm is stable | A1 | **6** | For showing the given result correctly |
| **(ii)** $mga(\theta\sin\theta + \cos\theta) + \frac{1}{6}mb^2\dot{\theta}^2 = K$ | B1 | | For correct statement of energy equation |
| Hence $(mga\theta\cos\theta)\dot{\theta} + \frac{1}{3}mb^2\dot{\theta}\ddot{\theta} = 0$ | M1 | | For attempt to differentiate w.r.t. *t* |
| | A1√ | | For correct derivative of PE term |
| | A1 | | For correct derivative of KE term |
| For small $\theta$, $mga\theta + \frac{1}{3}mb^2\ddot{\theta} \approx 0 \Rightarrow \ddot{\theta} \approx -\frac{3ga}{b^2}\theta$ | M1 | | For use of $\cos\theta \approx 1$ and simplifying |
| Motion is approximate SHM with period $\frac{2\pi b}{\sqrt{(3ga)}}$ | M1 | | For use of $\frac{2\pi}{\omega}$ from standard SHM form |
| | A1 | **7** | For showing the given answer correctly |
| | | **13** | |
| **7** **(i)** **(a)**  | B1 | | For correct triangle for at least one case |
| | B1 | | For both triangle (together or separately) |
| $\frac{\sin\theta}{220} = \frac{\sin 40°}{160}$ | M1 | | For a method for finding a relevant angle |
| Hence $\theta = 62.1°$, $\phi = 117.9°$ | A1 | | For either angle correct |
| Required bearings are 012.1° and 067.9° | A1 | | For one correct bearing |
| | A1 | **6** | For the other correct bearing |
| **(b)** Shorter time occurs for $\theta = 62.1°$ | B1√ | | For selecting the appropriate case |
| $\frac{v}{\sin 77.9°} = \frac{160}{\sin 40°} \Rightarrow v = 243.4$ | M1 | | For finding the relative speed, or equivalent |
| | A1 | | For correct value 243.4 |
| Hence time is $\frac{15000}{243.4} = 61.6$ s | M1 | | For calculation of the time taken |
| | A1√ | **5** | For correct value 61.6 |
| **(ii)** For closest approach, $\sin\alpha = \frac{130}{220} \Rightarrow \alpha = 36.2°$ | M1 | | For use of correct velocity triangle |
| | A1 | | For correct angle |
| Hence min distance is $15000\sin(40 - \alpha) \approx 988$ m | M1 | | For use of correct displacement triangle |
| | A1 | **4** | For showing given answer correctly |
| | | **15** | |