OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4731

Mechanics 4

Wednesday **21 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 A straight rod AB of length a has variable density. At a distance x from A its mass per unit length is $k(a + 2x)$, where k is a positive constant. Find the distance from A of the centre of mass of the rod. [5]

2 A flywheel takes the form of a uniform disc of mass 8 kg and radius 0.15 m. It rotates freely about an axis passing through its centre and perpendicular to the disc. A couple of constant moment is applied to the flywheel. The flywheel turns through an angle of 75 radians while its angular speed increases from $10\,\text{rad s}^{-1}$ to $25\,\text{rad s}^{-1}$.

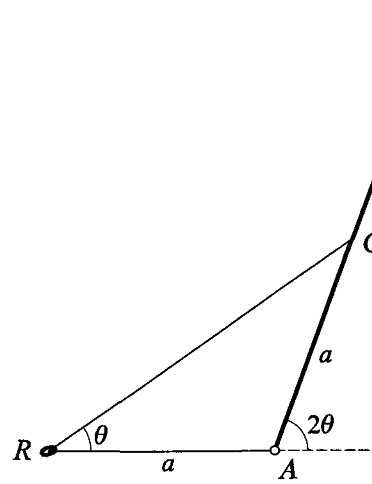
(i) Find the moment of the couple about the axis. [5]

When the flywheel is rotating with angular speed $25\,\text{rad s}^{-1}$, it locks together with a second flywheel which is mounted on the same axis and is at rest. Immediately afterwards, both flywheels rotate together with the same angular speed $9\,\text{rad s}^{-1}$.

(ii) Find the moment of inertia of the second flywheel about the axis. [3]

3 The region bounded by the x-axis, the lines $x = 1$ and $x = 2$ and the curve $y = \dfrac{1}{x^2}$ for $1 \leqslant x \leqslant 2$, is occupied by a uniform lamina of mass 24 kg. The unit of length is the metre. Find the moment of inertia of this lamina about the x-axis. [8]

4



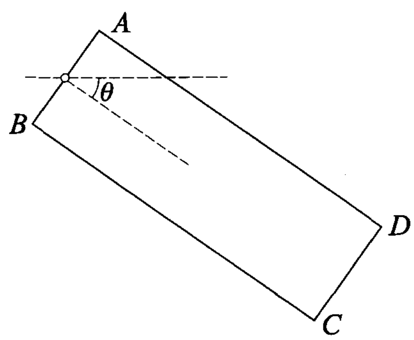
A uniform rod AB, of mass m and length $2a$, is freely hinged to a fixed point at A. A particle of mass $2m$ is attached to the rod at B. A light elastic string, with natural length a and modulus of elasticity $5mg$, passes through a fixed smooth ring R. One end of the string is fixed to A and the other end is fixed to the mid-point C of AB. The ring R is at the same horizontal level as A, and is at a distance a from A. The rod AB and the ring R are in a vertical plane, and RC is at an angle θ above the horizontal, where $0 < \theta < \frac{1}{4}\pi$, so that the acute angle between AB and the horizontal is 2θ (see diagram).

(i) By considering the energy of the system, find the value of θ for which the system is in equilibrium. [7]

(ii) Determine whether this position of equilibrium is stable or unstable. [3]

5 A uniform rectangular lamina $ABCD$ has mass 20 kg and sides of lengths $AB = 0.6$ m and $BC = 1.8$ m. It rotates in its own vertical plane about a fixed horizontal axis which is perpendicular to the lamina and passes through the mid-point of AB.

(i) Show that the moment of inertia of the lamina about the axis is $22.2\,\text{kg m}^2$. [3]

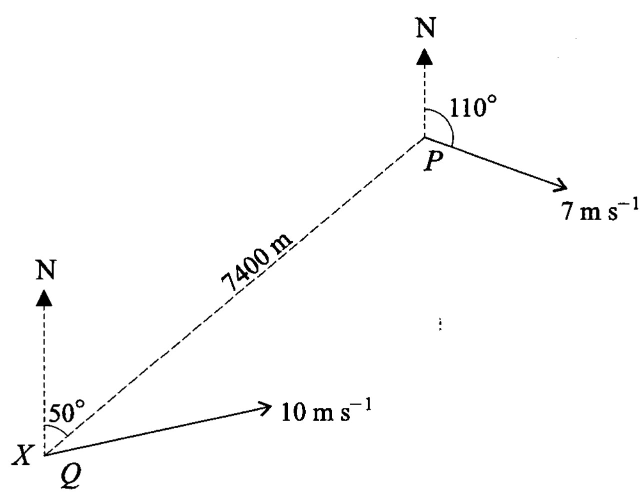


The lamina is released from rest with BC horizontal and below the level of the axis. Air resistance may be neglected, but a frictional couple opposes the motion. The couple has constant moment 44.1 N m about the axis. The angle through which the lamina has turned is denoted by θ (see diagram).

(ii) Show that the angular acceleration is zero when $\cos\theta = 0.25$. [3]

(iii) Hence find the maximum angular speed of the lamina. [5]

6



A ship P is moving with constant velocity $7\,\text{m s}^{-1}$ in the direction with bearing 110°. A second ship Q is moving with constant speed $10\,\text{m s}^{-1}$ in a straight line. At one instant Q is at the point X, and P is 7400 m from Q on a bearing of 050° (see diagram). In the subsequent motion, the shortest distance between P and Q is 1790 m.

(i) Show that one possible direction for the velocity of Q relative to P has bearing 036°, to the nearest degree, and find the bearing of the other possible direction of this relative velocity. [3]

Given that the velocity of Q relative to P has bearing 036°, find

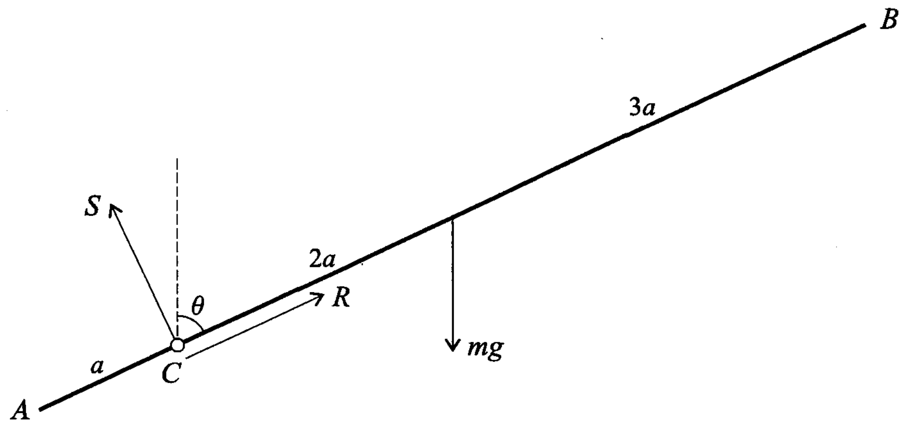
(ii) the bearing of the direction in which Q is moving, [4]

(iii) the magnitude of the velocity of Q relative to P, [2]

(iv) the time taken for Q to travel from X to the position where the two ships are closest together, [3]

(v) the bearing of P from Q when the two ships are closest together. [1]

7



A uniform rod AB has mass m and length $6a$. It is free to rotate in a vertical plane about a smooth fixed horizontal axis passing through the point C on the rod, where $AC = a$. The angle between AB and the upward vertical is θ, and the force acting on the rod at C has components R parallel to AB and S perpendicular to AB (see diagram). The rod is released from rest in the position where $\theta = \frac{1}{3}\pi$. Air resistance may be neglected.

(i) Find the angular acceleration of the rod in terms of a, g and θ. [4]

(ii) Show that the angular speed of the rod is $\sqrt{\dfrac{2g(1 - 2\cos\theta)}{7a}}$. [3]

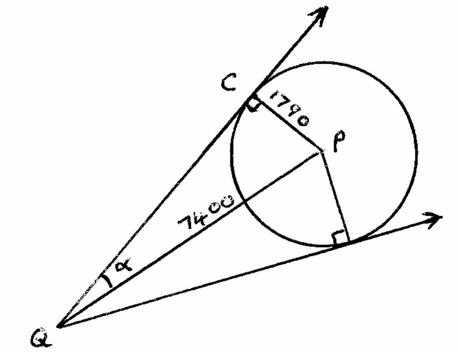
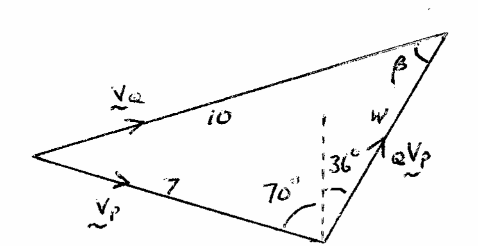
(iii) Find R and S in terms of m, g and θ. [6]

(iv) When $\cos\theta = \frac{1}{3}$, show that the force acting on the rod at C is vertical, and find its magnitude. [4]

MARKING SCHEME

1	$\int x\rho\,dx = \int_0^a k(a+2x)\,x\,dx$	M1	for $\int ...(a+2x)x\,dx$
	$= k\left[\tfrac{1}{2}ax^2 + \tfrac{2}{3}x^3\right]_0^a \quad (=\tfrac{7}{6}ka^3)$	A1	
	$\int \rho\,dx = k\int_0^a (a+2x)\,dx \quad = k\left[ax + x^2\right]_0^a$ $= 2ka^2$	B1	for ... $\left[ax + x^2\right]_0^a$
	$\bar{x} = \dfrac{\tfrac{7}{6}ka^3}{2ka^2}$	M1	*Dependent on first M1*
	$= \tfrac{7}{12}a$	A1 **5**	Accept $0.583a$
2 (i)	$I = \tfrac{1}{2}\times 8\times 0.15^2 \quad (=0.09\text{ kg m}^2)$	B1	
	Using $\omega_2^{\,2} = \omega_1^{\,2} + 2\alpha\theta$ $25^2 = 10^2 + 2\alpha\times 75$ $\alpha = 3.5\text{ rad s}^{-2}$	M1A1	
	Couple is $I\alpha = 0.09\times 3.5$	M1	
	$= 0.315\text{ N m}$	A1 ft **5**	ft from wrong I and / or α, but ft requires M1M1
	OR Increase in KE is $\tfrac{1}{2}\times 0.09\times(25^2 - 10^2)$ M1A1 ft $= 23.625\text{ J}$		
	M1		WD by couple is $L\times 75$
	Couple is $\dfrac{23.625}{75} = 0.315\text{ N m}$ A1 ft		ft requires M1M1
(ii)	By conservation of angular momentum	M1	Using angular momentum
	$(0.09 + I_2)\times 9 = 0.09\times 25$	A1 ft	
	$I_2 = 0.16\text{ kg m}^2$	A1 **3**	
3	$\int_1^2 \dfrac{1}{x^2}\,dx = \left[-\dfrac{1}{x}\right]_1^2$	M1	
	$= \tfrac{1}{2}$	A1	
	Mass per unit area $\rho = 48\text{ kg m}^{-2}$	B1	
	$I = \int \tfrac{4}{3}(\rho\,y\,\delta x)(\tfrac{1}{2}y)^2$	M1	For integral of y^3
	$= \int \tfrac{1}{3}\rho\,y^3\,dx$	A1	
	$= \tfrac{1}{3}\rho\int_1^2 \dfrac{1}{x^6}\,dx$	A1 ft	
	$= \tfrac{1}{3}\rho\left[-\dfrac{1}{5x^5}\right]_1^2$	A1	For correct integration of $\dfrac{1}{x^6}$
	$= \dfrac{31}{480}\rho = \dfrac{31}{480}\times 48$		
	$= 3.1\text{ kg m}^2$	A1 **8**	

4 (i)	$RC = 2a\cos\theta$	B1	or $RC^2 = 2a^2 + 2a^2\cos 2\theta$
	EPE $= \frac{5mg}{2a}(2a\cos\theta)^2$	M1	
	GPE $= mga\sin 2\theta + 2mg(2a\sin 2\theta)$	M1	One term sufficient for M1
	$V = 10mga\cos^2\theta + 5mga\sin 2\theta$	A1	
	$\frac{dV}{d\theta} = -20mga\cos\theta\sin\theta + 10mga\cos 2\theta$ $= -10mga\sin 2\theta + 10mga\cos 2\theta$	B1	Correct differentiation of $\cos^2\theta$ (or $\cos 2\theta$) and $\sin 2\theta$
	For equilibrium, $10mga(\cos 2\theta - \sin 2\theta) = 0$ $\tan 2\theta = 1$	M1	For using $\frac{dV}{d\theta} = 0$
	$\theta = \frac{1}{8}\pi$	A1 **7**	Accept $22\frac{1}{2}°$, 0.393
(ii)	$\frac{d^2V}{d\theta^2} = -20mga\cos 2\theta - 20mga\sin 2\theta$	B1 ft	
	When $\theta = \frac{1}{8}\pi$, $\frac{d^2V}{d\theta^2}$ $(= -20\sqrt{2}\,mga) < 0$	M1	Determining the sign of V''
	Hence the equilibrium is unstable	A1 **3**	Correctly shown
	OR Other method for determining whether V has a maximum or a minimum — M1 Correct determination — A1 ft Equilibrium is unstable — A1		Correctly shown
5 (i)	$I = \frac{1}{3}(20)(0.3^2 + 0.9^2) + 20\times 0.9^2$	M1 M1	MI of lamina about any axis Use of parallel (or perp) axes rule
	$= 22.2\ \text{kg m}^2$	A1 (ag) **3**	Correctly obtained
	OR $I = \frac{1}{3}\times 20\times 0.3^2 + \frac{4}{3}\times 20\times 0.9^2$ — M1M1 $= 22.2\ \text{kg m}^2$ — A1		As above
(ii)	Total moment is $20\times 9.8\times 0.9\cos\theta - 44.1$	M1	
	Angular acceleration is zero when moment is zero	M1	
	$\cos\theta = \frac{44.1}{20\times 9.8\times 0.9} = 0.25$	A1 (ag) **3**	
(iii)	Maximum angular speed when $\cos\theta = 0.25$ $\theta = 1.318$	M1	
	Work done against couple is 44.1×1.318	A1	
	By work energy principle,	M1	Equation involving work, KE and PE
	$\frac{1}{2}I\omega^2 = 20\times 9.8\times 0.9\sin\theta - 44.1\theta$	A1 ft	
	$\omega = 3.19\ \text{rad s}^{-1}$	A1 **5**	

6 (i)	As viewed from P 		
	$\sin\alpha = \dfrac{1790}{7400}$	M1	
	$\alpha = 14.0°$		
	Bearing of relative velocity is $50 - \alpha = 036°$	A1 (ag)	
	or $50 + \alpha = 064°$	B1 ft	For 64 or ft $50+\alpha$
		3	
(ii)	Velocity diagram 	B1	Correct diagram *(may be implied)*
	$\dfrac{\sin\beta}{7} = \dfrac{\sin 106}{10}$	M1	*Correct triangle must be intended*
	$\beta = 42.3°$	A1	
	Bearing of $\mathbf{v}_Q$ is $36 + \beta = 078.3°$	A1	Accept 78°
		4	
(iii)	$\dfrac{w}{\sin 31.7} = \dfrac{10}{\sin 106}$	M1	*If cosine rule is used, M1 also requires an attempt at solving the quadratic*
	$w = 5.47 \text{ m s}^{-1}$	A1	
		2	
	Alternative for (ii) and (iii)		
	$\begin{pmatrix} w\sin 36 \\ w\cos 36 \end{pmatrix} = \begin{pmatrix} 10\sin\theta \\ 10\cos\theta \end{pmatrix} - \begin{pmatrix} 7\sin 110 \\ 7\cos 110 \end{pmatrix}$ B1		
	Obtaining an equation in θ only, and solving it M1		e.g. $10\sin\theta - 7.2654\cos\theta = 8.3173$
	$\theta = 78.3°$ A2		or A1A1 if another angle found first
	Obtaining an equation in w only, and solving it M1		
	$w = 5.47 \text{ m s}^{-1}$ A1		
(iv)	$QC = \sqrt{7400^2 - 1790^2} = 7180$ m	M1	(Or M2 for other complete method for finding the time)
	Time taken is $\dfrac{7180}{5.468}$	M1	For attempt at relative distance $\div\, w$ (not awarded for $7400 \div w$)
	$= 1310$ s	A1 ft	or 21.9 minutes ft is $7180 \div w$
		3	
(v)	Bearing of CP is $90 + 36 = 126°$	B1	
		1	

7 (i)	$I = \frac{1}{3}m(3a)^2 + m(2a)^2$ $= 7ma^2$ $mg(2a\sin\theta) = I\alpha$ $\alpha = \dfrac{2g\sin\theta}{7a}$	M1 A1 M1 A1 **4**	Using parallel axes rule
(ii)	By conservation of energy $\frac{1}{2}I\omega^2 = mg(2a\cos\frac{1}{3}\pi - 2a\cos\theta)$ $\frac{7}{2}ma^2\omega^2 = mga(1-2\cos\theta)$ $\omega = \sqrt{\dfrac{2g(1-2\cos\theta)}{7a}}$	M1 A1 A1 (ag) **3**	Equation involving KE and PE Need to see how $\frac{1}{3}\pi$ is used Correctly obtained
(iii)	$mg\cos\theta - R = m(2a\omega^2)$ $R = mg\cos\theta - \frac{4}{7}mg(1-2\cos\theta)$ $= \frac{1}{7}mg(15\cos\theta - 4)$	M1 A1 A1	For radial acceleration $r\omega^2$
	$mg\sin\theta - S = m(2a\alpha)$ $S = mg\sin\theta - \frac{4}{7}mg\sin\theta$ $= \frac{3}{7}mg\sin\theta$	M1 A1 A1 **6**	For transverse acceleration $r\alpha$
	OR $S(2a) = I_G\alpha = (3ma^2)\alpha$ M1A1 $S = \frac{3}{7}mg\sin\theta$ A1		Must use I_G
(iv)	When $\cos\theta = \frac{1}{3}$, $\sin\theta = \frac{\sqrt{8}}{3}$, $\tan\theta = \sqrt{8}$ $R = \frac{1}{7}mg$, $S = \frac{\sqrt{8}}{7}mg$ Angle with R is $\tan^{-1}\dfrac{S}{R} = \tan^{-1}\sqrt{8} = \theta$ so the resultant force is vertical Magnitude is $\sqrt{R^2 + S^2}$ $= \frac{1}{7}mg\sqrt{1+8} = \frac{3}{7}mg$	M1 A1 M1 A1 **4**	
	OR When resultant force is F vertically upwards $S = F\sin\theta$, hence $F = \frac{3}{7}mg$ M1A1 $R = F\cos\theta$, so $\frac{1}{7}mg(15\cos\theta - 4) = \frac{3}{7}mg\cos\theta$ M1 $\cos\theta = \frac{1}{3}$ A1		
	OR Horizontal force is $R\sin\theta - S\cos\theta$ $= \frac{1}{7}mg(15\cos\theta - 4)\sin\theta - \frac{3}{7}mg\sin\theta\cos\theta$ M1 $= \frac{1}{7}mg\sin\theta(12\cos\theta - 4)$ $= 0$ when $\cos\theta = \frac{1}{3}$ A1 Vertical force is $R\cos\theta + S\sin\theta$ $= \frac{1}{7}mg \times \frac{1}{3} + \frac{3}{7}mg \times \frac{8}{9} = \frac{3}{7}mg$ M1A1		