OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Mechanics 3

4730

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Monday 25 January 2010
Morning

Duration: 1 hour 30 minutes

* 4 7 3 0 *

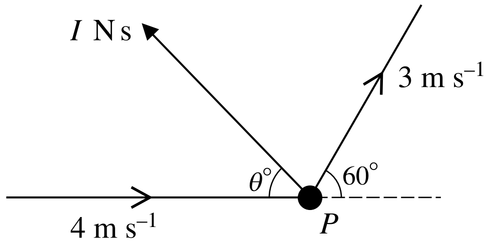
INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

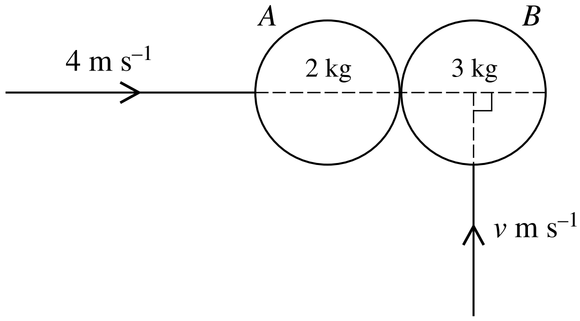
- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1



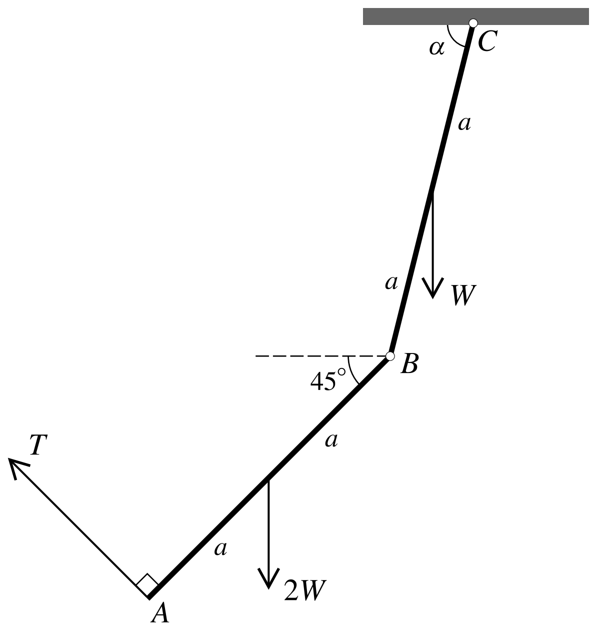
A particle P of mass 0.4 kg is moving horizontally with speed 4 m s^{-1} when it receives an impulse of magnitude I N s, in a direction which makes an angle $(180 - \theta)°$ with the direction of motion of P. Immediately after the impulse acts P moves horizontally with speed 3 m s^{-1}. The direction of motion of P is turned through an angle of 60° by the impulse (see diagram). Find I and θ. **[7]**

2



Two uniform smooth spheres A and B, of equal radius, have masses 2 kg and 3 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision, A has speed 4 m s^{-1} and is moving along the line of centres, and B has speed v m s^{-1} and is moving perpendicular to the line of centres (see diagram). The coefficient of restitution is 0.6. The direction of motion of B after the collision makes an angle of 45° with the line of centres. Find the value of v. **[7]**

3



Two uniform rods AB and BC, each of length $2a$, have weights $2W$ and W respectively. The rods are freely jointed to each other at B, and BC is freely jointed to a fixed point at C. The rods are held in equilibrium in a vertical plane by a light string attached to A and perpendicular to AB. The rods AB and BC make angles $45°$ and α, respectively, with the horizontal. The tension in the string is T (see diagram).

(i) By taking moments about B for AB, show that $W = \sqrt{2}T$. **[3]**

(ii) Find the value of $\tan\alpha$. **[6]**

4 A particle P of mass 0.2 kg travels in a straight line on a horizontal surface. It passes through a point O on the surface with speed $2\,\text{m s}^{-1}$. A resistive force of magnitude $0.2(v + v^2)$ N acts on P in the direction opposite to its motion, where $v\,\text{m s}^{-1}$ is the speed of P when it is at a distance x m from O.

(i) Show that $\frac{1}{1+v}\frac{dv}{dx} = -1$. **[3]**

(ii) By solving the differential equation in part **(i)** show that $\frac{-e^x}{3-e^x}\frac{dx}{dt} = -1$, where t s is the time taken for P to travel x m from O. **[5]**

(iii) Hence find the value of t when $x = 1$. **[3]**

5 A light elastic string of natural length 1.6 m has modulus of elasticity 120 N. One end of the string is attached to a fixed point O and the other end is attached to a particle P of weight 1.5 N. The particle is released from rest at the point A, which is 2.1 m vertically below O. It comes instantaneously to rest at B, which is vertically above O.

(i) Verify that the distance AB is 4 m. **[4]**

(ii) Find the maximum speed of P during its upward motion from A to B. **[7]**

6

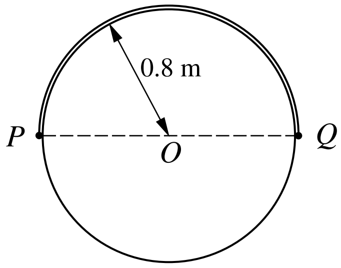


Fig. 1

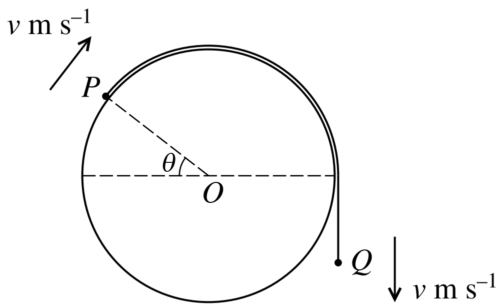


Fig. 2

A light inextensible string of length 0.8π m has particles P and Q, of masses 0.4 kg and 0.58 kg respectively, attached to its ends. The string passes over a smooth horizontal cylinder of radius 0.8 m, which is fixed with its axis horizontal and passing through a fixed point O. The string is held at rest in a vertical plane perpendicular to the axis of the cylinder, with P and Q at opposite ends of the horizontal diameter of the cylinder through O (see Fig. 1). The string is released and Q begins to descend. When OP has rotated through θ radians, with P remaining in contact with the cylinder, the speed of each particle is $v\,\mathrm{m\,s^{-1}}$ (see Fig. 2).

(i) By considering the total energy of the system, obtain an expression for v^2 in terms of θ. **[5]**

(ii) Show that the magnitude of the force exerted on P by the cylinder is $(7.12\sin\theta - 4.64\theta)$ N. **[4]**

(iii) Given that P leaves the surface of the cylinder when $\theta = \alpha$, show that $1.53 < \alpha < 1.54$. **[4]**

7 A particle P of mass 0.5 kg is attached to one end of each of two identical light elastic strings of natural length 1.6 m and modulus of elasticity 19.6 N. The other ends of the strings are attached to fixed points A and B on a line of greatest slope of a smooth plane inclined at 30° to the horizontal. The distance AB is 4.8 m and A is higher than B.

(i) Find the distance AP for which P is in equilibrium on the line AB. **[5]**

P is released from rest at a point on AB where both strings are taut. The strings remain taut during the subsequent motion of P and t seconds after release the distance AP is $(2.5 + x)$ m.

(ii) Use Newton's second law to obtain an equation of the form $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = kx$. State the property of the constant k for which the equation indicates that P's motion is simple harmonic, and find the period of this motion. **[5]**

(iii) Given that $x = 0.5$ when $t = 0$, find the values of x for which the speed of P is $2.8\,\mathrm{m\,s^{-1}}$. **[4]**

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1		M1	For using I = Δmv in one direction
	$0.4(3\cos 60^o - 4) = -I\cos\theta$ (= -1)	A1	
	$0.4(3\sin 60^o) = I\sin\theta$ (= 1.03920)	A1	SR: Allow B1 (max 1/3) for $3\cos 60^o - 4 = -I\cos\theta$ and $3\sin 60^o = I\sin\theta$
	[$\tan\theta = -1.5\sqrt{3}/(1.5-4)$; $I^2 = 0.4^2[(1.5-4)^2 + (1.5\sqrt{3})^2]$]	M1	For eliminating I or θ (allow following SR case)
	θ = 46.1 or I = 1.44	A1	Allow for θ (only) following SR case.
		M1	For substituting for θ or for I (allow following SR case)
	I = 1.44 or θ = 46.1	A1ft [7]	ft incorrect θ or I; allow for θ (only) following SR case.
	Alternatively		
	$I^2 = 1.2^2 + 1.6^2 - 2\times1.2\times1.6\cos 60^o$ or ${}'V'^2 = 3^2 + 4^2 - 2\times3\times4\cos 60^o$	M1	For use of cosine rule
		A1	
		M1	For correct use of factor 0.4 (= m)
	I = 1.44	A1	
		M1	For use of sine rule
	$\frac{\sin\theta}{3(or 1.2)} = \frac{\sin 60}{\sqrt{13}(or 2.08)}$ or		
	$\frac{\sin\alpha}{4(or 1.6)} = \frac{\sin 60}{\sqrt{13}(or 2.08)}$ and $\theta = 120 - \alpha$	A1ft	α must be angle opposite 1.6; (α = 73.9) ft value of I or 'V'
	θ = 46.1	A1 [7]	
2		M1	For using the principle of conservation of momentum
	2a + 3b = 2×4	A1	
		M1	For using NEL
	b – a = 0.6×4	A1	
	[2(b - 2.4) + 3b = 8]	M1	For eliminating a
	b = 2.56	A1	
	v = 2.56	B1ft [7]	ft v = b
3(i)		M1	For using 'mmt of 2W = mmt of T'
	$2W(a\cos 45^o) = T(2a)$	A1	
	$W = \sqrt{2}\,T$	A1 [3]	AG
(ii)	Components (H, V) of force on BC at B are $H = -T/\sqrt{2}$ and $V = T/\sqrt{2} - 2W$	B1	
		M1	For taking moments about C for BC
	W(a cosα) + H(2a sinα) = V(2a cosα)	A1	
	[$W\cos\alpha - T\sqrt{2}\sin\alpha = T\sqrt{2}\cos\alpha - 4W\cos\alpha$]	M1	For substituting for H and V and reducing equation to the form X sinα = Y cosα
	$T\sqrt{2}\sin\alpha = (5W - T\sqrt{2})\cos\alpha$	A1ft	
	tan α = 4	A1 [6]	

	Alternatively for part **(ii)** anticlockwise mmt = W(a cosα) +2W(2a cosα + a cos45°) = T[2a cos(α – 45°) + 2a] [5W cosα + $\sqrt{2}$ W = T($\sqrt{2}$ cosα + $\sqrt{2}$ sinα) + 2] T$\sqrt{2}$ sin α = (5W - T$\sqrt{2}$) cos α tan α = 4	M1 A1 A1 M1 A1ft A1 [6]	For taking moments about C for the whole For reducing equation to the form X sinα = Y cosα
4(i)	[-0.2(v + v^2) = 0.2a] [v dv/dx = -(v + v^2) [1/(1 + v)] dv/dx = -1	M1 M1 A1 [3]	For using Newton's second law For using a = v dv/dx AG
(ii)	 ln (1 + v) = -x (+ C) ln(1+ v) = -x + ln3 [(1 + dx/dt)/3 = e^{-x} ➔ dx/dt = 3e^{-x} -1 ➔e^x dx/dt = 3 – e^x] [-e^x/(3 – e^x)] dx/dt = -1	M1 A1 A1 M1 A1 [5]	For integrating For transposing for v and using v = dx/dt AG
(iii)	[ln(3 – e^x) = -t + ln2] ln(3 – e^x) = -t + ln2 Value of t is 1.96 (or ln{2 ÷ (3 – e)}	M1 A1 A1 [3]	For integrating and using x(0) = 0
5(i)	 Loss of EE = 120(0.5^2 – 0.3^2)/(2×1.6) and gain in PE = 1.5×4 v = 0 at B and loss of EE = gain in PE (= 6) ➔distance AB is 4m	M1 A1 M1 A1 [4]	For using EE = λx^2/2L and PE = Wh For comparing EE loss and PE gain AG
(ii)	[120e/1.6 = 1.5] e = 0.02 Loss of EE = 120(0.5^2 – 0.02^2)/(2×1.6) (or 120(0.3^2 – 0.02^2)/(2×1.6)) Gain in PE = 1.5(2.1- 1.6 – 0.02) (or 1.5(1.9 + 1.6 + 0.02) loss) [KE at max speed = 9.36 – 0.72 (or 3.36 + 5.28)] ½ (1.5/9.8)v^2 = 9.36 – 0.72 Maximum speed is 10.6 ms^{-1}	M1 A1 B1ft B1ft M1 A1 A1 [7]	For using T = mg and T = λx/L ft incorrect e only ft incorrect e only For using KE at max speed = Loss of EE – Gain (or + loss) in PE
	First alternative for **(ii)** x is distance AP [½ (1.5/9.8)v^2 + 1.5x + 120(0.5 – x)^2/3.2 = 120 x0.5^2/3.2] KE and PE terms correct EE terms correct v^2 = 470.4x – 490x^2 [470.4 – 980x = 0] x = 0.48 Maximum speed is 10.6 ms^{-1}	 M1 A1 A1 A1 M1 A1 A1	 For using energy at P = energy at A For attempting to solve dv^2/dx = 0

	Second alternative for **(ii)** [120e/1.6 = 1.5] e = 0.02 [1.5 – 120(0.02 + x)/1.6 = 1.5 $\ddot{x}$/g] n = $\sqrt{490}$ a = 0.48 Maximum speed is 10.6 ms^{-1}	M1 A1 M1 M1 A1 A1 A1	For using T = mg and T = λx/L For using Newton's second law For obtaining the equation in the form $\ddot{x}$ = $-n^2x$, using (AB – L – e_{equil}) for amplitude and using v_{max} = na.
6(i)	PE gain by P = 0.4g × 0.8 sin θ PE loss by Q = 0.58g × 0.8 θ ½ (0.4 + 0.58)v^2 = g × 0.8(0.58 θ –0.4sin θ) v^2 = 9.28 θ - 6.4sin θ	B1 B1 M1 A1ft A1 [5]	 For using KE gain = PE loss AEF
(ii)	 0.4g sin θ – R = 0.4v^2/0.8 [0.4g sin θ – R = 4.64 θ – 3.2 sin θ] R = 7.12 sin θ – 4.64 θ	M1 A1 M1 A1 [4]	For applying Newton's second law to P and using a = v^2/r For substituting for v^2 AG
(iii)	R(1.53) = 0.01(48...), R(1.54) = -0.02(9...) or simply R(1.53) > 0 and R(1.54) < 0 R(1.53) × R(1.54) < 0 ➔1.53 < α < 1.54	M1 A1 M1 A1 [4]	For substituting 1.53 and 1.54 into R(θ) For using the idea that if R(1.53) and R(1.54) are of opposite signs then R is zero (and thus P leaves the surface) for some value of θ between 1.53 and 1.54. AG
7(i)	 T_{AP} = 19.6e/1.6 and T_{BP} = 19.6(1.6-e)/1.6 0.5g sin30° + 12.25(1.6 – e) = 12.25e Distance AP is 2.5m	M1 A1 M1 A1ft A1 [5]	For using T = λe/L For resolving forces parallel to the plane
(ii)	Extensions of AP and BP are 0.9 + x and 0.7 – x respectively 0.5g sin 30° + 19.6(0.7 – x)/1.6 – 19.6(0.9 + x)/1.6 = 0.5 $\ddot{x}$ $\ddot{x}$ = -49x Period is 0.898 s	B1 B1ft B1 M1 A1 [5]	 AG For stating k < 0 and using T = 2π/$\sqrt{-k}$
(iii)	 $2.8^2 = 49(0.5^2 – x^2)$ x^2 = 0.09 x = 0.3 and -0.3	M1 A1ft A1 A1ft [4]	For using $v^2 = \omega^2(A^2 – x^2)$ where ω^2 = -k ft incorrect value of k May be implied by a value of x ft incorrect value of k or incorrect value of x^2 (stated)