OCR
Oxford Cambridge and RSA

Wednesday 13 May 2015 – Morning

A2 GCE MATHEMATICS

4729/01 Mechanics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4729/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A cyclist travels along a straight horizontal road. The total mass of the cyclist and her bicycle is 80 kg and the resistance to motion is a constant 60 N.

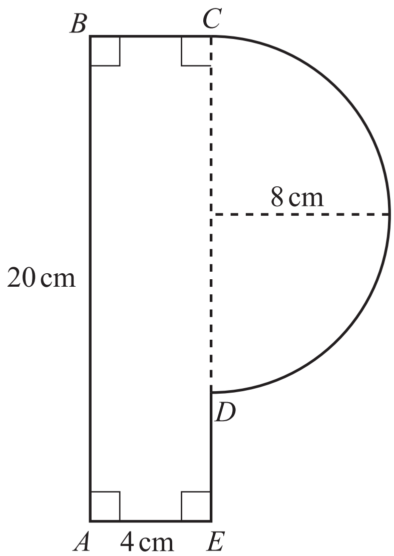
(i) The cyclist travels at a constant speed working at a constant rate of 480 W. Find the speed at which she travels. **[3]**

(ii) The cyclist now instantaneously increases her power to 600 W. After travelling at this power for 14.2 s her speed reaches 9.4 m s^{-1}. Find the distance travelled at this power. **[4]**

2 A particle of mass 0.3 kg is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point A. The particle moves in a horizontal circle of radius 0.343 m, with centre vertically below A, at a constant angular speed of 6 rad s^{-1}. Find the tension in the string and the angle at which the string is inclined to the vertical. **[6]**

3 A car of mass 1500 kg travels along a straight horizontal road with its engine working at a constant rate of P W. There is a constant resistance to motion of R N. Points A and B are on the road. At point A the car's speed is 16 m s^{-1} and its acceleration is 0.3875 m s^{-2}. At point B the car's speed is 25 m s^{-1} and its acceleration is 0.2 m s^{-2}. Find the values of P and R. **[6]**

4



A uniform solid prism has cross-section $ABCDE$ in the shape of a rectangle measuring 20 cm by 4 cm joined to a semicircle of radius 8 cm as shown in the diagram. The centre of mass of the solid lies in this cross-section.

(i) Find the distance of the centre of mass of the solid from AB. **[5]**

The solid is placed with AE on rough horizontal ground (so the object does not slide) and is in equilibrium with a horizontal force of magnitude 4 N applied along CB.

(ii) Find the greatest and least possible values for the weight of the solid. **[5]**

5 A small sphere of mass 0.2 kg is projected vertically downwards with a speed of $5\,\mathrm{m\,s^{-1}}$ from a height of 1.6 m above horizontal ground. It hits the ground and rebounds vertically upwards coming to instantaneous rest at a height of h m above the ground. The coefficient of restitution between the sphere and the ground is 0.7.

(i) Find h. **[4]**

(ii) Find the magnitude and direction of the impulse exerted on the sphere by the ground. **[3]**

(iii) Find the loss of energy of the sphere between the instant of projection and the instant it comes to instantaneous rest at height h m. **[3]**

6 A particle is projected with speed $v\,\mathrm{m\,s^{-1}}$ from a point O on horizontal ground. The angle of projection is $\theta°$ above the horizontal. At time t seconds after the instant of projection the horizontal displacement of the particle from O is x m and the upward vertical displacement from O is y m.

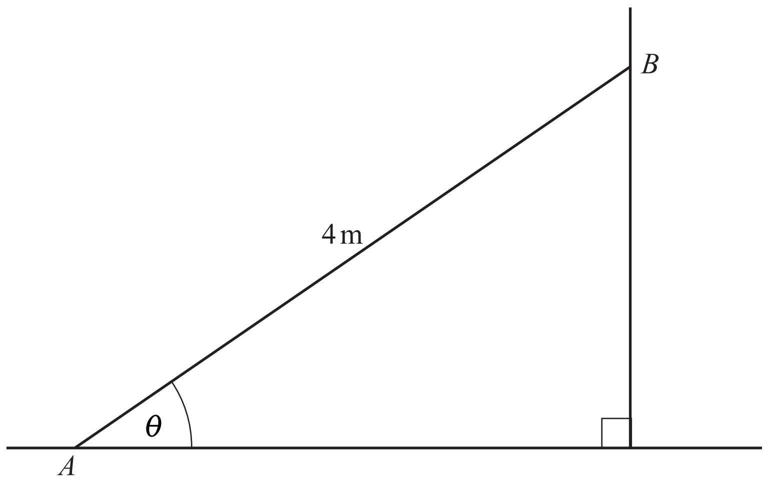
(i) Show that

$$y = x\tan\theta - \frac{4.9x^2}{v^2\cos^2\theta}.$$ **[4]**

A stone is thrown from the top of a vertical cliff 100 m high. The initial speed of the stone is $16\,\mathrm{m\,s^{-1}}$ and the angle of projection is $\theta°$ to the horizontal. The stone hits the sea 40 m from the foot of the cliff.

(ii) Find the two possible values of θ. **[6]**

7



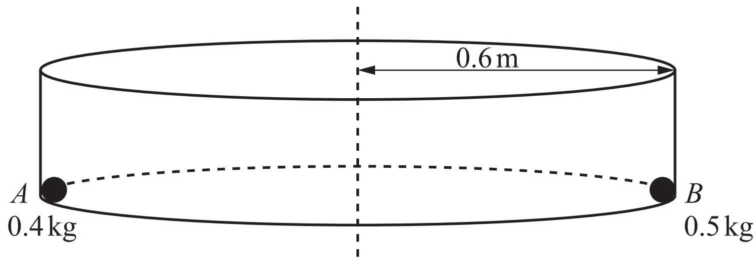
A uniform ladder AB of weight W N and length 4 m rests with its end A on rough horizontal ground and its end B against a smooth vertical wall. The ladder is inclined at an angle θ to the horizontal where $\tan\theta = \frac{1}{2}$ (see diagram). A small object S of weight $2W$ N is placed on the ladder at a point C, which is 1 m from A. The coefficient of friction between the ladder and the ground is μ and the system is in limiting equilibrium.

(i) Show that $\mu = \frac{2}{3}$. **[6]**

A small object of weight aW N is placed on the ladder at its mid-point and the object S of weight $2W$ N is placed on the ladder at its lowest point A.

(ii) Given that the system is in equilibrium, find the set of possible values of a. **[5]**

8



Two small spheres, A and B, are free to move on the inside of a smooth hollow cylinder, in such a way that they remain in contact with both the curved surface of the cylinder and its horizontal base. The mass of A is 0.4 kg, the mass of B is 0.5 kg and the radius of the cylinder is 0.6 m (see diagram). The coefficient of restitution between A and B is 0.35. Initially, A and B are at opposite ends of a diameter of the base of the cylinder with A travelling at a constant speed of $v\,\mathrm{m\,s^{-1}}$ and B stationary. The magnitude of the force exerted on A by the curved surface of the cylinder is 6 N.

(i) Show that $v = 3$. **[2]**

(ii) Calculate the speeds of the particles after A's first impact with B. **[6]**

Sphere B is removed from the cylinder and sphere A is now set in motion with constant angular speed $\omega\,\mathrm{rad\,s^{-1}}$. The magnitude of the total force exerted on A by the cylinder is 4.9 N.

(iii) Find ω. **[4]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance
1	(i)		$D = 480/v$ $480/v - 60 = 0$ $v = 8$ m s^{-1}	B1 M1 A1 **[3]**	Use of $D = P/v$ Use of N2L with 2 terms to find v
	(ii)		600×14.2 $½(80)(9.4^2 - 8^2)$ $8520 = 974.4 + 60d$ $d = 126$ m	B1 B1ft M1 A1 **[4]**	WD by cyclist (8520 J) ft v from (i); KE gained (974.4 J); may be implied in energy equation Attempt at energy equation with all terms Exact 125.76
2			$T\cos\theta = 0.3g$ (= 2.94) $T\sin\theta = 0.3(0.343)(6^2)$ (= 3.7044) $T = 4.73$ N $\theta = 51.6°$	B1 M1 A1 M1 A1 A1 **[6]**	Resolve vertically or $T\sin a = 0.3g$; component of T required Attempt at use of N2L with $a = r\omega^2$ or $a = v^2/r$ Resolve horizontally or $T\cos a = 0.3(0.343)(6^2)$; component of T required Attempt to solve for T or θ; component of T required in both equations SC if 38.4° then B1M1A1M1A1A0 5/6 Max
3			$P/16$ or $P/25$ seen $P/16 - R = 1500(0.3875)$ or $P/25 - R = 1500(0.2)$ $P/16 - R = 1500(0.3875)$ and $P/25 - R = 1500(0.2)$ $P = 12\ 500$ $R = 200$	B1 M1 A1 M1 A1 A1 **[6]**	Use of Driving force = P/v Attempt to use N2L once, can be with D at this stage Both equations correct, may have consistent wrong sign with R Attempt to solve for P or R Allow 12.5 kW Allow if –ve, and justified why +ve (eg. comment on direction)
4	(i)		$2(8)\sin(\pi/2)/(3\pi/2)$ $(80 + 32\pi)\ x_G$ $=80(2) + 32\pi(4 + \text{cv(CoM)})$ $x_G = 5(.00)$ cm	B1 M1 A1 A1ft A1 **[5]**	CoM of semi-circle (3.395305….); ($32/3\pi$) Table of values idea to get an equation/expression, using any fixed axis Relative to the axis they are using; ft their CoM (5.004444492 cm)
	(ii)		 $4(20) = W(x_G$ from (i) – 4)	M1 A1ft	Moments about E or A; allow wrong distance for either force ft on cv(x_G); mg OK here; if $x_G < 4$ need (4 - x_G from (i))

Question			Answer	Marks	Guidance
			$4(20) = W(x_G$ from (i))	A1ft	ft on cv(x_G); mg OK here
			(Greatest) $W = 80$ N	A1	79.64601393 from exact x_G; allow anything between 79.6 and 80
			(Least) $W = 16$ N	A1	15.98579026 from exact x_G
				[5]	SC for masses found only, 4/5 for both 8.13 to 8.16 and 1.63
5	**(i)**		$v^2 = 5^2 + 2g(1.6)$	B1	Complete method to find v (= 7.507…)
			$0.7 \times 7.507\ldots$ (= 5.255…)	B1	$0.7 \times$ cv(v), **but not cv(v) = 5**; may be seen in (ii)
			$(0.7 \times \text{cv}(v))^2 = 2gh$	M1	Complete method to find h, with final speed 0; **allow cv(v) = 5** for method
			$h = 1.41$ m	A1	Exact 1.409
				[4]	
	(ii)			M1	Change in momentum found, with relevant velocities ie cv(v) and $0.7 \times$ cv(v) but not cv(v) =5
			$0.2(7.507\ldots)(0.7) - (-0.2)(7.507\ldots)$	A1ft	This may be negative; ft on their v found in (i)
			Impulse = 2.55 N s, upwards	A1	(2.5524…) Must have direction also.
				[3]	
	(iii)			M1	Change in energy found, all energy terms needed and no extra terms
			$0.2(9.8)(1.6) + ½(0.2)(5^2) - 0.2(9.8)(\text{cv}(h))$		This may be negative
			OR $½(0.2)(7.507\ldots^2) - ½(0.2)(0.7 \times 7.507\ldots)^2$	A1ft	
			Loss of energy = 2.87 J	A1	(2.87436 exact); art 2.87; allow -2.87
				[3]	
6	**(i)**		$x = vt\cos\theta$	B1	aef
			$y = vt\sin\theta - ½gt^2$	B1	aef; may see this with t already eliminated
				M1	Eliminate t
			$y = x\tan\theta - \dfrac{4.9x^2}{v^2\cos^2\theta}$	A1	www; **AG**
				[4]	
	(ii)			M1	Attempt to substitute values into trajectory equation
			$-100 = 40\tan\theta - 4.9 \times 40^2/(16^2 \times \cos^2\theta)$	A1	
			$(30.625\tan^2\theta - 40\tan\theta - 69.375 = 0)$	A1	aef; obtain correct quadratic in $\tan\theta$, may be unsimplified

Question			Answer	Marks	Guidance
			($\tan\theta = 2.2937\ldots$ or $-0.9876\ldots$) $\theta = 66.4$ $\theta = -44.6$	M1 A1 A1 **[6]**	Attempt to solve quadratic in $\tan\theta$ Allow 44.6 below the horizontal
7	(i)		 $2W(1\cos\theta) + W(2\cos\theta) =$ $R_B(4\sin\theta)$ $R_A = 3W$ $\mu = 2/3$	*M1 A1 A1 B1 dep*M1 A1 **[6]**	Moments about A, all terms needed and no extra; dimensionally correct; each term must include $\sin\theta$, or $\cos\theta$ or $\tan\theta$ May have Fr for R_B Use of $Fr = \mu R_A$ to get an equation in W (or R_A) only **AG**; must come from exact θ
	OR		 $W(2\cos\theta) + 2W(3\cos\theta)$ $+ Fr(4\sin\theta) = R_A(4\cos\theta)$ $R_A = 3W$ $\mu = 2/3$	*M1 A1 A1 B1 dep*M1 A1 **[6]**	Moments about B, all terms needed and no extra; dimensionally correct; each term must include $\sin\theta$, or $\cos\theta$ or $\tan\theta$ Use of $Fr = \mu R_A$ to get an equation in W (or R_A) only **AG**; must come from exact θ
	(ii)		 $(W + aW)(2\cos\theta) = R_B(4\sin\theta)$ $R_A = 3W + aW$ $a \le 3$	*M1 A1 B1 dep*M1 A1 **[5]**	Moments about A, all terms needed and no extra; dimensionally correct; each term must include $\sin\theta$, or $\cos\theta$ or $\tan\theta$ May have Fr for R_B Resolve vertically $Fr \le \mu R_A$ to get an inequality in W and a only; allow equality here Allow a < 3.If using equality, correct inequality need not be justified.
	OR		For first 2 marks $(W + aW)(2\cos\theta)+2W(4\cos\theta)+ Fr(4\sin\theta)=R_A(4\cos\theta)$	*M1 A1	Moments about B, all terms needed and no extra; dimensionally correct; each term must include $\sin\theta$, or $\cos\theta$ or $\tan\theta$
8	(i)		 $0.4v^2/0.6 = 6$, $v = 3$	M1 A1 **[2]**	Attempt at use of N2L with $a = v^2/r$ or $a = r\omega^2$; allow verification. **AG** If +/-3 then -3 must be rejected for A1.
	(ii)			*M1	Attempt at use of conservation of linear momentum

Question			Answer	Marks	Guidance
			$3(0.4) = 0.4v_A + 0.5v_B$	A1	
				*M1	Attempt at use of restitution equation, must be correct way round
			$v_B - v_A = 3(0.35)$	A1	Must be consistent with the directions used for CoLM
				dep*M1	Solving simultaneous equations
			$v_A = 0.75$ m s^{-1} and $v_B = 1.8$ m s^{-1}	A1	Both values positive as final answer.
				[6]	
	(iii)		$Y = 0.4g$ (= 3.92)	B1	Resolve vertically
			$(4.9)^2 - (0.4g)^2$ (= 2.94^2)	B1	Use of Pythagoras
			$0.4(0.6)\omega^2 = \text{cv}(2.94)$	M1	Resolve horizontally, cv(2.94) from a legitimate attempt at 2.94
			$\omega = 3.5$	A1	Exact
				[4]	
	OR		$4.9\cos\theta = 0.4g$	B1	Resolve vertically, θ angle with vertical
			$\cos\theta = 0.8$ or $\theta = 36.9$	B1	
			$0.4(0.6)\omega^2 = 4.9\sin\theta$	M1	Resolve horizontally, θ substituted and from a legitimate attempt
			$\omega = 3.5$	A1	Exact
				[4]	
	OR		$Y = 0.4g$ (= 3.92)	B1	Resolve vertically
			$X = 0.4(0.6)\omega^2$ (= $0.24\omega^2$)	B1	Resolve horizontally
			$(0.4g)^2 + (0.4(0.6)\omega^2)^2 = 4.9^2$	M1	Attempt at use of Pythagoras, from a legitimate attempt at X
			$\omega = 3.5$	A1	Exact
				[4]	