OCR

RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

MATHEMATICS

4729/01

Mechanics 2

WEDNESDAY 20 JUNE 2007

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

[Turn over

1 A man drags a sack at constant speed in a straight line along horizontal ground by means of a rope attached to the sack. The rope makes an angle of 35° with the horizontal and the tension in the rope is 40 N. Calculate the work done in moving the sack 100 m. [3]

2 Calculate the range on a horizontal plane of a small stone projected from a point on the plane with speed $12\,\mathrm{m\,s^{-1}}$ at an angle of elevation of 27°. [4]

3 A rocket of mass 250 kg is moving in a straight line in space. There is no resistance to motion, and the mass of the rocket is assumed to be constant. With its motor working at a constant rate of 450 kW the rocket's speed increases from $100\,\mathrm{m\,s^{-1}}$ to $150\,\mathrm{m\,s^{-1}}$ in a time t seconds.

(i) Calculate the value of t. [4]

(ii) Calculate the acceleration of the rocket at the instant when its speed is $120\,\mathrm{m\,s^{-1}}$. [4]

4 A ball is projected from a point O on the edge of a vertical cliff. The horizontal and vertically upward components of the initial velocity are $7\,\mathrm{m\,s^{-1}}$ and $21\,\mathrm{m\,s^{-1}}$ respectively. At time t seconds after projection the ball is at the point (x, y) referred to horizontal and vertically upward axes through O. Air resistance may be neglected.

(i) Express x and y in terms of t, and hence show that $y = 3x - \frac{1}{10}x^2$. [5]

The ball hits the sea at a point which is 25 m below the level of O.

(ii) Find the horizontal distance between the cliff and the point where the ball hits the sea. [3]

5 A cyclist and her bicycle have a combined mass of 70 kg. The cyclist ascends a straight hill AB of constant slope, starting from rest at A and reaching a speed of $4\,\mathrm{m\,s^{-1}}$ at B. The level of B is 6 m above the level of A. For the cyclist's motion from A to B, find

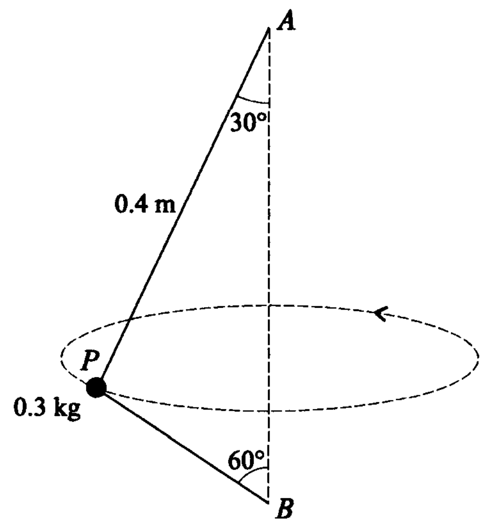
(i) the increase in kinetic energy, [2]

(ii) the increase in gravitational potential energy. [2]

During the ascent the resistance to motion is constant and has magnitude 60 N. The work done by the cyclist in moving from A to B is 8000 J.

(iii) Calculate the distance AB. [4]

6



A particle P of mass 0.3 kg is attached to one end of each of two light inextensible strings. The other end of the longer string is attached to a fixed point A and the other end of the shorter string is attached to a fixed point B, which is vertically below A. AP makes an angle of 30° with the vertical and is 0.4 m long. PB makes an angle of 60° with the vertical. The particle moves in a horizontal circle with constant angular speed and with both strings taut (see diagram). The tension in the string AP is 5 N.

Calculate

(i) the tension in the string PB, [3]

(ii) the angular speed of P, [3]

(iii) the kinetic energy of P. [3]

7 Two small spheres A and B, with masses 0.3 kg and m kg respectively, lie at rest on a smooth horizontal surface. A is projected directly towards B with speed $6\,\text{m}\,\text{s}^{-1}$ and hits B. The direction of motion of A is reversed in the collision. The speeds of A and B after the collision are $1\,\text{m}\,\text{s}^{-1}$ and $3\,\text{m}\,\text{s}^{-1}$ respectively. The coefficient of restitution between A and B is e.

(i) Show that $m = 0.7$. [2]

(ii) Find e. [2]

B continues to move at $3\,\text{m}\,\text{s}^{-1}$ and strikes a vertical wall at right angles. The coefficient of restitution between B and the wall is f.

(iii) Find the range of values of f for which there will be a second collision between A and B. [2]

(iv) Find, in terms of f, the magnitude of the impulse that the wall exerts on B. [3]

(v) Given that $f = \frac{3}{4}$, calculate the final speeds of A and B, correct to 1 decimal place. [7]

[Question 8 is printed overleaf.]

8

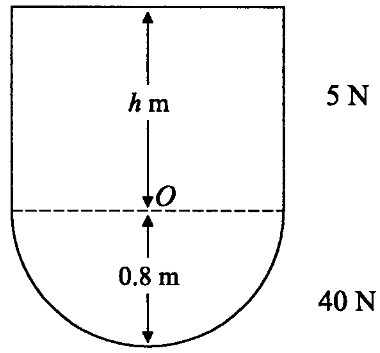


Fig. 1

An object consists of a uniform solid hemisphere of weight 40 N and a uniform solid cylinder of weight 5 N. The cylinder has height h m. The solids have the same base radius 0.8 m and are joined so that the hemisphere's plane face coincides with one of the cylinder's faces. The centre of the common face is the point O (see Fig. 1). The centre of mass of the object lies inside the hemisphere and is at a distance of 0.2 m from O.

(i) Show that $h = 1.2$. [6]

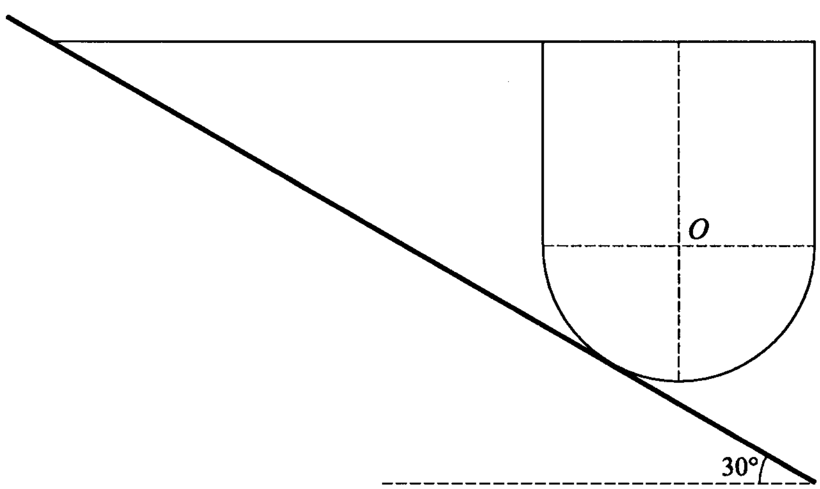


Fig. 2

One end of a light inextensible string is attached to a point on the circumference of the upper face of the cylinder. The string is horizontal and its other end is tied to a fixed point on a rough plane. The object rests in equilibrium on the plane with its axis of symmetry vertical. The plane makes an angle of 30° with the horizontal (see Fig. 2). The tension in the string is T N and the frictional force acting on the object is F N.

(ii) By taking moments about O, express F in terms of T. [4]

(iii) Find another equation connecting T and F. Hence calculate the tension and the frictional force. [6]

1	40 cos35°	B1	
	WD = 40cos35° x 100	M1	
	3280 J	A1 **3**	ignore units **3**

2	$0 = 12\sin27°t - 4.9t^2$ any correct.	M1	**or** $R = u^2\sin2\theta/g$ (B2)
	t = 1.11 …..method for total time	A1	correct formula only
	R = 12cos27° x t	M1	12^2 x sin54° / 9.8 sub in values
	11.9	A1 **4**	11.9 **4**

3 (i)	WD = $½x250x150^2 – ½x250x100^2$	M1	
	1 560 000	A1	1 562 500
	450 000 = 1 560 000/t	M1	
	3.47	A1 **4**	
(ii)	F = 450 000/120	M1	
	3750	A1	
	3750 = 250a	M1	
	15 ms^{-2}	A1 **4**	**8**

4 (i)	$x = 7t$	B1	
	$y = 21t - 4.9t^2$	M1	**or** – g/2
		A1	
	$y = 21.x/7 - 4.9\,x^2/49$	M1	
	$y = 3x - x^2/10$	A1 **5**	**AG**
(ii)	$-25 = 3x - x^2/10$ (must be -25)	M1	**or** method for total time (5.26)
	solving quadratic	M1	**or** 7 x total time
	36.8 m	A1 **3**	**8**

5(i)	$½ . 70 .4^2$	M1	
	560 J	A1 **2**	
(ii)	70 x 9.8 x 6	M1	
	4120	A1 **2**	4116
(iii)	60d	B1	
	8000 = 560 + 4120 + 60d	M1	4 terms
		A1 √	√ their KE and PE
	55.4 m	A1 **4**	**8**

6 (i)	5cos30° = 0.3x9.8 + Scos60°	M1	res. vertically (3 parts with comps)
		A1	
	2.78 N	A1 **3**	
(ii)	r = 0.4sin30° = 0.2	B1	may be on diagram
	5sin30° + Ssin60° =0.3 x 0.2 x ω^2	M1	res. horizontally (3 parts with comps)
	9.04 rads^{-1}	A1 **3**	
(iii)	v = 0.2 x 9.04	M1	**or** previous v via mv^2/r
	KE = ½ x 0.3 x $(0.2 \text{ x } 9.04)^2$	M1	
	0.491 J or 0.49	A1√ **3**	√ their ω^2 x 0.006 **9**

7 (i)	1.8 = –0.3 + 3m	M1	
	m = 0.7	A1 **2**	**AG**
(ii)	e = 4/6	M1	accept 2/6 for M1
	2/3	A1 **2**	accept 0.67
(iii)	± 3f	B1	
	1/3 👍 f (🛇 1)	B1 **2**	
(iv)	I = 3f x 0.7 – – 3 x 0.7	M1	ok for only one minus sign for M1
		A1	
	I = 2.1 (f + 1)	A1 **3**	aef 2 marks only for -2.1(f + 1)
(v)	0.3 + 6.3/4 = 0.3*a* + 0.7*b*	M1	can be – 0.7*b*
	3*a* + 7*b* = 18.75	A1 *	aef
	2/3 = (*a* – *b*)/ 5/4	M1	allow e=3/4 or their e for M1
	3*a* – 3*b* = 5/2	A1 *	aef * means dependent.
	solve	M1	
	a = 2.5	A1	(2.46) allow ± (59/24)
	b = 1.6	A1 **7**	(1.625) allow ± (13/8) **16**

8 (i)	com of hemisphere 0.3 from *O*	B1	**or** 0.5 from base
	com of cylinder *h*/2 from *O*	B1	
	0.6x45 = 40x0.5 + (0.8+*h*/2) x 5 **or**	M1	**or** 40x0.3 – 5xh/2 = 45 x 0.2
	45(h+0.2) = 5h/2 +40(h+0.3)	A1	**or** 5(0.2 + h/2) = 40x0.1
	27 = 20 + (0.8+*h*/2) x 5	M1	solving
	h = 1.2	A1 **6**	**AG**
(ii)	1.2 T	B1	
	0.8 F	B1	
	0.8F = 1.2T	M1	
	F = 3T/2	A1 **4**	aef
(iii)	F + Tcos30°	B1	**or** 45 x 0.8 sin30°
	45sin30° must be involved in res.	B1	T x (1.2 + 0.8cos30°)
	resolving parallel to the slope	M1	mom. about point of contact
	F + Tcos30° = 45sin30° aef	A1	45.0.8sin30°=T(1.2+0.8cos30°)
	T = 9.51	A1	
	F = 14.3	A1 **6**	**16**
or	T + Fcos30° = Rsin30°	B1	res. horizontally
(iii)	Rcos30° + Fsin30° = 45	B1	res. vertically
	tan30°=(T+Fcos30°)/(45-Fsin30°)	M1	eliminating R