OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4728

Mechanics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4728
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Monday 24 January 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

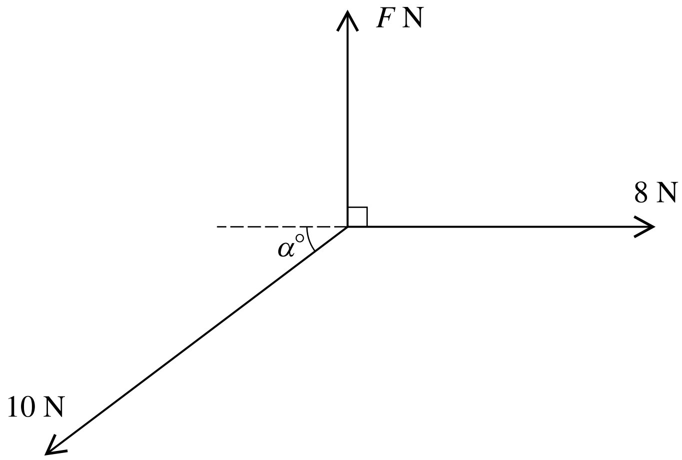
- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 Two particles P and Q are projected directly towards each other on a smooth horizontal surface. P has mass 0.5 kg and initial speed $2.4\,\text{m s}^{-1}$, and Q has mass 0.8 kg and initial speed $1.5\,\text{m s}^{-1}$. After a collision between P and Q, the speed of P is $0.2\,\text{m s}^{-1}$ and the direction of its motion is reversed. Calculate

(i) the change in the momentum of P, **[2]**

(ii) the speed of Q after the collision. **[4]**

2



Three horizontal forces of magnitudes F N, 8 N and 10 N act at a point and are in equilibrium. The F N and 8 N forces are perpendicular to each other, and the 10 N force acts at an obtuse angle $(90 + \alpha)°$ to the F N force (see diagram). Calculate

(i) α, **[3]**

(ii) F. **[3]**

3 A particle is projected vertically upwards with velocity $5\,\text{m s}^{-1}$ from a point 2.5 m above the ground.

(i) Calculate the speed of the particle when it strikes the ground. **[3]**

(ii) Calculate the time after projection when the particle reaches the ground. **[3]**

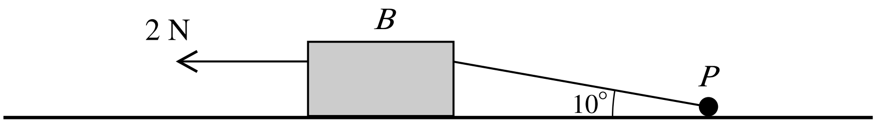
(iii) Sketch on separate diagrams

(a) the (t, v) graph,

(b) the (t, x) graph,

representing the motion of the particle. **[4]**

4

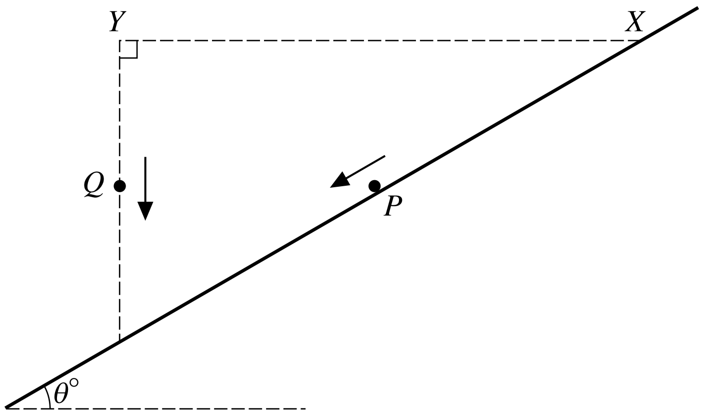


A block B of mass 0.8 kg and a particle P of mass 0.3 kg are connected by a light inextensible string inclined at 10° to the horizontal. They are pulled across a horizontal surface with acceleration $0.2\,\mathrm{m\,s^{-2}}$, by a horizontal force of 2 N applied to B (see diagram).

(i) Given that contact between B and the surface is smooth, calculate the tension in the string. **[3]**

(ii) Calculate the coefficient of friction between P and the surface. **[7]**

5



X is a point on a smooth plane inclined at $\theta°$ to the horizontal. Y is a point directly above the line of greatest slope passing through X, and XY is horizontal. A particle P is projected from X with initial speed $4.9\,\mathrm{m\,s^{-1}}$ down the line of greatest slope, and simultaneously a particle Q is released from rest at Y. P moves with acceleration $4.9\,\mathrm{m\,s^{-2}}$, and Q descends freely under gravity (see diagram). The two particles collide at the point on the plane directly below Y at time T s after being set in motion.

(i) **(a)** Express in terms of T the distances travelled by the particles before the collision. **[3]**

(b) Calculate θ. **[2]**

(c) Using the answers to parts **(a)** and **(b)**, show that $T = \frac{2}{3}$. **[3]**

(ii) Calculate the speeds of the particles immediately before they collide. **[3]**

6 The velocity $v\,\mathrm{m\,s^{-1}}$ of a particle at time t s is given by $v = t^2 - 9$. The particle travels in a straight line and passes through a fixed point O when $t = 2$.

(i) Find the displacement of the particle from O when $t = 0$. **[4]**

(ii) Calculate the distance the particle travels from its position at $t = 0$ until it changes its direction of motion. **[6]**

(iii) Calculate the distance of the particle from O when the acceleration of the particle is $10\,\mathrm{m\,s^{-2}}$. **[5]**

[Question 7 is printed overleaf.]

7 A particle P of mass 0.6 kg is projected up a line of greatest slope of a plane inclined at 30° to the horizontal. P moves with deceleration $10\,\text{m}\,\text{s}^{-2}$ and comes to rest before reaching the top of the plane.

(i) Calculate the frictional force acting on P, and the coefficient of friction between P and the plane. **[7]**

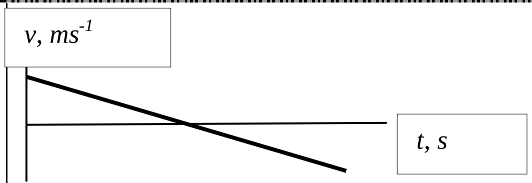
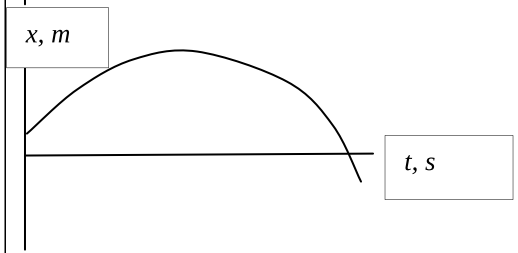
(ii) Find the magnitude of the contact force exerted on P by the plane and the angle between the contact force and the upward direction of the line of greatest slope,

(a) when P is in motion, **[5]**

(b) when P is at rest. **[2]**

1 i	ΔMom P = 0.5(2.4 + 0.2) ΔMom P = +/-1.3 $kgms^{-1}$	M1 A1 [2]	+/- 0.5(2.4 ± 0.2)	MR P/Q +/-0.8(1.5+/-0.2) M1A0
ii	Momentum before = 0.5x2.4 – 0.8x1.5 0.5x2.4+/ – 0.8x1.5 = +/-(-0.5x0.2 +/- 0.8v) Speed = 0.125 ms^{-1} OR ΔMom Q =+/- (+/-0.8v – 0.8x1.5) 1.3 = +/-(0.8v – 0.8x1.5) Speed = 0.125 ms^{-1}	B1 M1 A1ft A1 [4] B1 M1 A1ft A1	+/-(0.5x2.4 – 0.8x1.5) Uses mom before = mom after Cv(Expression for before momentum) 1/8, +ve (not 0.13) Uses ΔMom P = ΔMom Q Cv(ans(i)) = +/-(+/-0.8v – 0.8x1.5) 1/8, +ve (not 0.13)	Cont MR 0.5x2.4-0.8x1.5 Uses mom before = mom after 0.5x2.4+/-0.8x1.5 = +/-(0.8x0.2+/-0.5v) 0.32 B1 M1A1A1 ft

2 i	10CorSα = 8 10cosα = 8 α = 36.9° OR 10CorSα = F 10sinα = 6 α = 36.9° OR tanθ = F/8 tanα = 6/8 α = 36.9°	M1 A1 A1 [3] M1 A1ft A1 M1 A1ft A1	Component of 10 = 8 Accept 37 36.8 and 37 from 36.7 Using value of F(ii) Using F(=6) from (ii) OR tanθ = 8/F, using value of F from (ii)	CorS is Cos or Sin (passim) Do not accept 36.7
ii	 F = 10sin36.9 F = 6 N OR $F^2 + 8^2 = 10^2$ F = 6 N	M1 A1ft A1 [3] M1 A1 A1	F = 10CorSα Allow 10Cos53.1 Accept 6.01 (or from 10Cos53.1) or 6.0 Pythagoras, 3 squared terms	 anything rounding to 6.0 from correct working. Accept $F^2 = 8^2 + 10^2$

3 i	 $v^2 = (+/-5)^2 + 2x9.8x2.5$ Speed (or v) = 8.6(0) ms^{-1} OR $0 = 5^2 – 2x9.8xs$ with $v^2 = (0) +2x9.8(s+2.5)$ $v^2 = 2x9.8x(2.5+1.28)$ Speed = 8.6(0) ms^{-1}	M1 A1 A1 [3] M1 A1 A1	Uses $v^2 = u^2 \pm 2gs$, u non-zero Accept √74 Do not accept -8.6(0) s = 1.2755... 19.8x3.7755.. Or rounds to 8.6	It is common to see the upwards and downwards motion treated separately. Both parts must be attempted for M1, and both parts must be attempted accurately with cvs for the A1
ii	 8.6 = -5 + 9.8t Time = 1.39 s OR $9.8t^2 - 10t - 5 = 0$ Time = 1.39 s OR 2.5 = (8.6-5)t/2 Time = 1.39 s OR t = 5/9.8 + 8.6/9.8 Time = 1.39	M1 A1ft A1 [3] M1 A1 A1 M1 A1ft A1 M1 A1ft A1	Uses v(from (i)) = +/-5 +/- 9.8t Cv(8.60 from (i)) $+/-2.5 = 5t +/- gt^2/2$ 2.5 = +/- (5 - Speed from (i)) x t / 2 Cv(8.60 from (i)) Times to top and ground found and added Cv(8.60 from (i))	It is common to see the upwards and downwards motion treated separately. Both parts must be attempted for M1, and both parts must be attempted accurately with cvs for the A1
iii a) b)	 	B1 B1 B1 B1 [4]	Straight descending line to t axis Continues straight below t axis Inverted “parabolic” curve, starts anywhere on t=0 Ends below t = 0 level, need not be below t axis	Ignore values written on diagrams

4 i	2 – F = 0.8x0.2 F = Tcos10 T = 1.87 N OR 2 – Tcos10 = 0.8x0.2 T = 1.87 N	M1 M1 A1 [3] M1 M1 A1	N2L 2 force terms and ma (F = 1.84 N) F = TCorS10 1.8683.. N2L 2 force terms and ma TCorS10	m is the block mass, award if T not F
ii	R - 0.3x9.8 + TCorS10 = 0 R = 0.3x9.8 – 1.87sin10 R = 2.62 Tcos10 – Fr = 0.3x0.2 Fr = 1.78 μ = 1.78 / 2.62 OR 1.78 = 2.62 μ μ = 0.68	M1 A1ft A1ft M1 A1ft M1 A1 [7]	3 term equation, vertically cv(T(i)) 2.61(5..) seen or implied N2L 2 forces for P, component of T cv(T(i)) seen or implied both terms same sign	Treat as a mis-read R-0.8x9.8-TCorS10 = 0 leading to R=8.16 (i.e.works on block[2/3] OR N2L 2 forces for P+Q: 2 – Fr = (0.8+0.3)x0.2 R, Fr unequal to T From correct value of T = 1.87 only

5 ia b c	 s(P) = 4.9T + 0.5x $4.9T^2$ y(Q) = (0) + 0.5x9.8T^2 (m)x4.9 = (m)gsinθ θ = 30 y(Q)/s(P) = sinθ OR y(Q) = s(P) sinθ $0.5x9.8(2/3)^2$ / $(4.9x2/3 + 2.45(2/3)^2$ = 0.5 OR 0.5x9.8T^2 / (4.9T + 2.45T^2) = sin30 T= 2/3 s AG	M1 A1 A1 [3] M1* A1 [2] M1 D*M1 A1 [3]	s=ut+0.5at^2 used along plane or vertically, with u = 4.9 or 0, and a = 4.9 or 9.8 appropriately Accept use of t or T Allow g in Y(Q) Allow CorSθ Uses appropriate trigonometry to relate distances Verification needs explicit value of sin(cv(θib)) Ratio of distances considered using cv (30)	 Sinθ =(0.5x9.8T^2)/(4.9T + 0.5x 4.9T^2) gets M1, but in ic. Beware circular argument. This may appear in b) $0.5x9.8(2/3)^2$ = $(4.9x2/3 + 2.45(2/3)^2$ x 0.5 OR 0.5x9.8T^2 = (4.9T + 2.45T^2) x sin30
ii	v = 4.9 + 4.9x2/3 OR v = (0) + 9.8x2/3 v = 8.17 ms^{-1} w = 9.8x2/3 = 6.53 ms^{-1}	M1 A1 A1 [3]	Uses v = u + at, with appropriate u, a values once 8.2 6.5	

6 i	$x = \int t^2 - 9\,dt$ $x = t^3/3 - 9t\ (+c)$ Finds x(2) Displacement = 15⅓ m OR $x(2) = [t^3/3 - 9t]_0^{\,2}$ Displacement = 15⅓ m	M1* A1 D*M1 B1 [4] D*M1 B1	Uses integration of v(t) Award if +c omitted Allow + c or c omitted Accept 15.3, 46/3. Must be +ve Uses limits$[\,]_0^{\,2}$ on integrated x(t) Must be +ve	 Awarded if c omitted or assumed 0
ii	t=0 s=0 or s=46/3 hence x(0) or c= 0 or 46/3 Solves $t^2 - 9 = 0$ $t = (\pm)3$ $x(3) = 3^3/3 - 9\times3\ (+ 15.3)$ x(3) = -18 (or -2.67) Dist = 18 m	B1* M1* A1 D*M1 M1 D*B1 [6]	Needs explanation, may be seen in part i May be implied Value of t when direction of motion changes Substitutes cv(t) >2 in integrated x(t) Evaluates c – 18 may be implied award if .. Accept 18(.0) [c=0 assumed]	B1* awarded if limits 0 and 3 used correctly Awarded if limits used correctly
iii	$a = d(t^2 - 9)/dt$ a = 2t 10 = 2t t = 5 $x(5)\ (= 5^3/3 - 9\times5 + 15.3) = 12$ m OR $[t^3/3 - 9t]_2^{\,5} = 12$ m	M1* A1 D*M1 A1 A1 [5] A1	Uses differentiation of v(t)	

7 i	Wt cmpts: // plane 0.6gsin30 Perp plane 0.6gcos30 0.6gsin30 +/- X = 0.6x10 X = +/-3.06 μ = 3.06 / 5.09(22..) μ = 0.601 OR 3.06 = μ x 5.09(22..) μ = 0.601	B1 B1 M1 A1ft A1 M1 A1 [7] M1 A1	+/-2.94 +/-5.09(22.) = R N2L // plane, 2 force terms and ma (allow no g) Both weight cmpt and accn signs same May be implied (Fr =0.6x10-0.6gsin30 used) Uses μ = Fr/R both terms same sign 0.6 Uses Fr = μR both terms same sign 0.6	 Accept Fr for X Accept Fr = |X| Accept Fr = |X|
ii a)	$C^2 = 3.06^2 + 5.09^2$ C = 5.94 N tanθ = 3.06/5.09(22..) Angle = (31) + 90 Angle = 121° OR tanφ = 5.09(22..)/3.06 Angle = 180 – (59) Angle = 121°	M1 A1 M1* D*M1 A1 [5] M1* D*M1 A1	Pythagoras with Fr and R, to find hypotenuse Accept 5.9, 5.95 but not 6(.0) Or tanθ = μ Not 120 tanφ = 1/ μ Not 120	
b)	C (= 0.6x9.8) = 5.88 N Angle = 60°	B1 B1 [2]	5.9	No working needed as C is vertical No working needed as C is vertical