OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4728

Mechanics 1

Thursday **16 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

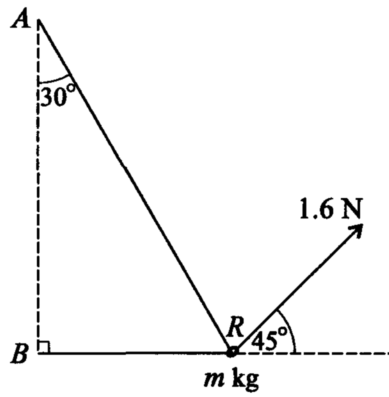
- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1



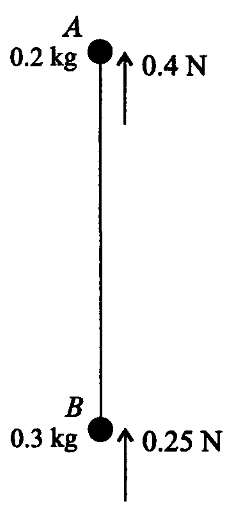
A light inextensible string has its ends attached to two fixed points A and B. The point A is vertically above B. A smooth ring R of mass m kg is threaded on the string and is pulled by a force of magnitude 1.6 N acting upwards at 45° to the horizontal. The section AR of the string makes an angle of 30° with the downward vertical and the section BR is horizontal (see diagram). The ring is in equilibrium with the string taut.

(i) Give a reason why the tension in the part AR of the string is the same as that in the part BR. [1]

(ii) Show that the tension in the string is 0.754 N, correct to 3 significant figures. [3]

(iii) Find the value of m. [3]

2



Particles A and B, of masses 0.2 kg and 0.3 kg respectively, are attached to the ends of a light inextensible string. Particle A is held at rest at a fixed point and B hangs vertically below A. Particle A is now released. As the particles fall the air resistance acting on A is 0.4 N and the air resistance acting on B is 0.25 N (see diagram). The downward acceleration of each of the particles is a m s^{-2} and the tension in the string is T N.

(i) Write down two equations in a and T obtained by applying Newton's second law to A and to B. [4]

(ii) Find the values of a and T. [3]

3 Two small spheres P and Q have masses 0.1 kg and 0.2 kg respectively. The spheres are moving directly towards each other on a horizontal plane and collide. Immediately before the collision P has speed $4\,\text{m}\,\text{s}^{-1}$ and Q has speed $3\,\text{m}\,\text{s}^{-1}$. Immediately after the collision the spheres move away from each other, P with speed $u\,\text{m}\,\text{s}^{-1}$ and Q with speed $(3.5 - u)\,\text{m}\,\text{s}^{-1}$.

(i) Find the value of u. [4]

After the collision the spheres both move with deceleration of magnitude $5\,\text{m}\,\text{s}^{-2}$ until they come to rest on the plane.

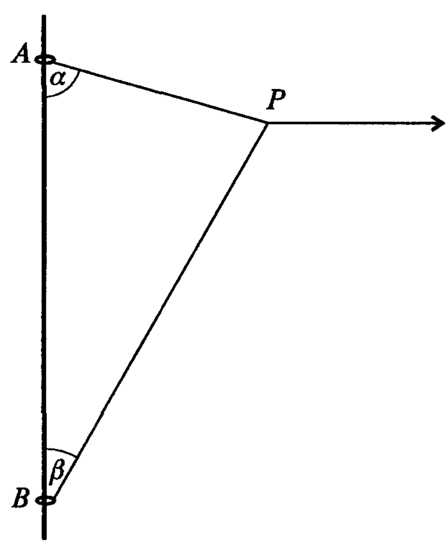
(ii) Find the distance PQ when both P and Q are at rest. [4]

4 A particle moves downwards on a smooth plane inclined at an angle α to the horizontal. The particle passes through the point P with speed $u\,\text{m}\,\text{s}^{-1}$. The particle travels 2 m during the first 0.8 s after passing through P, then a further 6 m in the next 1.2 s. Find

(i) the value of u and the acceleration of the particle, [7]

(ii) the value of α in degrees. [2]

5



Two small rings A and B are attached to opposite ends of a light inextensible string. The rings are threaded on a rough wire which is fixed vertically. A is above B. A horizontal force is applied to a point P of the string. Both parts AP and BP of the string are taut. The system is in equilibrium with angle $BAP = \alpha$ and angle $ABP = \beta$ (see diagram). The weight of A is 2 N and the tensions in the parts AP and BP of the string are 7 N and T N respectively. It is given that $\cos\alpha = 0.28$ and $\sin\alpha = 0.96$, and that A is in limiting equilibrium.

(i) Find the coefficient of friction between the wire and the ring A. [7]

(ii) By considering the forces acting at P, show that $T\cos\beta = 1.96$. [2]

(iii) Given that there is no frictional force acting on B, find the mass of B. [3]

6 A particle of mass 0.04 kg is acted on by a force of magnitude P N in a direction at an angle α to the upward vertical.

(i) The resultant of the weight of the particle and the force applied to the particle acts horizontally. Given that $\alpha = 20°$ find

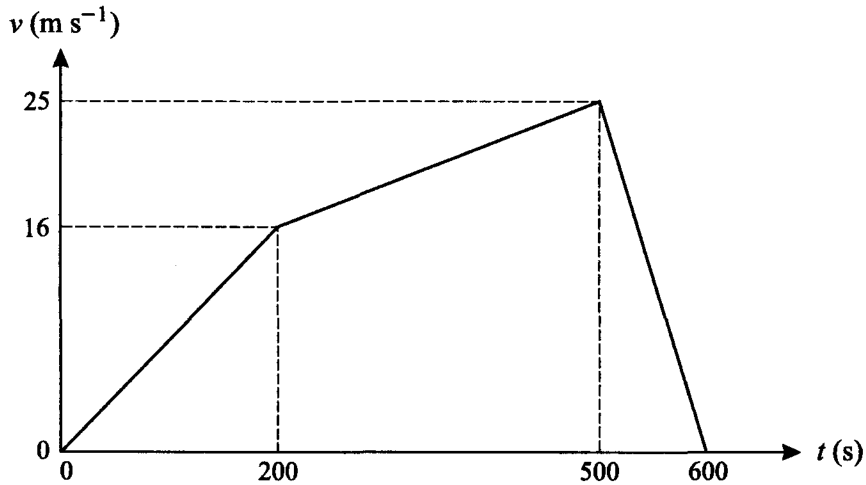
(a) the value of P, [3]

(b) the magnitude of the resultant, [2]

(c) the magnitude of the acceleration of the particle. [2]

(ii) It is given instead that $P = 0.08$ and $\alpha = 90°$. Find the magnitude and direction of the resultant force on the particle. [5]

7



A car P starts from rest and travels along a straight road for 600 s. The (t, v) graph for the journey is shown in the diagram. This graph consists of three straight line segments. Find

(i) the distance travelled by P, [3]

(ii) the deceleration of P during the interval $500 < t < 600$. [2]

Another car Q starts from rest at the same instant as P and travels in the same direction along the same road for 600 s. At time t s after starting the velocity of Q is $(600t^2 - t^3) \times 10^{-6}$ m s^{-1}.

(iii) Find an expression in terms of t for the acceleration of Q. [2]

(iv) Find how much less Q's deceleration is than P's when $t = 550$. [2]

(v) Show that Q has its maximum velocity when $t = 400$. [2]

(vi) Find how much further Q has travelled than P when $t = 400$. [6]

1	(i)	R is smooth	B1 1	
	(ii)	$T + T\cos60° = 1.6\cos45°$ Tension is 0.754 N **AG**	M1 A1 A1 3	For resolving forces horizontally to obtain an equation in T (requires 3 relevant terms and at least one force resolved)
	(iii)	$mg = T\sin60° + 1.6\sin 45°$ $m = 0.182$	M1 A1 ft A1 3	For resolving forces vertically to obtain an equation for m (requires 3 relevant terms with both T and the 1.6 N force resolved) ft sin/cos mix from (ii) SR $m = T\sin60° + 1.6\sin 45°$ M1 $m = 1.78$ B1

2	(i)	$0.2g + T - 0.4 = 0.2a$ $0.3g - T - 0.25 = 0.3a$	M1 A1 A1 A1 4	For applying $F = ma$ (requires at least ma, T and air resistance in linear combination in at least one equation). At least one equation with not more than one error. SR $0.2g - T - 0.4 = 0.2a$ **and** $0.3g + T - 0.25 = 0.3a$ B1
	(ii)	$0.5g - 0.65 = 0.5a$ or $5T - 0.7 = 0$ $a = 8.5$ **and** $T = 0.14$ (positive only)	M1 A1 ft A1 3	For obtaining an equation in T or a only, either by eliminating a or T from the equations in (i) or by applying $F = ma$ to the complete system For a correct equation in a only or T only ft opposite direction of T only

3	(i)	Momentum before=$0.1 \times 4 - 0.2 \times 3$ Momentum after = $-0.1u + 0.2(3.5 - u)$ $0.1 \times 4 - 0.2 \times 3 = -0.1u + 0.2(3.5 - u)$ $u = 3$ (positive value only)	B1 B1 M1 A1 4	or Loss by $P = 0.1 \times 4 + 0.1u$ or Gain by $Q = 0.2(3.5 - u) + 0.2 \times 3$ For using the principle of conservation of momentum
				SR If mgv used for momentum instead of mv, then $u = 3$ B1
	(ii)	 $0 = 3^2 - 10s_1$ and $0 = 0.5^2 - 10s_2$ $0.9 + 0.025$ Distance is 0.925 m cao	M1 A1 ft M1 A1 4	For using $v^2 = u^2 + 2as$ with $v = 0$ (either case) or equivalent equations ft value of u from (i) For using $PQ = s_1 + s_2$

4	(i) α	 $2 = 0.8u + ½ a(0.8)^2$ $8 = 2u + ½ a2^2$ or $6 = 1.2(u + 0.8a) + ½ a(1.2)^2$ or $6 = 1.2(2 \times 2 \div 0.8 - u) + ½ a(1.2)^2$ $u = 1.5$ Acceleration is 2.5 ms^{-2}	M1 A1 M1 A1 M1 A1 A1 7	For using $s = ut + ½ at^2$ for the first stage For obtaining another equation in u and a with relevant values of velocity, displacement and time For eliminating a or u
	(i) β	 $2 = 0.8v - ½ a(0.8)^2$ $6 = 1.2v + ½ a(1.2)^2$ Acceleration is 2.5 ms^{-2} (v = 3.5) $u = 1.5$	M1 A1 M1 A1 M1 A1 A1 7	For using $s = vt - ½ at^2$ for the first stage For using $s = ut + ½ at^2$ for the second stage For obtaining values of a and v and using $v = u + at$ for first stage to find u
	(i) γ	$2 \div 0.8$ ms^{-1} and $6 \div 1.2$ ms^{-1} = 2.5 ms^{-1} and 5ms^{-1} $t_1 = 0.4$ and $t_2 = (0.8 +) 0.6$ $5 = 2.5 + a(1.4 - 0.4)$ Acceleration is 2.5 ms^{-2}	M1 A1 B1 M1 A1	For finding average speeds in both intervals For finding mid-interval times For using $v = u + at$ between the mid-interval times

		$2.5 = u + 2.5 \times 0.4$ or $5 = u + 2.5 \times 1.4$	M1		
		$u = 1.5$	A1	7	For using $v = u + at$ between t = 0 and one of the mid-interval times
	(ii)	$2.5 = 9.8\sin\alpha$ $\alpha = 14.8°$	M1 A1ft	2	For using $(m)a = (m)g\sin\alpha$ ft value of acceleration

5	(i)		M1		For resolving forces on A vertically (3 terms)
		$F = 2 + 7\cos\alpha$ $F = 3.96$ (may be implied)	A1 A1		
		$N = 7\sin\alpha$	M1		For resolving forces on A horizontally (2 terms)
		$N = 6.72$ (may be implied)	A1		
		$3.96 = \mu\, 6.72$	M1		For using $F = \mu N$
		Coefficient is 0.589 or 33/56 cao	A1	7	
	(ii)	$T\cos\beta = 7\cos\alpha$	M1		For resolving forces at P vertically (2 terms)
		$T\cos\beta = 7 \times 0.28$ ($= 1.96$ **AG)**	A1	2	
	(iii)		M1		For resolving forces on B vertically (2 terms)
		$T\cos\beta - mg = 0$	A1		
		Mass is 0.2 kg	A1	3	

6	(i)(a)	$V = P\cos 20° - 0.04g$	B1		
			M1		For setting $V = 0$
		$P = 0.417$	A1	3	
	(i)(b)	$R = P\sin 20°$	M1		For using R = horizontal component of P
		Magnitude is 0.143 N	A1ft	2	ft value of P
	(i)(c)	$0.143 = 0.04a$ Acceleration is 3.57 ms^{-2}	M1 A1ft	2	For using Newton's second law ft magnitude of the resultant
	(ii)	$R^2 = 0.08^2 + (0.04g)^2$	M1		For using $R^2 = P^2 + W^2$
		Magnitude is 0.400 N (or 0.40 or 0.4)	A1		
		$\tan\theta = +/-0.04g/0.08$ or $\tan(90° - \theta) = +/-0.08/0.04g$	M1		For using $\tan\theta = Y/X$ or $\tan(90° - \theta) = X/Y$
		Angle made with horizontal is 78.5° or 1.37 radians, or angle made with vertical is 11.5° or 0.201 radians	A1		
		Downwards or below horizontal	B1	5	Direction may alternatively be shown clearly on a diagram or given as a bearing

7	(i)	$\frac{1}{2}\, 200\times 16 + 300\times \frac{1}{2}(16+25) +$ $\frac{1}{2}\, 100\times 25$ (=1600 + 6150 + 1250) Distance is 9000m	M1 A1 A1	3	For using the idea that the area of the quadrilateral represents distance
	(ii)	$a = (0-25)/(600-500)$ Deceleration is 0.25 ms^{-2}	M1 A1	2	For using the idea that gradient (= vel ÷ time) represents acceleration Or for using v = u + at Allow acceleration = - 0.25 ms^{-2}
	(iii)	Acceleration is $(1200t - 3t^2)\times 10^{-6}$	M1 A1	2	For using $a(t) = \dot{v}(t)$
	(iv)	0.25 – 0.2475 Amount is +/- 0.0025 ms^{-2}	M1 A1ft	2	For using 'ans(ii) – $\lvert a_Q(550)\rvert$' ft ans(ii) only
	(v)	$1200t - 3t^2 = 0$ t = (0 or) 400 **AG**	M1 A1	2	For solving $a_Q(t) = 0$ or for finding $a_Q(400)$ Or for obtaining $a_Q(400) = 0$
	(vi)	$\frac{1}{2}\, 200\times 16 + 200\times \frac{1}{2}(16+22)$ $s_Q(t) = (200t^3 - t^4/4)\times 10^{-6}$ $(+C)$ 6400 – 5400 Distance is 1000 m	M1 A1 M1 A1 M1 A1	6	For correct method for $s_P(400)$ For using $s_Q(t) = \int v_Q\, dt$ For using correct limits and finding $\lvert s_Q(400) - s_P(400)\rvert$