OCR
Oxford Cambridge and RSA

Tuesday 9 June 2015 – Morning

A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4727/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

1 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = \sin x.$$ **[8]**

2 The elements of a group G are polynomials of the form $a + bx + cx^2$, where $a, b, c \in \{0, 1, 2, 3, 4\}$. The group operation is addition, where the coefficients are added modulo 5.

(i) State the identity element. **[1]**

(ii) State the inverse of $3 + 2x + x^2$. **[2]**

(iii) State the order of G. **[1]**

The proper subgroup H contains $2 + x$ and $1 + x$.

(iv) Find the order of H, justifying your answer. **[4]**

3 The plane Π passes through the points $(1, 2, 1)$, $(2, 3, 6)$ and $(4, -1, 2)$.

(i) Find a cartesian equation of the plane Π. **[5]**

The line l has equation $\mathbf{r} = \begin{pmatrix} -1 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$.

(ii) Find the coordinates of the point of intersection of Π and l. **[3]**

(iii) Find the acute angle between Π and l. **[3]**

4 In an Argand diagram, the complex numbers $0, z$ and $ze^{\frac{1}{6}i\pi}$ are represented by the points O, A and B respectively.

(i) Sketch a possible Argand diagram showing the triangle OAB. Show that the triangle is isosceles and state the size of angle AOB. **[4]**

The complex numbers $1 + i$ and $5 + 2i$ are represented by the points C and D respectively. The complex number w is represented by the point E, such that $CD = CE$ and angle $DCE = \frac{1}{6}\pi$.

(ii) Calculate the possible values of w, giving your answers exactly in the form $a + bi$. **[5]**

5 Find the particular solution of the differential equation

$$x\frac{dy}{dx} + 3y = x^2 + x$$

for which $y = 1$ when $x = 1$, giving y in terms of x. **[8]**

6 Find the shortest distance between the lines with equations

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-5}{-1} \quad \text{and} \quad \frac{x-3}{4}=\frac{y-1}{-2}=\frac{z+1}{3}.$$ **[7]**

7 **(i)** Use de Moivre's theorem to show that $\tan 4\theta \equiv \dfrac{4\tan\theta - 4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}$. **[4]**

(ii) Hence find the exact roots of $t^4+4\sqrt{3}t^3-6t^2-4\sqrt{3}t+1=0$. **[5]**

8 Let G be any multiplicative group. H is a subset of G. H consists of all elements h such that $hg = gh$ for every element g in G.

(i) Prove that H is a subgroup of G. **[8]**

Now consider the case where G is given by the following table:

	e	p	q	r	s	t
e	e	p	q	r	s	t
p	p	q	e	s	t	r
q	q	e	p	t	r	s
r	r	t	s	e	q	p
s	s	r	t	p	e	q
t	t	s	r	q	p	e

(ii) Show that H consists of just the identity element. **[4]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			AE: $\lambda^2+4\lambda+13=0$ $\lambda=-2\pm 3\mathrm{i}$ CF: $\mathrm{e}^{-2x}\left(A\cos 3x+B\sin 3x\right)$ PI: $y=a\cos x+b\sin x$ $y'=-a\sin x+b\cos x,\ y''=-a\cos x-b\sin x$ in DE: $-a\cos x-b\sin x+4(-a\sin x+b\cos x)$ $+13\left(a\cos x+b\sin x\right)=\sin x$ $12a+4b=0$ $12b-4a=1$ $a=-\frac{1}{40},\ b=\frac{3}{40}$ GS: $y=\frac{1}{40}\left(3\sin x-\cos x\right)+\mathrm{e}^{-2x}\left(A\cos 3x+B\sin 3x\right)$	M1 A1 A1ft B1 M1 M1 A1 A1ft **[8]**	condone $Ae^{(-2+3i)x}+Be^{(-2-3i)x}$ Differentiate twice and substitute Compare	ft on complex λ only If wrong trial function can only gain a maximum of the next M1 and must use correct method to differentiate it ft must be of form y = "their CF+PI" and of form " $a\cos x+b\sin x$ with a or b nonzero plus standard CF form" with 2 constants and not in complex exponential form
2	(i)		0	B1 **[1]**	accept $0+0x+0x^2$	
2	(ii)		$2+3x+4x^2$	M1 A1 **[2]**	for 2 correct terms	
2	(iii)		125	B1 **[1]**	or 5^3	
2	(iv)		more than five elements are shown to be generated so $\|H\|>5$ $\|H\|$ is a factor of 125 proper so $\|H\|<125$ $\|H\|=25$	B1 B1 B1 B1 **[4]**	e.g. elements generated by $1+x$ are $\{1+x, 2+2x, 3+3x, 4+4x, 0\}$ which does not include $2+x$ or order subgroups 1, 5, 25 or 125 or order is (1), 5, 25	Insufficient to just reference Lagrange alone penalise use of H instead of $\|H\|$

Question			Answer	Marks	Guidance	
3	(i)		vectors in plane $\begin{pmatrix}2\\3\\6\end{pmatrix}-\begin{pmatrix}1\\2\\1\end{pmatrix}=\begin{pmatrix}1\\1\\5\end{pmatrix}$ and $\begin{pmatrix}4\\-1\\2\end{pmatrix}-\begin{pmatrix}1\\2\\1\end{pmatrix}=\begin{pmatrix}3\\-3\\1\end{pmatrix}$	M1*	or multiple(s)	or multiple of $\begin{pmatrix}-2\\4\\4\end{pmatrix}$
			$\begin{pmatrix}1\\1\\5\end{pmatrix}\times\begin{pmatrix}3\\-3\\1\end{pmatrix}=\begin{pmatrix}16\\14\\-6\end{pmatrix}=2\begin{pmatrix}8\\7\\-3\end{pmatrix}$	M1dep* A1	for **M1**, method shown or 2 correct elements	
			$\mathbf{r}\bullet\begin{pmatrix}8\\7\\-3\end{pmatrix}=\begin{pmatrix}1\\2\\1\end{pmatrix}\bullet\begin{pmatrix}8\\7\\-3\end{pmatrix}$	M1		
			$8x+7y-3z=19$	A1 **[5]**	AEF (Cartesian)	
3	(ii)		$x=-1+4\lambda, y=-2+3\lambda, z=6-2\lambda$			
			$8(-1+4\lambda)+7(-2+3\lambda)-3(6-2\lambda)=19$ $\Rightarrow \lambda=1$	M1 M1	solves and attempts substitution	
			intersect at (3, 1, 4)	A1 **[3]**	Accept vector form	
3	(iii)		$\cos(\alpha)=\dfrac{\left\|\begin{pmatrix}8\\7\\-3\end{pmatrix}\bullet\begin{pmatrix}4\\3\\-2\end{pmatrix}\right\|}{\sqrt{8^2+7^2+3^2}\sqrt{4^2+3^2+2^2}}=\dfrac{59}{\sqrt{122}\sqrt{29}}$	M1*		can be implied by 7.3° or 0.13 or $\cos\alpha=0.9919$ seen
			$\theta=\frac{1}{2}\pi-\alpha$	M1dep*	can use $\sin\theta$	consistent use of degrees or radians
			$\theta\approx1.44$ or $\theta\approx82.7°$	A1 **[3]**		
3						

Question			Answer	Marks	Guidance	
4	(i)		Diagram	B1		must have triangle where B is anticlockwise from A, looks isosceles, AOB < $\frac{\pi}{4}$, if axes labelled then must be correct
			$OB = \lvert z\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}\rvert = \lvert z\rvert\lvert \mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}\rvert = \lvert z\rvert .1 = \lvert z\rvert = OA$	M1		condone $OB = \lvert z\rvert = OA$
			So triangle is isosceles oe	A1	without contradictions	
			$\angle AOB = \frac{1}{6}\pi$	B1	or 30°	Can be just on diagram
				[4]		
4	(ii)		$w = (1+\mathrm{i}) + \left((5+2\mathrm{i}) - (1+\mathrm{i})\right)\mathrm{e}^{\pm\frac{1}{6}\pi\mathrm{i}}$	M1	Rotation of CD	Condone omission of ± in M marks
				M1 A1	Translation of attempted CE	
			$w = \frac{1}{2} + 2\sqrt{3} + \left(3 + \frac{1}{2}\sqrt{3}\right)\mathrm{i}$	M1	converts $\mathrm{e}^{\pm\frac{1}{6}\pi\mathrm{i}}$ into $a + bi$ form	
			or $\frac{3}{2} + 2\sqrt{3} + \left(-1 + \frac{1}{2}\sqrt{3}\right)\mathrm{i}$	A1		
				[5]		
			Alternative method: $CE = \begin{pmatrix} a \\ b \end{pmatrix}, CD = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$. Now use $CE \cdot CD = 17\cos(\pi/6)$ and $CE^2 = 17$			
			to obtain equations $4a + b = 17\sqrt{3}/2$ and $a^2 + b^2 = 17$ (or equivalent)	M1 A1	(for both).	
			Obtain 3-term quadratic in one variable and solve to get one correct value of a or b	M1	Quadratics are $a^2 - 4\sqrt{3}a + 47/4 = 0$ and $b^2 - \sqrt{3}b - 13/4 = 0$	
			$(a,b) = (2\sqrt{3} \pm \frac{1}{2}, \frac{1}{2}\sqrt{3} \mp 2)$ Final answer	A1 A1 **[5]**		

Question			Answer	Marks	Guidance	
5			$\frac{dy}{dx}+\frac{3}{x}y=x+1$	B1	Divide both sides by x	
			$I=\exp\left(\int \frac{3}{x}dx\right)=e^{3\ln x}$	M1		
			$=x^3$	A1		A0 means no further marks can be gained
			$x^3\frac{dy}{dx}+3x^2y=x^4+x^3$ $\frac{d}{dx}\left(x^3y\right)=\cdots$ $\cdots=x^4+x^3$	M1	Multiply and recognise derivative	
				M1	Integrate both sides (their two term polynomial)	condone absent A at this stage
			$x^3y=\frac{1}{5}x^5+\frac{1}{4}x^4+A$	A1		
			$x=1, y=1 \Rightarrow A=\frac{11}{20}$	M1	Use condition	
			$y=\frac{1}{5}x^2+\frac{1}{4}x+\frac{11}{20}x^{-3}$	A1		
				[8]		
6			$\begin{pmatrix}2\\3\\-1\end{pmatrix}\times\begin{pmatrix}4\\-2\\3\end{pmatrix}=\begin{pmatrix}7\\-10\\-16\end{pmatrix}$	M1* M1dep*	Direction vectors of lines Vector product	 condone 1 error
				A1		
			$\begin{pmatrix}3\\1\\-1\end{pmatrix}-\begin{pmatrix}1\\-2\\5\end{pmatrix}=\begin{pmatrix}2\\3\\-6\end{pmatrix}$	M1 A1	Vector between lines	
				M1	Component of their vector in their direction	
			shortest distance $=\frac{\left\lvert\begin{pmatrix}2\\3\\-6\end{pmatrix}\cdot\begin{pmatrix}7\\-10\\-16\end{pmatrix}\right\rvert}{\sqrt{7^2+10^2+16^2}}=\frac{80}{\sqrt{405}}\left(=\frac{16\sqrt{5}}{9}\right)$	A1	or 3.98	
				[7]		

Question			Answer	Marks	Guidance	
			Alternative method after 1st three marks: Forms general vector between lines, equates to $k(7i-10j-16k)$ solves to $k=16/81$ then shortest dist = $k\lvert 7i-10j-16k\rvert$ $=\frac{80}{\sqrt{405}}\left(=\frac{16\sqrt{5}}{9}\right)$	M1* A1 M1dep* A1 **[7]**	or 3.98	
7	(i)		$\cos 4\theta+\mathrm{i}\sin 4\theta=(\cos\theta+\mathrm{i}\sin\theta)^4$	B1	soi by at least $\tan 4\theta=\frac{im((\cos\theta+i\sin\theta)^4)}{re((\cos\theta+i\sin\theta)^4)}$	
			$=c^4+4\mathrm{i}c^3s-6c^2s^2-4\mathrm{i}cs^3+s^4$	B1		Can be broken down already but with i's in place
			Taking re and im parts $\cos 4\theta=c^4-6c^2s^2+s^4$ $\sin 4\theta=4c^3s-4cs^3$	M1	take real and imaginary parts	
			$\tan 4\theta=\frac{4\tan\theta-4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}$	A1 **[4]**	AG. Must show division of numerator and denominator by c^4 and must have been explicit about re and im	
	(ii)		Rearranging polynomial gives $1-6t^2+t^4=\sqrt{3}(4t-4t^3)$	M1		
			so $\tan 4\theta=\frac{1}{\sqrt{3}}$	A1		
			$4\theta=$ their "$\frac{1}{6}\pi$"$+n\pi$	B1		
			$t=\tan\theta=\tan\frac{1}{24}\pi,\ \tan\frac{7}{24}\pi,\ \tan\frac{13}{24}\pi,\ \tan\frac{19}{24}\pi$	B1	one correct	condone all angles seen and no extras, but t not given as equal to tan θ
				B1 **[5]**	all correct or (4) equivalent	

Question			Answer	Marks	Guidance	
8	(i)		$eg = ge$ so $e \in H$	B1	Showing identity in H	
			$hg = gh$ $\Rightarrow g = h^{-1}gh$ $\Rightarrow gh^{-1} = h^{-1}g$ $\Rightarrow h^{-1} \in H$	 M1 M1 A1		
			$h_1h_2g = h_1gh_2$ $= gh_1h_2$ so $h_1h_2 \in H$, so H closed	M1 M1 A1		
			so H is a subgroup of G	A1	For completing argument without considering other properties of H.	
				[8]		
	(ii)		Correctly evaluates first g_1g_2	B1*		where g_1, g_2 distinct and $\neq e$
			$g_1g_2 \neq g_2g_1$ for one correct pair	M1		
			$g_1g_2 \neq g_2g_1$ for sufficient pairs to cover all 5 elements and conclude that they are not in H	A1		
			so $H = \{e\}$	A1dep*	Complete argument	
				[4]		