RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4727

Further Pure Mathematics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Monday 24 May 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 The line l_1 passes through the points (0, 0, 10) and (7, 0, 0) and the line l_2 passes through the points (4, 6, 0) and (3, 3, 1). Find the shortest distance between l_1 and l_2. **[7]**

2 A multiplicative group with identity e contains distinct elements a and r, with the properties $r^6 = e$ and $ar = r^5a$.

(i) Prove that $rar = a$. **[2]**

(ii) Prove, by induction or otherwise, that $r^n a r^n = a$ for all positive integers n. **[4]**

3 In this question, w denotes the complex number $\cos\frac{2}{5}\pi + \mathrm{i}\sin\frac{2}{5}\pi$.

(i) Express w^2, w^3 and w^* in polar form, with arguments in the interval $0 \leqslant \theta < 2\pi$. **[4]**

(ii) The points in an Argand diagram which represent the numbers

$$1, \quad 1 + w, \quad 1 + w + w^2, \quad 1 + w + w^2 + w^3, \quad 1 + w + w^2 + w^3 + w^4$$

are denoted by A, B, C, D, E respectively. Sketch the Argand diagram to show these points and join them in the order stated. (Your diagram need not be exactly to scale, but it should show the important features.) **[4]**

(iii) Write down a polynomial equation of degree 5 which is satisfied by w. **[1]**

4 **(i)** Use the substitution $y = xz$ to find the general solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - y = x\cos\left(\frac{y}{x}\right),$$

giving your answer in a form without logarithms. (You may quote an appropriate result given in the List of Formulae (MF1).) **[6]**

(ii) Find the solution of the differential equation for which $y = \pi$ when $x = 4$. **[2]**

5 Convergent infinite series C and S are defined by

$$C = 1 + \tfrac{1}{2}\cos\theta + \tfrac{1}{4}\cos 2\theta + \tfrac{1}{8}\cos 3\theta + \dots\ ,$$
$$S = \tfrac{1}{2}\sin\theta + \tfrac{1}{4}\sin 2\theta + \tfrac{1}{8}\sin 3\theta + \dots\ .$$

(i) Show that $C + \mathrm{i}S = \dfrac{2}{2 - \mathrm{e}^{\mathrm{i}\theta}}$. **[4]**

(ii) Hence show that $C = \dfrac{4 - 2\cos\theta}{5 - 4\cos\theta}$, and find a similar expression for S. **[4]**

6 **(i)** Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 17y = 17x + 36.$$ **[7]**

(ii) Show that, when x is large and positive, the solution approximates to a linear function, and state its equation. **[2]**

7 A line l has equation $\mathbf{r} = \begin{pmatrix} -7 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$. A plane Π passes through the points (1, 3, 5) and (5, 2, 5), and is parallel to l.

(i) Find an equation of Π, giving your answer in the form $\mathbf{r.n} = p$. **[4]**

(ii) Find the distance between l and Π. **[4]**

(iii) Find an equation of the line which is the reflection of l in Π, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. **[4]**

8 A set of matrices M is defined by

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\quad B = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix},\quad C = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix},\quad D = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\quad E = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix},\quad F = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix},$$

where ω and ω^2 are the complex cube roots of 1. It is given that M is a group under matrix multiplication.

(i) Write down the elements of a subgroup of order 2. **[1]**

(ii) Explain why there is no element X of the group, other than A, which satisfies the equation $X^5 = A$. **[2]**

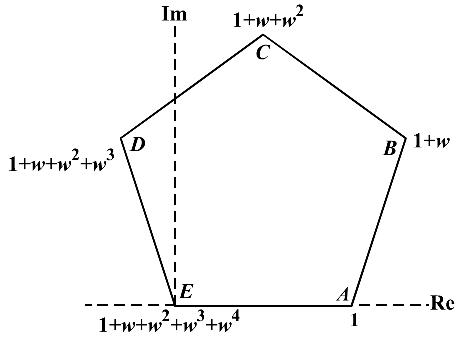
(iii) By finding BE and EB, verify the closure property for the pair of elements B and E. **[4]**

(iv) Find the inverses of B and E. **[3]**

(v) Determine whether the group M is isomorphic to the group N which is defined as the set of numbers {1, 2, 4, 8, 7, 5} under multiplication modulo 9. Justify your answer clearly. **[3]**

1	Direction of $l_1 = k[7, 0, -10]$ Direction of $l_2 = k[1, 3, -1]$	B1	For both directions
	EITHER $\mathbf{n} = [7, 0, -10] \times [1, 3, -1]$ *OR* $\begin{cases} [x, y, z]\,.\,[7, 0, -10] = 0 \ \Rightarrow\ 7x - 10z = 0 \\ [x, y, z]\,.\,[1, 3, -1] = 0 \ \Rightarrow\ x + 3y - z = 0 \end{cases}$	M1	For finding vector product of directions of l_1 and l_2 *OR* for using 2 scalar products and obtaining equations
	$\Rightarrow \mathbf{n} = k[10, -1, 7]$	A1	For correct **n**
	METHOD 1 Vector $(\mathbf{a} - \mathbf{b})$ from l_1 to $l_2 = \pm[4, 6, -10]$		
	OR $\pm[-4, 3, 1]$ *OR* $\pm[3, 3, -9]$ *OR* $\pm[-3, 6, 0]$	B1	For a correct vector
	$d = \dfrac{\lvert(\mathbf{a}-\mathbf{b})\,.\,\mathbf{n}\rvert}{\lvert\mathbf{n}\rvert} = \dfrac{36}{\sqrt{150}}$	M1*	For finding $(\mathbf{a} - \mathbf{b})\,.\,\mathbf{n}$
		M1 (*dep)	For $\lvert\mathbf{n}\rvert$ in denominator *OR* for using $\hat{\mathbf{n}}$
	$d = \frac{6}{5}\sqrt{6} \approx 2.94$	A1 **7**	For correct distance **AEF**
	METHOD 2 Planes containing l_1 and l_2 perp. to **n**	M1*	For finding planes and $p_1 - p_2$ seen
	are $\mathbf{r}\,.[10, -1, 7] = p_1 = 70$, $\mathbf{r}\,.[10, -1, 7] = p_2 = 34$	B1	For $p_1 = 70k$ and $p_2 = 34k$
	$\Rightarrow d = \dfrac{\lvert 70 - 34\rvert}{\sqrt{150}} = \dfrac{36}{\sqrt{150}} = \frac{6}{5}\sqrt{6} \approx 2.94$	M1 (*dep)	For $\lvert\mathbf{n}\rvert$ in denominator *OR* for using $\hat{\mathbf{n}}$
		A1	For correct distance **AEF**
	METHOD 3 $\mathbf{r}_1 = [7\lambda, 0, 10 - 10\lambda]$ *OR* $[7 + 7\lambda, 0, -10\lambda]$ $\mathbf{r}_2 = [4 + \mu, 6 + 3\mu, -\mu]$ *OR* $[3 + \mu, 3 + 3\mu, 1 - \mu]$	B1	For correct points on l_1 and l_2 using different parameters
	$\begin{array}{rc\|c\|c\|c\|c\|} 7\lambda + 10\alpha - \mu & = & 4 & -3 & 3 & -4 \\ -\alpha - 3\mu & = & 6 & 6 & 3 & 3 \\ -10\lambda + 7\alpha + \mu & = & -10 & 0 & -9 & 1 \end{array}$ $\Rightarrow \alpha = -\frac{6}{25}$	M1*	For setting up 3 linear equations from $\mathbf{r}_1 + \alpha\mathbf{n} = \mathbf{r}_2$ and solving for α
	$\lvert\mathbf{n}\rvert = \sqrt{150}$	M1 (*dep)	For $\lvert\mathbf{n}\rvert$ seen multiplying α
	$\Rightarrow d = \frac{6}{25}\sqrt{150} = \frac{6}{5}\sqrt{6} \approx 2.94$	A1	For correct distance **AEF**
		$\boxed{7}$	

2	**(i)**	$a\,r = r^5 a \Rightarrow r\,a\,r = r^6 a$	M1		Pre-multiply $a\,r = r^5 a$ by r
		$r^6 = e \Rightarrow r\,a\,r = a$	A1	**2**	Use $r^6 = e$ and obtain answer **AG**
	(ii)	METHOD 1			
		For $n = 1$, $r\,a\,r = a$ *OR* For $n = 0$, $r^0\,a\,r^0 = a$	B1		For stating true for $n = 1$ *OR* for $n = 0$
		Assume $r^k a\,r^k = a$			
		EITHER Assumption $\Rightarrow$ $r^{k+1} a\,r^{k+1} = rar = a$	M1		For attempt to prove true for $k + 1$
		OR $r^{k+1} a\,r^{k+1} = r.r^k a\,r^k.r = rar = a$			
		OR $r^{k+1} a\,r^{k+1} = r^k.ra\,r.r^k = r^k\,ar^k = a$	A1		For obtaining correct form
		Hence true for all $n \in \mathbb{Z}^+$	A1	**4**	For statement of induction conclusion
		METHOD 2			
		$r^2 ar^2 = r.rar.r = ra\,r = a$, similarly for $r^3 a\,r^3 = a$	M1		For attempt to prove for n = 2, 3
		$r^4 ar^4 = r.r^3 ar^3.r = ra\,r = a$, similarly for $r^5 a\,r^5 = a$	A1		For proving true for n = 2, 3, 4, 5
		$r^6 a\,r^6 = e\,a\,e = a$	B1		For showing true for n = 6
		For $n > 6$, $r^n = r^{n \bmod 6}$, hence true for all $n \in \mathbb{Z}^+$	A1		For using n mod 6 and correct conclusion
		METHOD 3			
		$r^n a\,r^n = r^{n-1}.rar.r^{n-1}$ *OR* $r^n a\,r^n = r^n.r^5 a.r^{n-1} = r^{n+5} a\,r^{n-1}$	M1		Starting from n, for attempt to prove true for $n-1$
		$= r^{n-1} a\,r^{n-1}$	A1		For proving true for $n-1$
		$= r^{n-2} a\,r^{n-2} = \ldots$	A1		For continuation from $n-2$ downwards
		$= r\,a\,r = a$	B1		For final use of $r\,a\,r = a$ **SR** can be done in reverse
		METHOD 4			
		$a\,r = r^5 a \Rightarrow a\,r^2 = r^5 a\,r = r^{10} a$ etc.	M1		For attempt to derive $a\,r^n = r^{5n} a$
		$\Rightarrow a\,r^n = r^{5n} a$	A1		For correct equation **SR** may be stated without proof
		$\Rightarrow r^n ar^n = r^{6n} a$	B1		For pre-multiplication by r^n
		$= ea = a$	A1		For obtaining a ($r^6 = e$ may be implied)
			$\boxed{6}$		

3					Allow $\text{cis}\frac{k}{5}\pi$ and $\text{e}^{\frac{k}{5}\pi\text{i}}$ throughout
	(i)	$w^2=\cos\frac{4}{5}\pi+\text{i}\sin\frac{4}{5}\pi$	B1		For correct value
		$w^3=\cos\frac{6}{5}\pi+\text{i}\sin\frac{6}{5}\pi$	B1		For correct value
		$w^*=\cos\frac{2}{5}\pi-\text{i}\sin\frac{2}{5}\pi$	B1		For w^* seen or implied
		$=\cos\frac{8}{5}\pi+\text{i}\sin\frac{8}{5}\pi$	B1	4	For correct value **SR** For exponential form with i missing, award B0 first time, allow others
	(ii)		B1*		For $1+w$ in approximately correct position
			B1 (*dep)		For $AB\approx BC\approx CD$
			B1 (*dep)		For BC, CD equally inclined to Im axis
			B1	4	For E at the origin Allow points joined by arcs, or not joined Labels not essential
	(iii)	$z^5-1=0$ *OR* $z^5+z^4+z^3+z^2+z=0$	B1	1	For correct equation **AEF** (in any variable) Allow factorised forms using w, exp or trig
			$\boxed{9}$		
4	**(i)**	$y=xz\Rightarrow\frac{\text{d}y}{\text{d}x}=z+x\frac{\text{d}z}{\text{d}x}$	B1		For correct differentiation of substitution
		$\Rightarrow xz+x^2\frac{\text{d}z}{\text{d}x}-xz=x\cos z\Rightarrow x\frac{\text{d}z}{\text{d}x}=\cos z$	M1 A1		For substituting into DE For DE in variables separable form
		$\Rightarrow\int\sec z\,\text{d}z=\int\frac{1}{x}\text{d}x$	M1		For attempt at integration to ln form on LHS
		$\Rightarrow\ln(\sec z+\tan z)=\ln kx$ *OR* $\ln\tan\left(\frac{1}{2}z+\frac{1}{4}\pi\right)=\ln kx$	A1		For correct integration (k not required here)
		$\Rightarrow\sec\left(\frac{y}{x}\right)+\tan\left(\frac{y}{x}\right)=kx$ *OR* $\tan\left(\frac{y}{2x}+\frac{1}{4}\pi\right)=kx$	A1	6	For correct solution **AEF** including $\text{RHS}=\text{e}^{(\ln x)+c}$
	(ii)	$(4,\pi)\Rightarrow\sec\frac{1}{4}\pi+\tan\frac{1}{4}\pi=4k$ *OR* $\tan\left(\frac{1}{8}\pi+\frac{1}{4}\pi\right)=4k$	M1		For substituting $(4,\pi)$ into their solution (with k)
		$\Rightarrow\sec\left(\frac{y}{x}\right)+\tan\left(\frac{y}{x}\right)=\frac{1}{4}\left(1+\sqrt{2}\right)x$ *OR* $\tan\left(\frac{y}{2x}+\frac{1}{4}\pi\right)=\left(\frac{1}{4}\tan\frac{3}{8}\pi\right)x$ *or* $\frac{1}{4}\left(1+\sqrt{2}\right)x$	A1	2	For correct solution **AEF** Allow decimal equivalent $0.60355x$ Allow $\text{e}^{\ln x}$ for x
			$\boxed{8}$		

5	(i)	$C+\mathrm{i}S=1+\frac{1}{2}e^{i\theta}+\frac{1}{4}e^{2i\theta}+\frac{1}{8}e^{3i\theta}+\ldots$	M1		For using $\cos n\theta+\mathrm{i}\sin n\theta=e^{\mathrm{i}n\theta}$ at least once for $n\geqslant 2$
			A1		For correct series
		$=\dfrac{1}{1-\frac{1}{2}e^{i\theta}}=\dfrac{2}{2-e^{i\theta}}$	M1		For using sum of infinite GP
			A1	4	For correct expression **AG** **SR** For omission of 1st stage award up to M0 A0 M1 A1 **OEW**
	(ii)	$C+\mathrm{i}S=\dfrac{2\left(2-e^{-i\theta}\right)}{\left(2-e^{i\theta}\right)\left(2-e^{-i\theta}\right)}$	M1		For multiplying top and bottom by complex conjugate
		$=\dfrac{4-2e^{-i\theta}}{4-2\left(e^{i\theta}+e^{-i\theta}\right)+1}=\dfrac{4-2\cos\theta+2\mathrm{i}\sin\theta}{4-4\cos\theta+1}$	M1		For reverting to cosθ and sinθ and equating Re *OR* Im parts
		$\Rightarrow C=\dfrac{4-2\cos\theta}{5-4\cos\theta},\quad S=\dfrac{2\sin\theta}{5-4\cos\theta}$	A1		For correct expression for C **AG**
			A1	4	For correct expression for S
			$\boxed{8}$		
6	(i)	Aux. equation $m^2+2m+17(=0)$	M1		For attempting to solve correct auxiliary equation
		$\Rightarrow m=-1\pm 4\mathrm{i}$	A1		For correct roots
		CF $(y=)\ e^{-x}\left(A\cos 4x+B\sin 4x\right)$	A1√		For correct CF (allow $A\,{\cos \atop \sin}(4x+\varepsilon)$) (trig terms required, not $e^{\pm 4\mathrm{i}x}$) f.t. from their m with 2 arbitrary constants
		PI $(y=)\ px+q\ \Rightarrow\ 2p+17(px+q)=17x+36$	M1		For stating and substituting PI of correct form
		$\Rightarrow\ p=1$	A1		For correct value of p
		and $q=2$	A1		For correct value of q
		GS $y=e^{-x}\left(A\cos 4x+B\sin 4x\right)+x+2$	B1√	7	For GS. f.t. from their CF+PI with 2 arbitrary constants in CF and none in PI. Requires $\boxed{y=}$.
	(ii)	$x\gg 0\Rightarrow e^{-x}\to 0$ *OR* very small	B1		For correct statement. Allow graph
		$\Rightarrow y=x+2$ approximately	B1√	2	For correct equation Allow ≈ , → and in words Allow relevant f.t. from linear part of GS
			$\boxed{9}$		

7	**(i)**	$(1, 3, 5)$ and $(5, 2, 5) \Rightarrow \pm[4, -1, 0]$ in Π	M1		For finding a vector in Π
		$\mathbf{n} = [2, -2, 3] \times [4, -1, 0] = k[1, 4, 2]$	M1		For finding vector product of direction vectors of l and a line in Π
			A1		For correct **n**
		$\Rightarrow \mathbf{r} \, . [1, 4, 2] = 23$	A1	4	For correct equation. Allow multiples
	(ii)	METHOD 1			
		Perpendicular to Π through $(-7, -3, 0)$ meets Π	M1		For using perpendicular from point on l to Π Award mark for $k\mathbf{n}$ used
		where $(-7+k)+4(-3+4k)+2(2k)=23$	M1		For substituting parametric line coords into Π
		$\Rightarrow k=2 \quad \Rightarrow d = 2\sqrt{1^2+4^2+2^2} = 2\sqrt{21} \approx 9.165$	M1		For normalising the **n** used in this part
			A1	4	For correct distance **AEF**
		METHOD 2			
		Π is $x+4y+2z=23$	M1		For attempt to use formula for perpendicular distance
		$\Rightarrow d = \dfrac{\lvert(-7)+4(-3)+2(0)-23\rvert}{\sqrt{1^2+4^2+2^2}} = 2\sqrt{21} \approx 9.165$	M1		For substituting a point on l into plane equation
			M1		For normalising the **n** used in this part
			A1		For correct distance **AEF**
		METHOD 3			
		$\mathbf{m} = [1, 3, 5] - [-7, -3, 0] = (\pm)[8, 6, 5]$	M1		For finding a vector from l to Π
		OR $= [5, 2, 5] - [-7, -3, 0] = (\pm)[12, 5, 5]$			
		$\Rightarrow d = \dfrac{\mathbf{m} \, . [1, 4, 2]}{\sqrt{1^2+4^2+2^2}} = \dfrac{42}{\sqrt{21}} = 2\sqrt{21} \approx 9.165$	M1		For finding $\mathbf{m} \, . \, \mathbf{n}$
			M1		For normalising the **n** used in this part
			A1		For correct distance **AEF**
		METHOD 4			As Method 1, using parametric form of Π
		$[-7, -3, 0] + k[1, 4, 2] = [1, 3, 5] + s[2, -2, 3] + t[4, -1, 0]$	M1		For using perpendicular from point on l to Π Award mark for $k\mathbf{n}$ used
		$\left.\begin{aligned} k-2s-4t &= 8 \\ 4k+2s+t &= 6 \\ 2k-3s &= 5 \end{aligned}\right\} \Rightarrow k=2 \quad \left(s=-\tfrac{1}{3},\ t=-\tfrac{4}{3}\right)$	M1		For setting up and solving 3 equations
		$\Rightarrow d = 2\sqrt{1^2+4^2+2^2} = 2\sqrt{21} \approx 9.165$	M1		For normalising the **n** used in this part
			A1		For correct distance **AEF**
		METHOD 5			
		$d_1 = \dfrac{23}{\sqrt{1^2+4^2+2^2}} = \dfrac{23}{\sqrt{21}}$	M1		For attempt to find distance from O to Π *OR* from O to parallel plane containing l
		$d_2 = \dfrac{[-7, -3, 0] \, . [1, 4, 2]}{\sqrt{1^2+4^2+2^2}} = \dfrac{-19}{\sqrt{21}}$	M1		For normalising the **n** used in this part
		$\Rightarrow d_1 - d_2 = d = \dfrac{23-(-19)}{\sqrt{21}} = 2\sqrt{21} \approx 9.165$	M1		For finding $d_1 - d_2$
			A1		For correct distance **AEF**
	(iii)	$(-7, -3, 0) + k(1, 4, 2)$	M1		State or imply coordinates of a point on the reflected line
		Use $k=4$	M1		State or imply 2 × distance from **(ii)** Allow $k = \pm 4$ *OR* $\pm 4\sqrt{21}$ f.t. from **(ii)**
		$\mathbf{b} = [2, -2, 3]$	B1		For stating correct direction
		$\mathbf{a} = [-3, 13, 8]$	A1	4	For correct point seen in equation $\mathbf{r} = \mathbf{a} + t\mathbf{b}$
		$\mathbf{r} = [-3, 13, 8] + t[2, -2, 3]$			**AEF** in this form
				$\boxed{12}$	

<table>
<tr><td>8 (i)</td><td>$\{A, D\}$ *OR* $\{A, E\}$ *OR* $\{A, F\}$</td><td>B1</td><td>1</td><td>For stating any one subgroup</td></tr>
<tr><td>(ii)</td><td>A is the identity
5 is not a factor of 6
OR elements can be only of order 1, 2, 3, 6</td><td>B1
B1</td><td>
2</td><td>For identifying A as the identity
For reference to factors of 6</td></tr>
<tr><td>(iii)</td><td>$BE = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = D$, $EB = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = F$
$D \text{ or } \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, F \text{ or } \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} \in M$
⇒ closure property satisfied</td><td>M1
A1
A1
A1</td><td>

4</td><td>For finding BE and EB AND using $\omega^3 = 1$
For correct BE (D *or* matrix)
For correct EB (F *or* matrix)
For justifying closure</td></tr>
<tr><td>(iv)</td><td>$B^{-1} = \frac{1}{1}\begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = C$
$E^{-1} = \frac{1}{-1}\begin{pmatrix} 0 & -\omega^2 \\ -\omega & 0 \end{pmatrix} = E$</td><td>M1
A1
A1</td><td>

3</td><td>For correct method of finding either inverse
For correct $B^{-1} = C$ Allow $\begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}$
For correct $E^{-1} = E$ Allow $\begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}$</td></tr>
<tr><td>(v)</td><td>METHOD 1
M is not commutative
e.g. from $BE \neq EB$ in part **(iii)**
N is commutative (as × mod 9 is commutative)
⇒ M and N not isomorphic</td><td>B1

B1
B1#</td><td>

3</td><td>For justification of M being not commutative
For statement that N is commutative
For correct conclusion</td></tr>
<tr><td></td><td>METHOD 2
Elements of M have orders 1, 3, 3, 2, 2, 2
Elements of N have orders 1, 6, 3, 2, 3, 6
Different orders *OR* self-inverse elements
⇒ M and N not isomorphic</td><td>B1*
B1 (*dep)
B1#</td><td></td><td>For all orders of one group correct
For sufficient orders of the other group correct
For correct conclusion
SR Award up to B1 B1 B1 if the self-inverse elements are sufficiently well identified for the groups to be non-isomorphic</td></tr>
<tr><td></td><td>METHOD 3
M has no generator
since there is no element of order 6
N has 2 *OR* 5 as a generator
⇒ M and N not isomorphic</td><td>B1

B1
B1#</td><td></td><td>For all orders of M shown correctly
For stating that N has generator 2 *OR* 5
For correct conclusion</td></tr>
<tr><td></td><td>METHOD 4
$\begin{array}{c|cccccc} M & A & B & C & D & E & F \\ \hline A & A & B & C & D & E & F \\ B & B & C & A & F & D & E \\ C & C & A & B & E & F & D \\ D & D & E & F & A & B & C \\ E & E & F & D & C & A & B \\ F & F & D & E & B & C & A \end{array}$
$\begin{array}{c|cccccc} N & 1 & 2 & 4 & 8 & 7 & 5 \\ \hline 1 & 1 & 2 & 4 & 8 & 7 & 5 \\ 2 & 2 & 4 & 8 & 7 & 5 & 1 \\ 4 & 4 & 8 & 7 & 5 & 1 & 2 \\ 8 & 8 & 7 & 5 & 1 & 2 & 4 \\ 7 & 7 & 5 & 1 & 2 & 4 & 8 \\ 5 & 5 & 1 & 2 & 4 & 8 & 7 \end{array}$
⇒ M and N not isomorphic</td><td>B1*
B1 (*dep)
B1#</td><td></td><td>For stating correctly all 6 squared elements of one group
For stating correctly sufficient squared elements of the other group
For correct conclusion</td></tr>
<tr><td></td><td></td><td></td><td></td><td># In all Methods, the last B1 is dependent on at least one preceding B1</td></tr>
<tr><td></td><td></td><td>$\boxed{13}$</td><td></td><td></td></tr>
</table>