OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4726

Further Pure Mathematics 2

Monday **16 JANUARY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Question **4**.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages and an insert.

1 (i) Write down and simplify the first three non-zero terms of the Maclaurin series for $\ln(1 + 3x)$. [3]

(ii) Hence find the first three non-zero terms of the Maclaurin series for

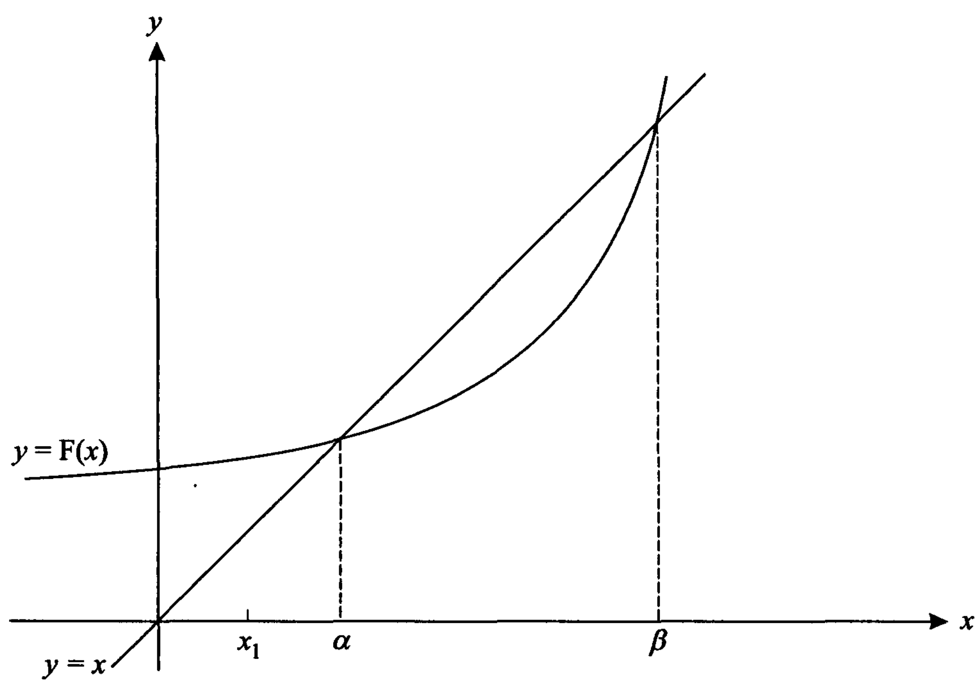
$$e^x \ln(1 + 3x),$$

simplifying the coefficients. [3]

2 Use the Newton-Raphson method to find the root of the equation $e^{-x} = x$ which is close to $x = 0.5$. Give the root correct to 3 decimal places. [5]

3 Express $\dfrac{x+6}{x(x^2+2)}$ in partial fractions. [5]

4 **Answer the whole of this question on the insert provided.**



The sketch shows the curve with equation $y = F(x)$ and the line $y = x$. The equation $x = F(x)$ has roots $x = \alpha$ and $x = \beta$ as shown.

(i) Use the copy of the sketch on the insert to show how an iteration of the form $x_{n+1} = F(x_n)$, with starting value x_1 such that $0 < x_1 < \alpha$ as shown, converges to the root $x = \alpha$. [3]

(ii) State what happens in the iteration in the following two cases.

(a) x_1 is chosen such that $\alpha < x_1 < \beta$.

(b) x_1 is chosen such that $x_1 > \beta$.

[3]

5 (i) Find the equations of the asymptotes of the curve with equation

$$y = \frac{x^2 + 3x + 3}{x + 2}.$$ [3]

(ii) Show that y cannot take values between -3 and 1. [5]

6 (i) It is given that, for non-negative integers n,

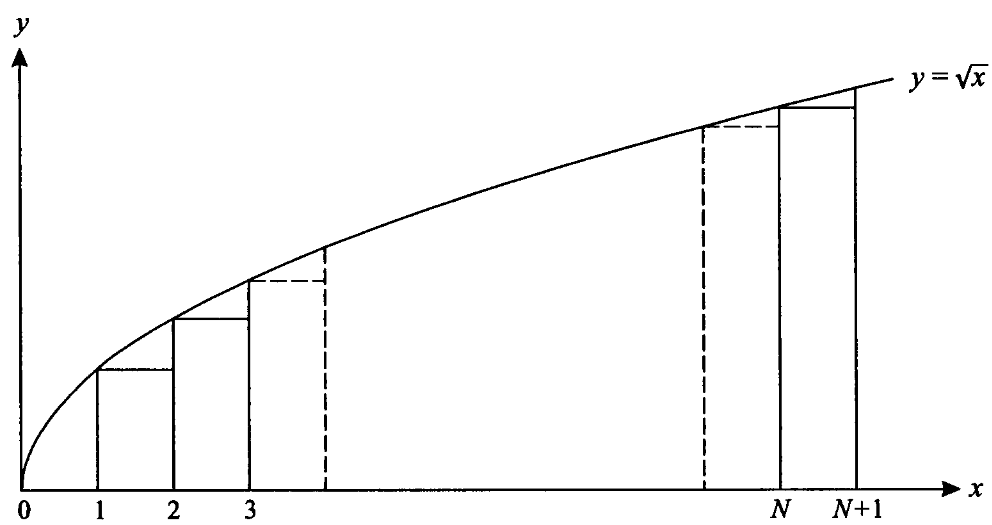
$$I_n = \int_0^1 e^{-x} x^n \, dx.$$

Prove that, for $n \geqslant 1$,

$$I_n = nI_{n-1} - e^{-1}.$$ [4]

(ii) Evaluate I_3, giving the answer in terms of e. [4]

7



The diagram shows the curve with equation $y = \sqrt{x}$. A set of N rectangles of unit width is drawn, starting at $x = 1$ and ending at $x = N + 1$, where N is an integer (see diagram).

(i) By considering the areas of these rectangles, explain why

$$\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{N} < \int_1^{N+1} \sqrt{x} \, dx.$$ [3]

(ii) By considering the areas of another set of rectangles, explain why

$$\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{N} > \int_0^{N} \sqrt{x} \, dx.$$ [3]

(iii) Hence find, in terms of N, limits between which $\sum_{r=1}^{N} \sqrt{r}$ lies. [3]

8 The equation of a curve, in polar coordinates, is

$$r = 1 + \cos 2\theta, \quad \text{for } 0 \leqslant \theta < 2\pi.$$

(i) State the greatest value of r and the corresponding values of θ. [2]

(ii) Find the equations of the tangents at the pole. [2]

(iii) Find the exact area enclosed by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{2}\pi$. [5]

(iv) Find, in simplified form, the cartesian equation of the curve. [4]

9 **(i)** Using the definitions of $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x}, prove that

$$\sinh 2x = 2 \sinh x \cosh x.$$ [4]

(ii) Show that the curve with equation

$$y = \cosh 2x - 6 \sinh x$$

has just one stationary point, and find its x-coordinate in logarithmic form. Determine the nature of the stationary point. [8]

Candidate Name	Centre Number	Candidate Number

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MATHEMATICS **4726**

Further Pure Mathematics 2

INSERT for Question 4

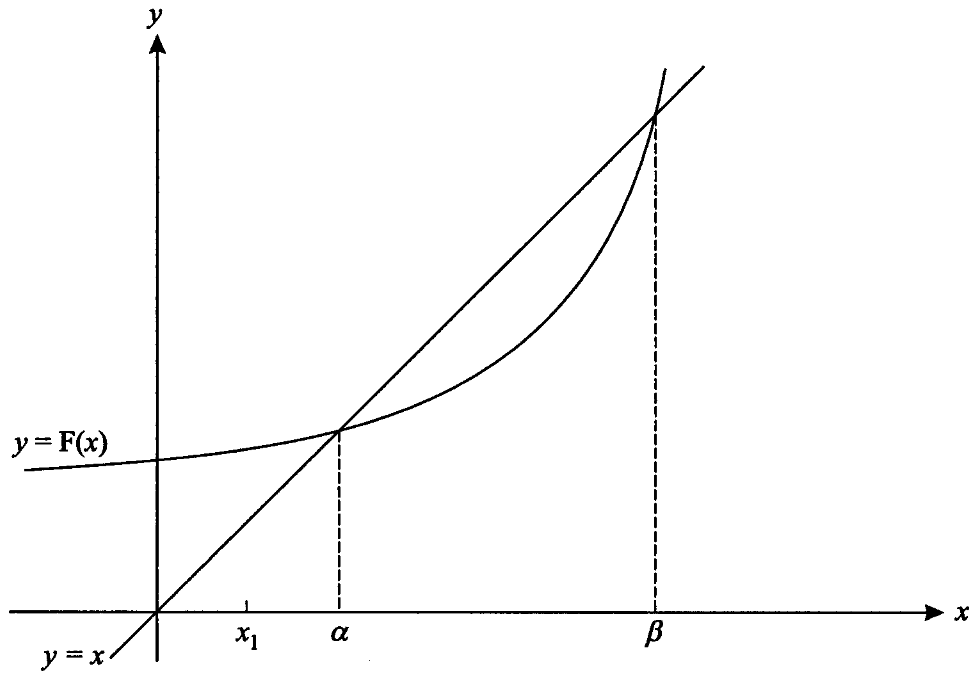
Monday **16 JANUARY 2006** Morning 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- This insert should be used to answer Question **4**.
- Write your name, centre number and candidate number in the spaces provided at the top of this page.
- Write your answers to Question **4** in the spaces provided in this insert, and attach it to your answer booklet.

This insert consists of 2 printed pages.

4 **(i)**



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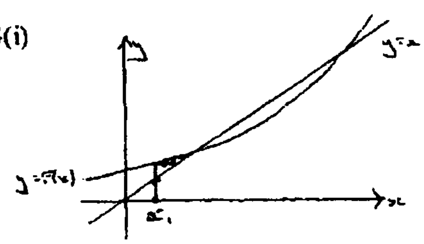
(ii) **(a)** ....

(b) ....

4726 FP2 MARK SCHEME January 2006 Final Draft

1(i) Use standard $\ln(1+3x) = 3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3}$	M1	Allow e.g. $3x^2$, 2! etc.
	M1	Attempt to simplify $(3x)^2$ etc.
$= 3x - 9x^2/2 + 9x^3$	A1	cao
(ii) Produce $(1 + x + x^2/2)$	B1	
	M1	Mult. 2 reasonable attempts, each of 3 terms (non-zero)
Get $3x - 3x^2/2 + 6x^3$	A1√	From their series
	SC M1	Reasonable attempt at diff. and replace $x = 0$ (2 correct)
	M1√	Put their values into correct Maclaurin expansion
	A1	cao
		(Applies to either/both parts)
2 Write as $f(x) = \pm(x - e^{-x})$	B1	Or equivalent
So $f'(x) = \pm(1 + e^{-x})$	B1	Correct from their $f(x)$
Use $x_{n+1} = x_n - f(x_n)/f'(x_n)$ with $x_0 = 0.5$	M1	Clear evidence of N-R on their f, f'
Get $x_1 = 0.56631$, $x_2 = 0.56714$	A1√	At least one to 4d.p.
Get $x_3 = 0.567(1)$	A1	cao to 3 d.p.
3 Use $A/x + (Bx + C)/(x^2 + 2)$	B1	
Equate $x+6$ to $A(x^2 + 2) + (Bx+C)x$ (or equiv.)	M1√	Equate to their P.F. (e.g. if B = 0 or C = 0 used)
Use $x = 0$ or equiv. for A (or equate coeff.etc.)	M1√	Include cover-up
Correctly find one of B,C	A1	
Get A=3, B=-3, C=1	A1	
4(i) 	B1	Line from x_1 to curve
	B1	Then to line
	B1	Clear explanation; allow use of step/staircase
(ii)(a) Converges to $x=\alpha$	B1, B1	
(b) Diverges (does not give either root)	B1	
5 (i) Give $x = -2$	B1	
Attempt to divide out	M1	Giving $y = x+k$; allow $k = 0$ here
Get $y = x + 1$	A1	Must be =
(ii) Write as quad. $x^2 + x(3 - y) + (3 - 2y) = 0$	M1	SC Differentiate M1
Use for real x, $b^2 - 4ac \geq 0$	M1	Solve $dy/dx=0$ M1
Produce quad. inequality in y	M1	Get 2 x,y values correct A1
Attempt to solve quad. inequality	M1	Attempt at max/min M1
Get A.G. clearly e.g. graph	A1	Justify, e.g. graph, constraints on y A1

6 (i) Use parts to $(-e^{-x}.x^n - \int -e^{-x}.nx^{n-1}\,dx)$	M1 Reasonable attempt e.g. $+e^{-x}$
	A1 cao
Use limits to get e^{-1}	B1 Allow ±
Tidy correctly to A.G.	A1
(ii) Use $I_3 = 3I_2 - e^{-1}$	B1 One such seen
$I_2 = 2I_1 - e^{-1}$	
$I_1 = I_0 - e^{-1}$	
Work out $I_0 = 1 - e^{-1}$ or $I_1 = 1 - 2e^{-1}$	M1,A1
Get $6 - 16e^{-1}$	A1
7 (i) Area under graph = $\int \sqrt{x}\,dx$	B1 Explain RHS (limits need not be specified)
> Sum of areas of rectangles from 1 to $N+1$	B1
Area of each rect. = Width x Height = $1 \times \sqrt{x}$	B1
(ii) Similarly, area under curve from 0 to N	B1
< sum of areas of rect. from 0 to N	B1
Clear explanation of A.G.	B1
(iii) Integrate $x^{0.5}$ and use 2 different sets of limits	M1,M1
Get area between $^2/_3((N+1)^{1.5}-1)$ and $^2/_3N^{1.5}$	A1
8 (i) Max. $r = 2$ at $\theta = 0$ and π	B1,B1 Two θ needed (rads only); ignore θ out of range
(ii) Solve $r = 0$ for θ, giving $\theta = \frac{1}{2}\pi$ and $^3/_2\pi$	M1,A1 Two θ needed (rads only); ignore θ out of range
(iii) Use correct formula with correct r	M1
Expand r	M1
Get $\int A + B\cos 2\theta + C\cos 4\theta\,d\theta$	M1 $C \neq 0$
Integrate their expression correctly	M1√
Get $3\pi/8$	A1 cao
(iv) Express $\cos 2\theta = \cos^2\theta - \sin^2\theta$ or similar	M1
Use $\cos\theta = x/r$ and/or $\sin\theta = y/r$	M1
Simplify to $(x^2 + y^2)^{1.5} = 2x^2$ or similar	M1,A1
9 (i) Correct def^n of cosh x and sinh x	B1,B1
Expand $2.\frac{1}{2}(e^x - e^{-x}).\frac{1}{2}(e^x + e^{-x})$	M1 Reasonable attempt
Clearly get $\frac{1}{2}(e^{2x} - e^{-2x})$ to A.G.	A1
(ii) Attempt to diff. and solve $dy/dx = 0$	M1 Reasonable attempt
Use (ii) to get A cosh x (B sinh x+ C)=0	M1
Clearly see cosh $x > 0$ or similar for one useable factor only	B1
Attempt to solve sinh x = - C/B	M1 Quote or via e^{-x} correctly
Get $x = \ln((3+\sqrt{13})/2)$	A1
Justify one answer only for sinh x = -C/B	B1
Accurate test for MINIMUM	B1 First or second diff^l test with numeric evidence
	B1 Correct value(s) for min.