OCR
RECOGNISING ACHIEVEMENT

Monday 10 June 2013 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

4733320613

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 The complex number $3 + ai$, where a is real, is denoted by z. Given that $\arg z = \frac{1}{6}\pi$, find the value of a and hence find $|z|$ and $z^* - 3$. **[6]**

2 The matrices **A**, **B** and **C** are given by $\mathbf{A} = (5 \quad 1)$, $\mathbf{B} = (2 \quad -5)$ and $\mathbf{C} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

(i) Find $3\mathbf{A} - 4\mathbf{B}$. **[2]**

(ii) Find **CB**. Determine whether **CB** is singular or non-singular, giving a reason for your answer. **[5]**

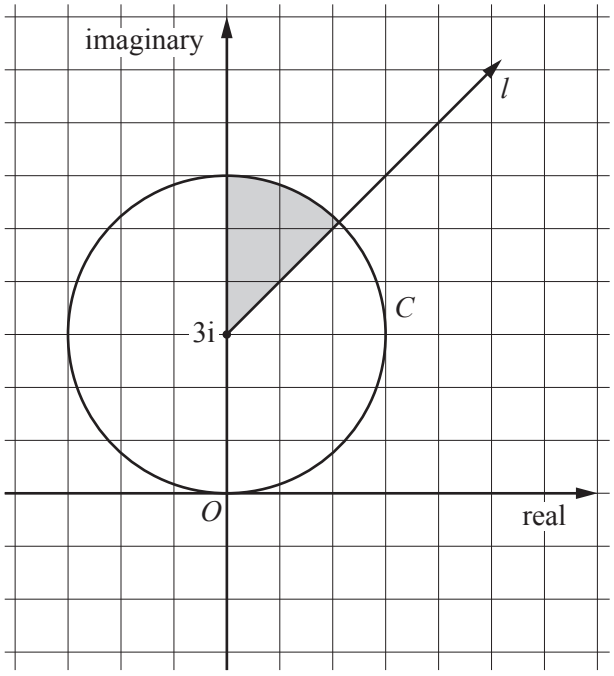
3 Use an algebraic method to find the square roots of $11 + (12\sqrt{5})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. **[6]**

4 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \geqslant 1$,

$$\mathbf{M}^n = \begin{pmatrix} 2^n & 2^{n+1} - 2 \\ 0 & 1 \end{pmatrix}.$$ **[6]**

5 Find $\sum_{r=1}^{n} (4r^3 - 3r^2 + r)$, giving your answer in a fully factorised form. **[6]**

6



The Argand diagram above shows a half-line l and a circle C. The circle has centre 3i and passes through the origin.

(i) Write down, in complex number form, the equations of l and C. **[4]**

(ii) Write down inequalities that define the region shaded in the diagram. [The shaded region includes the boundaries.] **[3]**

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7 **(i)** Find the matrix that represents a rotation through 90° clockwise about the origin. **[2]**

(ii) Find the matrix that represents a reflection in the x-axis. **[2]**

(iii) Hence find the matrix that represents a rotation through 90° clockwise about the origin, followed by a reflection in the x-axis. **[2]**

(iv) Describe a **single** transformation that is represented by your answer to part **(iii)**. **[2]**

8 The cubic equation $kx^3 + 6x^2 + x - 3 = 0$, where k is a non-zero constant, has roots α, β and γ.

Find the value of $(\alpha + 1)(\beta + 1) + (\beta + 1)(\gamma + 1) + (\gamma + 1)(\alpha + 1)$ in terms of k. **[6]**

9 **(i)** Show that $\dfrac{1}{3r-1} - \dfrac{1}{3r+2} \equiv \dfrac{3}{(3r-1)(3r+2)}$. **[2]**

(ii) Hence show that $\displaystyle\sum_{r=1}^{2n} \frac{1}{(3r-1)(3r+2)} = \frac{n}{2(3n+1)}$. **[6]**

10 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 2 & 1 \\ 1 & 3 & 2 \\ 4 & 1 & 1 \end{pmatrix}$.

(i) Find the value of a for which $\mathbf{A}$ is singular. **[5]**

(ii) Given that $\mathbf{A}$ is non-singular, find $\mathbf{A}^{-1}$ and hence solve the equations

$$\begin{aligned} ax + 2y + z &= 1, \\ x + 3y + 2z &= 2, \\ 4x + y + z &= 3. \end{aligned}$$

[7]

Question			Answer	Marks	Guidance
1				M1	Use correct trig expression
			$\sqrt{3}$	A1	Obtain correct answer
				M1	Correct expression for modulus
			$2\sqrt{3}$	A1FT	Obtain correct answer aef
			$3-\sqrt{3}\mathrm{i}$	B1FT	Correct conjugate seen or implied
			$-\sqrt{3}\,\mathrm{i}$	B1FT	Correct answer
				[6]	
2	(i)		$\begin{pmatrix}7 & 23\end{pmatrix}$	B1B1	Each element correct, missing brackets B1 only
				[2]	
2	(ii)		$\begin{pmatrix}6 & -15\\ 4 & -10\end{pmatrix}$	M1	Obtain 2 × 2 matrix
				A1	Obtain 2 correct elements
				A1	Obtain other 2 correct elements
			det **CB** = 0	A1FT	Obtain their det **CB,** must be a 2 × 2 matrix
			singular	A1FT	Correct conclusion from their det **CB**
				[5]	
3			$x^2-y^2=11$ and $xy=6\sqrt{5}$	M1	Attempt to equate real and imaginary parts of $(x+\mathrm{i}y)^2$ and $11+12\sqrt{5}$
				A1	Obtain both results cao
				M1*	Obtain a quadratic in x^2 or y^2
			$\pm(2\sqrt{5}+3\mathrm{i})$	DM1	Solve a 3 term quadratic to obtain a value for x or y
				A1	Obtain 1 correct answer as complex number
				A1	Obtain only the other correct answer
				[6]	
4				B1	Establish result true for $n = 1$ or $n = 2$
				M1	Multiply **M** and $\mathbf{M}^k$, either order
			$2\left(2^{k+1}-2\right)+2$ or $2^{k+1}+2^{k+1}-2$	A1	Obtain correct element
				A1	Obtain other 3 correct elements
				A1	Obtain $2^{k+2}-2$ convincingly
				B1	Specific statement of induction conclusion, provided 5/5 earned so far and verified for $n = 1$
				[6]	

Question			Answer	Marks	Guidance
5			$4\times\frac{1}{4}n^2(n+1)^2-3\times\frac{1}{6}n(n+1)(2n+1)+\frac{1}{2}n(n+1)$	M1	Express as sum of three series
				A1	Obtain 2 correct (unsimplified) terms
				A1	Obtain correct 3rd (unsimplified) term
			$n^3(n+1)$	M1	Attempt to factorise, at least factor of n
				A2	Obtain correct answer, A1 if not fully factorised
				[6]	
6	**(i)**		$\arg(z-3i)=\frac{1}{4}\pi$	M1	Use arg (z – a) = θ in equation for l condone missing brackets
				A1	Obtain correct answer
				M1	Use $\lvert z-a\rvert=k$ in equation for C, k must be real
			$\lvert z-3i\rvert=3$	A1	Obtain correct answer
				[4]	
	(ii)		$\lvert z-3i\rvert\le 3$ or e.g. $x^2+(y-3)^2\le 9$	B1	Obtain correct inequality, or answer consistent with sensible **(i)**
			$\frac{1}{4}\pi\le\arg(z-3i)\le\frac{1}{2}\pi$ or $y\ge x+3,\ x\ge 0$	B1 B1	Each correct single inequality, or answer consistent with sensible **(i)**
				[3]	**SC if < used consistently, but otherwise all correct, B2**
7	**(i)**		$\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$	B1B1	Each column correct
				[2]	
	(ii)		$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$	B1B1	Each column correct
				[2]	
	(iii)		$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$	M1	Attempt at matrix multiplication in correct order
				A1FT	Obtain correct answer from their **(i)** and **(ii)**
				[2]	
	(iv)		Reflection, in $y=x$	B1B1	Correct description of their **(iii) only**
				[2]	

Question			Answer	Marks	Guidance
8			*Either*		
			$\sum\alpha = -\frac{6}{k}, \sum\alpha\beta = \frac{1}{k}$	B1B1	Correct values stated or used
			$\sum\alpha\beta + 2\sum\alpha + 3$	M1 A1	Expand brackets Obtain correct expression aef
			$3 - \frac{11}{k}$	M1 A1 **[6]**	Use their values, in terms of k, for $\sum\alpha$ and $\sum\alpha\beta$ Obtain correct answer aef
			Or		
				B1	State or use substitution $x = u - 1$
				M1	Expand and attempt to simplify coefficients
			$ku^3 + (6-3k)u^2 + (3k-11)u + 2 - k = 0$	A1 A1	Obtain at least correct 1st and 3rd terms
				M1	Use their "$\frac{c}{a}$"
			$3 - \frac{11}{k}$	A1	Obtain correct answer a.e.f.
9	(i)			M1 A1 **[2]**	Use correct denominator or partial fractions Obtain **given** answer convincingly
	(ii)			M1 A1 M1	Express at least 1st two and last term using **(i)** All terms correct Show correct terms cancelling
			$\frac{1}{2} - \frac{1}{6n+2}$	A1	Obtain correct unsimplified answer
				M1	Include $\frac{1}{3}$ and combine their sum as a single fraction
				A1 **[6]**	Obtain **given** answer

Question			Answer	Marks	Guidance
10	(i)			M1	Show correct expansion process for 3 × 3
				M1	Correct evaluation of any 2 × 2
			$a + 3$	A1	Obtain correct answer
				M1	Use det **A** = 0
			$a = -3$	A1FT	Obtain correct answer from their det **A**
				[5]	
	(ii)		$\frac{1}{a+3}\begin{pmatrix} 1 & -1 & 1 \\ 7 & a-4 & 1-2a \\ -11 & 8-a & 3a-2 \end{pmatrix}$	M1	Show correct processes for adjoint entries
				A1	Obtain at least 4 correct entries in adjoint
				A1	Obtain completely correct adjoint
				B1	Divide adjoint by their det **A**
			$\frac{1}{a+3}\begin{pmatrix} 2 \\ 2-4a \\ 7a-1 \end{pmatrix}$	M1	Pre-multiply column matrix by their $\mathbf{A}^{-1}$
				A2	Obtain correct answer, A1 for 1 element correct
				[7]	