OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

4725

Further Pure Mathematics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes

* 4 7 2 5 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 2 \\ 3 & 4 \end{pmatrix}$ and **I** is the 2×2 identity matrix.

(i) Find $\mathbf{A} - 4\mathbf{I}$. **[2]**

(ii) Given that **A** is singular, find the value of a. **[3]**

2 The cubic equation $2x^3 + 3x - 3 = 0$ has roots α, β and γ.

(i) Use the substitution $x = u - 1$ to find a cubic equation in u with integer coefficients. **[3]**

(ii) Hence find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. **[2]**

3 The complex number z satisfies the equation $z + 2\mathrm{i}z^* = 12 + 9\mathrm{i}$. Find z, giving your answer in the form $x + \mathrm{i}y$. **[5]**

4 Find $\sum_{r=1}^{n} r(r+1)(r-2)$, expressing your answer in a fully factorised form. **[6]**

5 **(i)** The transformation T is represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Give a geometrical description of T. **[2]**

(ii) The transformation T is equivalent to a reflection in the line $y = -x$ followed by another transformation S. Give a geometrical description of S and find the matrix that represents S. **[4]**

6 One root of the cubic equation $x^3 + px^2 + 6x + q = 0$, where p and q are real, is the complex number $5 - \mathrm{i}$.

(i) Find the real root of the cubic equation. **[3]**

(ii) Find the values of p and q. **[4]**

7 **(i)** Show that $\dfrac{1}{r^2} - \dfrac{1}{(r+1)^2} \equiv \dfrac{2r+1}{r^2(r+1)^2}$. **[1]**

(ii) Hence find an expression, in terms of n, for $\sum_{r=1}^{n} \dfrac{2r+1}{r^2(r+1)^2}$. **[4]**

(iii) Find $\sum_{r=2}^{\infty} \dfrac{2r+1}{r^2(r+1)^2}$. **[2]**

8 The complex number a is such that $a^2 = 5 - 12\mathrm{i}$.

(i) Use an algebraic method to find the two possible values of a. **[5]**

(ii) Sketch on a single Argand diagram the two possible loci given by $|z - a| = |a|$. **[4]**

9 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & 1 \\ 1 & 1 & a \end{pmatrix}$, where $a \neq 1$.

(i) Find $\mathbf{A}^{-1}$. **[7]**

(ii) Hence, or otherwise, solve the equations

$$\begin{aligned} 2x - y + z &= 1, \\ 3y + z &= 2, \\ x + y + az &= 2. \end{aligned}$$

[4]

10 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(i) Find $\mathbf{M}^2$ and $\mathbf{M}^3$. **[3]**

(ii) Hence suggest a suitable form for the matrix $\mathbf{M}^n$. **[1]**

(iii) Use induction to prove that your answer to part **(ii)** is correct. **[4]**

(iv) Describe fully the single geometrical transformation represented by $\mathbf{M}^{10}$. **[3]**

4725 Further Pure Mathematics 1

1 (i)	$\begin{pmatrix} a-4 & 2 \\ 3 & 0 \end{pmatrix}$	B1		Two elements correct
		B1	**2**	Remaining elements correct
(ii)	$4a-6$	B1		Correct determinant
		M1		Equate det A to 0 and solve
	$a=\frac{3}{2}$	A1	**3**	Obtain correct answer a. e. f.
		5		
2 (i)	u^3-3u^2+3u-1	B1		Correct unsimplified expansion of $(u-1)^3$
		M1		Substitute for x
	$2u^3-6u^2+9u-8=0$	A1	**3**	Obtain correct **equation**
(ii)		M1		Use $(\pm)\frac{d}{a}$ of new equation
	4	A1ft	**2**	Obtain correct answer from their equation
		5		
3	$x-iy$	B1		Conjugate known
		M1		Equate real and imaginary parts
	$x+2y=12 \quad 2x+y=9$	A1		Obtain both equations, OK with factor of i
		M1		Solve pair of equations
	$z=2+5i$	A1	**5**	Obtain correct answer as a complex number **S.C.** Solving $z+2iz=12+9i$ can get max 4/5, not first B1
		5		
4		M1		Express as sum of three series
		M1		Use standard results
	$\frac{1}{4}n^2(n+1)^2-\frac{1}{6}n(n+1)(2n+1)-n(n+1)$	A1		Obtain correct unsimplified answer
		M1		Attempt to factorise
		A1		Obtain at least factor of $n(n+1)$
	$\frac{1}{12}n(n+1)(n+2)(3n-7)$	A1	**6**	Obtain fully factorised correct answer
		6		

5	**(i)**	B1		Rotation 90° (about origin)
		B1	**2**	Anticlockwise
	(ii) ***Either***	M1		Show image of unit square after reflection in $y = -x$
	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	A1		Deduce reflection in x-axis
		B1ft		Each column correct
		B1ft	**4**	ft for matrix of their transformation
	Or	M1		Post multiply by correct reflection matrix
		A1		Obtain correct answer
		B1B1		State reflection, in x-axis
			S.C.	If pre-multiplication, M0 but B1 B1 Available for correct description of their matrix
		6		
6	**(i)**	B1		State or use 5 + i as a root
		M1		Use $\sum \alpha\beta = 6$
	$x = -2$	A1	**3**	Obtain correct answer
	(ii) ***Either***	M1		Use $p = -\sum \alpha$
	$p = -8$	A1ft		Obtain correct answer, from their root
		M1		Use $q = -\alpha\beta\gamma$
	$q = 52$	A1ft	**4**	Obtain correct answer, from their root
	Or	M1		Attempt to find quadratic factor
		M1		Attempt to expand quadratic and linear
		A1A1		Obtain correct answers
	Or	M1		Substitute (5 – i)into equation
		M1		Equate real and imaginary parts
		A1		Obtain correct answer for p
		A1ft		Obtain correct answer for q, ft their p
		7		
7	**(i)**	B1	**1**	Obtain **given** answer correctly
	(ii)	M1		Express at least 1st two and last term using **(i)**
		A1		All terms correct
		M1		Show that correct terms cancel
	$1 - \frac{1}{(n+1)^2}$	A1	**4**	Obtain correct answer, in terms of n
	(iii) $\frac{1}{4}$	B1		Sum to infinity seen or implied
		B1	**2**	Obtain correct answer **S.C.** -¾ scores B1
		7		

8 (i)	M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ & $5 - 12i$
$x^2 - y^2 = 5$ and $xy = -6$	A1		Obtain both results, a.e.f
	M1		Obtain quadratic in x^2 or y^2
	M1		Solve to obtain $x = (\pm)3$ or $y = (\pm)2$
$\pm(3 - 2i)$	A1	5	Obtain correct answers as complex nos
(ii)	B1ft		Circle with centre at their square root
	B1		Circle passing through origin
	B1ft		2^{nd} circle centre correct relative to 1^{st}
	B1	4	Circle passing through origin
	9		
9 (i)	M1		Show correct expansion process for 3×3 or multiply adjoint by **A**
	M1		Correct evaluation of any 2×2 at any stage
det **A** = $\Delta = 6a - 6$	A1		Obtain correct answer
$\mathbf{A}^{-1} = \frac{1}{\Delta}\begin{pmatrix} 3a-1 & a+1 & -4 \\ 1 & 2a-1 & -2 \\ -3 & -3 & 6 \end{pmatrix}$	M1		Show correct process for adjoint entries
	A1		Obtain at least 4 correct entries in adjoint
	B1		Divide by their determinant
	A1	7	Obtain completely correct answer
(ii) $\frac{1}{\Delta}\begin{pmatrix} 5a-7 \\ 4a-5 \\ 3 \end{pmatrix}$	M1		Attempt product of form $\mathbf{A}^{-1}\mathbf{C}$ or eliminate to get 2 equations and solve
	A1A1A1 ft all 3		Obtain correct answer
		4	**S.C.** if det now omitted, allow max A2 ft
	11		
10 (i)	B1		Correct $\mathbf{M}^2$ seen
$\mathbf{M}^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$ $\mathbf{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$	M1		Convincing attempt at matrix multiplication for $\mathbf{M}^3$
	A1	3	Obtain correct answer
(ii) $\mathbf{M}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$	B1ft	1	State correct form, consistent with **(i)**

10 (iii)	M1		Correct attempt to multiply **M** & $\mathbf{M}^k$ or v.v.
	A1		Obtain element 2(k + 1)
	A1		Clear statement of induction step, from correct working
	B1	**4**	Clear statement of induction conclusion, following their working
(iv)	B1		Shear
	DB1		x-axis invariant
	DB1	**3**	e.g. (1, 1) $\rightarrow$ (21, 1) or equivalent using scale factor or angles
	11		