OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS 4724

Core Mathematics 4

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Friday 14 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(i)** Expand $(1-x)^{\frac{1}{2}}$ in ascending powers of x as far as the term in x^2. **[3]**

(ii) Hence expand $\left(1-2y+4y^2\right)^{\frac{1}{2}}$ in ascending powers of y as far as the term in y^2. **[3]**

2 **(i)** Express $\dfrac{7-2x}{(x-2)^2}$ in the form $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}$, where A and B are constants. **[3]**

(ii) Hence find the exact value of $\displaystyle\int_4^5 \frac{7-2x}{(x-2)^2}\,\mathrm{d}x$. **[4]**

3 **(i)** Show that the derivative of $\sec x$ can be written as $\sec x \tan x$. **[4]**

(ii) Find $\displaystyle\int \frac{\tan x}{\sqrt{1+\cos 2x}}\,\mathrm{d}x$. **[4]**

4 A curve has parametric equations

$$x = 2 + t^2, \qquad y = 4t.$$

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. **[2]**

(ii) Find the equation of the normal at the point where $t = 4$, giving your answer in the form $y = mx + c$. **[3]**

(iii) Find a cartesian equation of the curve. **[2]**

5 In this question, I denotes the definite integral $\displaystyle\int_2^5 \frac{5-x}{2+\sqrt{x-1}}\,\mathrm{d}x$. The value of I is to be found using two different methods.

(i) Show that the substitution $u=\sqrt{x-1}$ transforms I to $\displaystyle\int_1^2 (4u-2u^2)\,\mathrm{d}u$ and hence find the exact value of I. **[5]**

(ii) **(a)** Simplify $\left(2+\sqrt{x-1}\right)\left(2-\sqrt{x-1}\right)$. **[1]**

(b) By first multiplying the numerator and denominator of $\dfrac{5-x}{2+\sqrt{x-1}}$ by $2-\sqrt{x-1}$, find the exact value of I. **[3]**

6 The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + s\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$. The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$.

(i) Find the acute angle between l_1 and l_2. **[4]**

(ii) Show that l_1 and l_2 are skew. **[4]**

(iii) One of the numbers in the equation of line l_1 is changed so that the equation becomes $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ a \end{pmatrix} + s\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$. Given that l_1 and l_2 now intersect, find a. **[2]**

7 Show that $\int_0^{\pi} (x^2 + 5x + 7)\sin x \,\mathrm{d}x = \pi^2 + 5\pi + 10$. **[7]**

8 The points P and Q lie on the curve with equation

$$2x^2 - 5xy + y^2 + 9 = 0.$$

The tangents to the curve at P and Q are parallel, each having gradient $\frac{3}{8}$.

(i) Show that the x- and y-coordinates of P and Q are such that $x = 2y$. **[5]**

(ii) Hence find the coordinates of P and Q. **[3]**

9 Paraffin is stored in a tank with a horizontal base. At time t minutes, the depth of paraffin in the tank is x cm. When $t = 0$, $x = 72$. There is a tap in the side of the tank through which the paraffin can flow. When the tap is opened, the flow of the paraffin is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -4(x - 8)^{\frac{1}{3}}.$$

(i) How long does it take for the level of paraffin to fall from a depth of 72 cm to a depth of 35 cm? **[7]**

(ii) The tank is filled again to its original depth of 72 cm of paraffin and the tap is then opened. The paraffin flows out until it stops. How long does this take? **[3]**

1	**(i)**	First two terms are $1-\frac{1}{2}x$........	B1	
		Third term = $\frac{\frac{1}{2}.-\frac{1}{2}}{2}[(-x)^2 \text{ or } x^2 \text{ or } -x^2]$	M1	
		= $-\frac{1}{8}x^2$	A1 **3**	$-\frac{1}{8}x^2$ without work $\to$ M1 A1
	(ii)	Attempt to replace x by $2y-4y^2$ or $2y+4y^2$	M1	or write as $1-\left(2y-4y^2 \text{ or } 2y+4y^2\right)$
		First two terms are $1-y$	B1	
		Third term = $+\frac{3}{2}y^2$ or $\surd\ (4b+2)y^2$	A1√ **3**	where b = cf$\left(x^2\right)$ in part (i)
			6	
2	**(i)**	$A(x-2)+B=7-2x$	M1	or $A(x-2)^2+B(x-2)=(7-2x)(x-2)$
		$A=-2$	A1	
		$B=3$	A1 **3**	
	(ii)	$\int \frac{A}{x-2}\,dx=\left(A \text{ or } \frac{1}{A}\right)\ln(x-2)$	B1	Accept $\ln\|x-2\|$, $\ln\|2-x\|$, $\ln(2-x)$
		$\int \frac{B}{(x-2)^2}\,dx=-\left(B \text{ or } \frac{1}{B}\right).\frac{1}{x-2}$	B1	Negative sign <u>is</u> required
		Correct f.t. of A & B; $A\ln(x-2)-\frac{B}{x-2}$	B1√	Still accept lns as before
		Using limits = $-2\ln 3+2\ln 2+\frac{1}{2}$ ISW	B1 **4**	<u>No</u> indication of ln(negative)
			7	
3	**(i)**	State/imply $\frac{d}{dx}(\sec x)=\frac{d}{dx}\left(\frac{1}{\cos x}\right)$ or $\frac{d}{dx}(\cos x)^{-1}$	B1	Not just $\sec x=\frac{1}{\cos x}$
		Attempt quotient rule or chain rule to power -1	M1	Allow $\frac{u\,dv-v\,du}{v^2}$ & wrong trig signs
		Obtain $\frac{\sin x}{\cos^2 x}$ or $-.-(\sin x)(\cos x)^{-2}$	A1	No inaccuracy allowed here
		Simplify with suff evid to **AG** e.g. $\frac{1}{\cos x}.\frac{\sin x}{\cos x}$	A1 **4**	Or vice versa. <u>Not</u> just = $\sec x.\tan x$
	(ii)	Use $\cos 2x=+/-1+/-2\cos^2 x$ or $+/-1+/-2\sin^2 x$	M1	or $\pm\left(\cos^2 x-\sin^2 x\right)$
		Correct denominator = $\sqrt{2\cos^2 x}$	A1	$\sqrt{2-2\sin^2 x}$ needs simplifying
		Evidence that $\frac{\tan x}{\cos x}=\sec x\tan x$ or $\int\frac{\tan x}{\cos x}dx=\sec x$	B1	irrespective of any const multiples
		$\frac{1}{\sqrt{2}}\sec x$ (+ c)	A1 **4**	Condone θ for x except final line
			8	

4	(i)	Attempt to use $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ or $\frac{dy}{dt}.\frac{dt}{dx}$	M1		Not just quote formula
		$\frac{4}{2t}$ or $\frac{2}{t}$	A1	2	
	(ii)	Subst $t = 4$ into their (i), invert & change sign	M1		
		Subst $t = 4$ into (x,y) & use num grad for tgt/normal	M1		
		$y = -2x + 52$ AEF CAO (no f.t.)	A1	3	Only the eqn of normal accepted
	(iii)	Attempt to eliminate t from the 2 given equations	M1		
		$x = 2 + \frac{y^2}{16}$ or $y^2 = 16(x-2)$ AEF ISW	A1	2	Mark at earliest acceptable form.
					7
5	(i)	Attempt to connect dx and du	M1		Including $\frac{du}{dx} =$ or $du = ...dx$; not dx = du
		$5 - x = 4 - u^2$	B1		perhaps in conjunction with next line
		Show $\int \frac{4-u^2}{2+u}.2u\, du$ reduced to $\int 4u - 2u^2\, du$ **AG**	A1		In a fully satisfactory & acceptable manner
		Clear explanation of why limits change	B1		e.g. when $x = 2$, $u = 1$ <u>and</u> when $x = 5$, $u = 2$
		$\frac{4}{3}$	B1	5	not dependent on any of first 4 marks
	(ii)(a)	$5 - x$	*B1	1	Accept $4 - x - 1 = 5 - x$ (this is not **AG**)
	(b)	Show reduction to $2 - \sqrt{x-1}$	dep*B1		
		$\int \sqrt{x-1}\, dx = \frac{2}{3}(x-1)^{\frac{3}{2}}$	B1		Indep of other marks, seen anywhere in (b)
		$\left(10 - \frac{2}{3}.8\right) - \left(4 - \frac{2}{3}\right) = \frac{4}{3}$ or $4\frac{2}{3} - 3\frac{1}{3} = \frac{4}{3}$	B1	3	Working must be shown
					9
6	(i)	Work with correct pair of direction vectors	M1		
		Demonstrate correct <u>method</u> for finding scalar product	M1		Of <u>any</u> two 3x3 vectors rel to question
		Demonstrate correct <u>method</u> for finding modulus	M1		Of <u>any</u> vector relevant to question
		24, 24.0 (24.006363..) (degrees) 0.419 (0.41899..) (rad)	A1	4	Mark earliest value, allow trunc/rounding
	(ii)	Attempt to set up 3 equations	M1		Of type $3 + 2s = 5, 3s = 3 + t, -2 - 4s = 2 - 2t$
		Find correct values of $(s,t) = (1,0)$ or $(1,4)$ or $(5,12)$	A1		Or 2 diff values of s (or of t)
		Substitute their (s,t) into equation not used	M1		and make a relevant deduction
		<u>Correctly</u> demonstrate failure	A1	4	dep on all 3 prev marks
	(iii)	Subst their (s,t) from first 2 eqns into new 3rd eqn	M1		New 3rd eqn of type $a - 4s = 2 - 2t$
		$a = 6$	A1	2	
					10

7		Attempt parts with $u = x^2 + 5x + 7$, $dv = \sin x$	M1		as far as $f(x) +/- \int g(x)\,dx$
		1st stage = $-(x^2+5x+7)\cos x + \int (2x+5)\cos x\,dx$	A1		signs need not be amalgamated at this stage
		$\int (2x+5)\cos x\,dx = (2x+5)\sin x - \int 2\sin x\,dx$	B1		indep of previous A1 being awarded
		$= (2x+5)\sin x + 2\cos x$	B1		
		$I = -(x^2+5x+7)\cos x + (2x+5)\sin x + 2\cos x$	A1		WWW
		(Substitute $x = \pi$) $-$(Substitute $x = 0$)	M1		An attempt at subst $x = 0$ must be seen
		$\pi^2 + 5\pi + 10$ WWW **AG**	A1	**7**	
					7
8	(i)	$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1		
		$\frac{d}{dx}(-5xy) = (-)(5)x\frac{dy}{dx} + (-)(5)y$	M1		i.e. reasonably clear use of product rule
		LHS completely correct $4x - 5x\frac{dy}{dx} - 5y + 2y\frac{dy}{dx} (= 0)$	A1		Accept " $\frac{dy}{dx} =$ " provided it is not used
		Substitute $\frac{dy}{dx} = \frac{3}{8}$ or solve for $\frac{dy}{dx}$ & then equate to $\frac{3}{8}$	M1		Accuracy not required for "solve for $\frac{dy}{dx}$ "
		Produce $x = 2y$ WWW **AG** (Converse acceptable)	A1	**5**	Expect $17x = 34y$ and/or $\frac{dy}{dx} = \frac{5y-4x}{2y-5x}$
	(ii)	Substitute $2y$ for x or $\frac{1}{2}x$ for y in curve equation	M1		
		Produce either $x^2 = 36$ or $y^2 = 9$	A1		
		AEF of $(\pm 6, \pm 3)$	A1	**3**	ISW Any correct format acceptable
					8
9	(i)	Attempt to sep variables in the form $\int \frac{p}{(x-8)^{1/3}}\,dx = \int q\,dt$	M1		Or invert as $\frac{dt}{dx} = \frac{r}{(x-8)^{1/3}}$; p,q,r consts
		$\int \frac{1}{(x-8)^{1/3}}\,dx = k(x-8)^{2/3}$	A1		k const
		All correct (+ c)	A1		
		For equation containing 'c'; substitute $t = 0$, $x = 72$	M1		M2 for $\int_{72}^{35} = \int_0^t$ or $\int_{35}^{72} = \int_0^t$
		Correct corresponding value of c from correct eqn	A1		
		Subst their c & $x = 35$ back into eqn	M1		
		$t = \frac{21}{8}$ or 2.63 / 2.625 [C.A.O]	A1	**7**	A2: $t = \frac{21}{8}$ or 2.63 / 2.625 WWW
	(ii)	State/imply in some way that $x = 8$ when flow stops	B1		
		Substitute $x = 8$ back into equation containing numeric 'c'	M1		
		$t = 6$	A1	**3**	
					10