OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Core Mathematics 4

4724

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 5 June 2009
Afternoon

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

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1 Find the quotient and the remainder when $3x^4 - x^3 - 3x^2 - 14x - 8$ is divided by $x^2 + x + 2$. **[4]**

2 Use the substitution $x = \tan\theta$ to find the exact value of

$$\int_1^{\sqrt{3}} \frac{1 - x^2}{1 + x^2}\,dx.$$ [7]

3 **(i)** Expand $(a + x)^{-2}$ in ascending powers of x up to and including the term in x^2. **[4]**

(ii) When $(1 - x)(a + x)^{-2}$ is expanded, the coefficient of x^2 is 0. Find the value of a. **[3]**

4 **(i)** Differentiate $e^x(\sin 2x - 2\cos 2x)$, simplifying your answer. **[4]**

(ii) Hence find the exact value of $\int_0^{\frac{1}{4}\pi} e^x \sin 2x\,dx$. **[3]**

5 A curve has parametric equations

$$x = 2t + t^2, \quad y = 2t^2 + t^3.$$

(i) Express $\frac{dy}{dx}$ in terms of t and find the gradient of the curve at the point $(3, -9)$. **[5]**

(ii) By considering $\frac{y}{x}$, find a cartesian equation of the curve, giving your answer in a form not involving fractions. **[4]**

6 The expression $\frac{4x}{(x-5)(x-3)^2}$ is denoted by $f(x)$.

(i) Express $f(x)$ in the form $\frac{A}{x-5} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$, where A, B and C are constants. **[4]**

(ii) Hence find the exact value of $\int_1^2 f(x)\,dx$. **[5]**

7 **(i)** The vector $\mathbf{u} = \frac{3}{13}\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is perpendicular to the vector $4\mathbf{i} + \mathbf{k}$ and to the vector $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. Find the values of b and c, and show that $\mathbf{u}$ is a unit vector. **[6]**

(ii) Calculate, to the nearest degree, the angle between the vectors $4\mathbf{i} + \mathbf{k}$ and $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. **[3]**

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8 **(i)** Given that $14x^2 - 7xy + y^2 = 2$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{28x - 7y}{7x - 2y}$. **[4]**

(ii) The points L and M on the curve $14x^2 - 7xy + y^2 = 2$ each have x-coordinate 1. The tangents to the curve at L and M meet at N. Find the coordinates of N. **[6]**

9 A tank contains water which is heated by an electric water heater working under the action of a thermostat. The temperature of the water, θ °C, may be modelled as follows. When the water heater is first switched on, $\theta = 40$. The heater causes the temperature to increase at a rate k_1 °C per second, where k_1 is a constant, until $\theta = 60$. The heater then switches off.

(i) Write down, in terms of k_1, how long it takes for the temperature to increase from 40 °C to 60 °C. **[1]**

The temperature of the water then immediately starts to decrease at a variable rate $k_2(\theta - 20)$ °C per second, where k_2 is a constant, until $\theta = 40$.

(ii) Write down a differential equation to represent the situation as the temperature is decreasing. **[1]**

(iii) Find the total length of time for the temperature to increase from 40 °C to 60 °C and then decrease to 40 °C. Give your answer in terms of k_1 and k_2. **[8]**

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1	<u>Long Division</u> For leading term $3x^2$ in quotient	B1	
	Suff evid of div process (ax^2 , mult back, attempt sub)	M1	
	(Quotient) = $3x^2-4x-5$	A1	
	(Remainder) = $-x+2$	A1	
	<u>Identity</u> $3x^4-x^3-3x^2-14x-8=Q\left(x^2+x+2\right)+R$	*M1	
	$Q=ax^2+bx+c, R=dx+e$ & attempt ≥ 3 ops.	dep*M1	If $a=3$,this $\Rightarrow$ 1 operation
	$a=3, b=-4, c=-5$	A1	dep*M1; Q $=ax^2+bx+c$
	$d=-1, e=2$	A1	
	<u>Inspection</u> Use 'Identity' method; if R = e, check cf(x) correct before awarding 2nd M1		
		4	
2	<u>Indefinite Integral</u> Attempt to connect $\mathrm{d}x$ & $\mathrm{d}\theta$	*M1	Incl $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ or $\frac{\mathrm{d}\theta}{\mathrm{d}x}$; not $\mathrm{d}x=\mathrm{d}\theta$
	Reduce to $\int 1-\tan^2\theta\,(\mathrm{d}\theta)$	A1	A0 if $\frac{\mathrm{d}\theta}{\mathrm{d}x}=\sec^2\theta$; but allow all following A marks
	Use $\tan^2\theta=(1,-1)+\left(\sec^2\theta,-\sec^2\theta\right)$	dep*M1	
	Produce $\int 2-\sec^2\theta\,(\mathrm{d}\theta)$	A1	
	Correct √ integration of function of type $d+e\sec^2\theta$	√A1	including d = 0
	EITHER Attempt limits change (allow degrees here)	M1	(This is 'limits' aspect; the integ need not be accurate)
	OR Attempt integ, re-subst & use original ($\sqrt{3}$,1)		
	$\frac{1}{6}\pi-\sqrt{3}+1$ isw Exact answer required	A1	
		7	

3 (i)	$\left(1+\frac{x}{a}\right)^{-2}=1+(-2)\frac{x}{a}+\frac{-2.-3}{2}\left(\frac{x}{a}\right)^{2}+\ldots$	M1		Check 3rd term; accept $\frac{x^2}{a}$
	$=1-\dfrac{2x}{a}+\ldots$ or $1+\left(-\dfrac{2x}{a}\right)$	B1		or $1-2xa^{-1}$ (Ind of M1)
	$\ldots+\frac{3x^2}{a^2}+\ldots$ (or $3\left(\frac{x}{a}\right)^2$ or $3x^2a^{-2}$)	A1		Accept $\frac{6}{2}$ for 3
	$(a+x)^{-2}=\frac{1}{a^2}\left\{\text{their expansion of }\left(1+\frac{x}{a}\right)^{-2}\right\}$ mult out	√A1	**4**	$\frac{1}{a^2}-\frac{2x}{a^3}+\frac{3x^2}{a^4}$ **;** accept eg a^{-2}
(ii)	Mult out $(1-x)$(their exp) to produce all terms/cfs(x^2)	M1		Ignore other terms
	Produce $\frac{3}{a^2}+\frac{2}{a}(=0)$ or $\frac{3}{a^4}+\frac{2}{a^3}$ $(=0)$ or AEF	A1		Accept x^2 if in both terms
	$a=-\frac{3}{2}$ www seen anywhere in (i) or (ii)	A1	**3**	Disregard any ref to $a=0$
		7		
4 (i)	Differentiate as a product, $u\,\mathrm{d}v+v\,\mathrm{d}u$	M1		or as 2 separate products
	$\dfrac{\mathrm{d}}{\mathrm{d}x}(\sin 2x)=2\cos 2x$ <u>or</u> $\dfrac{\mathrm{d}}{\mathrm{d}x}(\cos 2x)=-2\sin 2x$	B1		
	$\mathrm{e}^x(2\cos 2x+4\sin 2x)+\mathrm{e}^x(\sin 2x-2\cos 2x)$	A1		terms may be in diff order
	Simplify to $5\,\mathrm{e}^x\sin 2x$ www	A1	**4**	Accept $10\mathrm{e}^x\sin x\cos x$
(ii)	Provided result (i) is of form $k\,\mathrm{e}^x\sin 2x$, k const			
	$\int \mathrm{e}^x\sin 2x\,\mathrm{d}x=\dfrac{1}{k}\mathrm{e}^x(\sin 2x-2\cos 2x)$	B1		
	$[\mathrm{e}^x(\sin 2x-2\cos 2x)]_0^{\frac{1}{4}\pi}=\mathrm{e}^{\frac{1}{4}\pi}+2$	B1		
	$\frac{1}{5}\left(\mathrm{e}^{\frac{1}{4}\pi}+2\right)$	B1	**3**	Exact form to be seen

SR Although 'Hence', award M2 for double integration by parts and solving + A1 for correct answer.

7

5 (i)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ aef used	M1	
	$= \frac{4t+3t^2}{2+2t}$	A1	
	Attempt to find t from one/both equations	M1	<u>or</u> diff (ii) cartesian eqn $\rightarrow$ M1
	State/imply $t=-3$ is only solution of both equations	A1	subst $(3,-9)$,solve for $\frac{dy}{dx} \rightarrow$ M1
	Gradient of curve = $-\frac{15}{4}$ or $\frac{-15}{4}$ or $\frac{15}{-4}$	A1 **5**	grad of curve = $-\frac{15}{4} \rightarrow$ A1
	[**SR** If $t=1$ is given as solution & not disqualified, award A0 + √A1 for grad = $-\frac{15}{4}$ & $\frac{7}{4}$;		
	If $t=1$ is given/used as only solution, award A0 + √A1 for grad = $\frac{7}{4}$]		
(ii)	$\frac{y}{x} = t$	B1	
	Substitute into either parametric eqn	M1	
	Final answer $x^3 = 2xy + y^2$	A2 **4**	
	[**SR** Any correct unsimplified form (involving fractions or common factors) $\rightarrow$ A1]		
		9	

6 (i)	$4x \equiv A(x-3)^2 + B(x-3)(x-5) + C(x-5)$	M1	
	$A = 5$	A1	'cover-up' rule, award B1
	$B = -5$	A1	
	$C = -6$	A1 **4**	'cover-up' rule, award B1
	Cands adopting other alg. manip. may be awarded M1 for a full satis method + 3 @ A1		
(ii)	$\int \frac{A}{x-5} dx = A\ln(5-x)$ or $A\ln\lvert 5-x\rvert$ or $A\ln\lvert x-5\rvert$	√B1	but <u>not</u> $A\ln(x-5)$
	$\int \frac{B}{x-3} dx = B\ln(3-x)$ or $B\ln\lvert 3-x\rvert$ or $B\ln\lvert x-3\rvert$	√B1	but <u>not</u> $B\ln(x-3)$
	<u>If candidate is awarded B0,B0</u>, then award **SR** √ B1 for $A\ln(x-5)$ **and** $B\ln(x-3)$		
	$\int \frac{C}{(x-3)^2} dx = -\frac{C}{x-3}$	√B1	
	$5\ln\frac{3}{4} + 5\ln 2$ aef, isw √ $A\ln\frac{3}{4} - B\ln 2$	√ B1	Allow if **SR** B1 awarded
	-3 √ $\frac{1}{2}C$	√B1 **5**	
	[Mark at earliest correct stage & isw; no ln 1]	**9**	

7	(i)	Attempt scalar prod {**u**.(4**i** + **k**) or **u**.(4**i** + 3**j** + 2**k**)}= 0	M1	where **u** is the given vector
		Obtain $\frac{12}{13}+c=0$ <u>or</u> $\frac{12}{13}+3b+2c=0$	A1	
		$c=-\frac{12}{13}$	A1	
		$b=\frac{4}{13}$	A1	cao No ft
		Evaluate $\left(\frac{3}{13}\right)^2+(\text{their } b)^2+(\text{their } c)^2$	M1	Ignore non-mention of $\sqrt{\ }$
		Obtain $\frac{9}{169}+\frac{144}{169}+\frac{16}{169}=1$ AG	A1 **6**	Ignore non-mention of $\sqrt{\ }$
	(ii)	Use $\cos\theta=\frac{\boldsymbol{x}.\boldsymbol{y}}{\lvert\boldsymbol{x}\rvert\lvert\boldsymbol{y}\rvert}$	M1	
		Correct method for finding scalar product	M1	
		36° (35.837653…) Accept 0.625 (rad)	A1 **3**	From $\frac{18}{\sqrt{17}\sqrt{29}}$

SR If 4**i**+**k** = (4,1,0) in (i) & (ii), mark as scheme but allow final A1 for 31°(31.160968) or 0.544

9

8	(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(y^2\right)=2y\frac{\mathrm{d}y}{\mathrm{d}x}$	B1	
		$\frac{\mathrm{d}}{\mathrm{d}x}(uv)=u\ \mathrm{d}v+v\ \mathrm{d}u$ used on $(-7)xy$	M1	
		$\frac{\mathrm{d}}{\mathrm{d}x}\left(14x^2-7xy+y^2\right)=28x-7x\frac{\mathrm{d}y}{\mathrm{d}x}-7y+2y\frac{\mathrm{d}y}{\mathrm{d}x}$	A1	(= 0)
		$2y\frac{\mathrm{d}y}{\mathrm{d}x}-7x\frac{\mathrm{d}y}{\mathrm{d}x}=7y-28x\rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{28x-7y}{7x-2y}$ www AG	A1 **4**	As AG, intermed step nec
	(ii)	Subst x = 1 into eqn curve & solve quadratic eqn in y	M1	$(y=3 \text{ or } 4)$
		Subst x = 1 and (one of) their y-value(s) into given $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=7 \text{ or } 0\right)$
		Find eqn of tgt, with their $\frac{\mathrm{d}y}{\mathrm{d}x}$, going through (1, their y)	*M1	using (one of) y value(s)
		Produce either $y=7x-4$ or $y=4$	A1	
		Solve simultaneously their two equations	dep*M1	provided they have two
		Produce $x=\frac{8}{7}$	A1 **6**	

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9	**(i)**	$\frac{20}{k_1}$ (seconds)	B1 **1**	
	(ii)	$\frac{d\theta}{dt} = -k_2(\theta - 20)$	B1 **1**	
	(iii)	Separate variables or invert each side	M1	Correct eqn or very similar
		Correct int of each side (+ c)	A1,A1	for each integration
		Subst $\theta = 60$ when $t = 0$ into eqn containing 'c'	M1	or $\theta = 60$ when t = their (**i**)
		c (or $-c$) = ln 40 or $\frac{1}{k_2}\ln 40$ or $\frac{1}{k_2}\ln 40k_2$	A1	Check carefully their 'c'
		Subst their value of c and $\theta = 40$ back into equation	M1	Use scheme on LHS
		$t = \frac{1}{k_2}\ln 2$	A1	Ignore scheme on LHS
		Total time = $\frac{1}{k_2}\ln 2 +$ their (i) (seconds)	√A1 **8**	

SR If the negative sign is omitted in part **(ii)**, allow all marks in **(iii)** with ln 2 replaced by $\ln\frac{1}{2}$.

SR If definite integrals used, allow M1 for eqn where $t = 0$ and $\theta = 60$ correspond; a second M1 for eqn where $t = t$ and $\theta = 40$ correspond & M1 for correct use of limits. Final answer scores 2.

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