RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

4724/01

MATHEMATICS

Core Mathematics 4

THURSDAY 14 JUNE 2007

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

 OCR is an exempt Charity **[Turn over**

1 The equation of a curve is $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{3x+1}{(x+2)(x-3)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [2]

(ii) Hence find $\mathrm{f}'(x)$ and deduce that the gradient of the curve is negative at all points on the curve. [3]

2 Find the exact value of $\displaystyle\int_0^1 x^2 \mathrm{e}^x \, \mathrm{d}x$. [6]

3 Find the exact volume generated when the region enclosed between the x-axis and the portion of the curve $y = \sin x$ between $x = 0$ and $x = \pi$ is rotated completely about the x-axis. [6]

4 **(i)** Expand $(2+x)^{-2}$ in ascending powers of x up to and including the term in x^3, and state the set of values of x for which the expansion is valid. [5]

(ii) Hence find the coefficient of x^3 in the expansion of $\dfrac{1+x^2}{(2+x)^2}$. [2]

5 A curve C has parametric equations

$$x = \cos t, \quad y = 3 + 2\cos 2t, \quad \text{where } 0 \leqslant t \leqslant \pi.$$

(i) Express $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t and hence show that the gradient at any point on C cannot exceed 8. [4]

(ii) Show that all points on C satisfy the cartesian equation $y = 4x^2 + 1$. [3]

(iii) Sketch the curve $y = 4x^2 + 1$ and indicate on your sketch the part which represents C. [2]

6 The equation of a curve is $x^2 + 3xy + 4y^2 = 58$. Find the equation of the normal at the point (2, 3) on the curve, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [8]

7 **(i)** Find the quotient and the remainder when $2x^3 + 3x^2 + 9x + 12$ is divided by $x^2 + 4$. [4]

(ii) Hence express $\dfrac{2x^3 + 3x^2 + 9x + 12}{x^2+4}$ in the form $Ax + B + \dfrac{Cx+D}{x^2+4}$, where the values of the constants A, B, C and D are to be stated. [1]

(iii) Use the result of part **(ii)** to find the exact value of $\displaystyle\int_1^3 \frac{2x^3 + 3x^2 + 9x + 12}{x^2+4} \, \mathrm{d}x$. [5]

8 The height, h metres, of a shrub t years after planting is given by the differential equation

$$\frac{dh}{dt} = \frac{6-h}{20}.$$

A shrub is planted when its height is 1 m.

(i) Show by integration that $t = 20\ln\left(\frac{5}{6-h}\right)$. [6]

(ii) How long after planting will the shrub reach a height of 2 m? [1]

(iii) Find the height of the shrub 10 years after planting. [2]

(iv) State the maximum possible height of the shrub. [1]

9 Lines L_1, L_2 and L_3 have vector equations

$$L_1: \ \mathbf{r} = (5\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + s(-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}),$$
$$L_2: \ \mathbf{r} = (3\mathbf{i} - 8\mathbf{j}) + t(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}),$$
$$L_3: \ \mathbf{r} = (2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + u(3\mathbf{i} + c\mathbf{j} + \mathbf{k}).$$

(i) Calculate the acute angle between L_1 and L_2. [4]

(ii) Given that L_1 and L_3 are parallel, find the value of c. [2]

(iii) Given instead that L_2 and L_3 intersect, find the value of c. [5]

Qu	Answer	Mark		Guidance
1	(i) Correct format $\frac{A}{x+2}+\frac{B}{x-3}$	M1		s.o.i. in answer
	$A=1$ and $B=2$	A1	2	for both
	(ii) $-A(x+2)^{-2}-B(x-3)^{-2}$ f.t.	√A1		
	Convincing statement that each denom > 0	B1		accept ≥ 0. Do not accept $x^2>0$.
	State whole exp < 0 **AG**	B1	3	Dep on previous 4 marks.
				5
2	Use parts with $u=x^2, dv=e^x$	*M1		obtaining a result $f(x)+/-\int g(x)(dx)$
	Obtain $x^2e^x-\int 2xe^x\,(dx)$	A1		
	Attempt parts again with $u=(-)(2)x, dv=e^x$	M1		
	Final = $(x^2-2x+2)e^x$ AEF incl brackets	A1		s.o.i. eg $e+(-2x+2)e^x$
	Use limits correctly throughout	dep*M1		Tolerate (their value for $x=1$) (-0)
	$e^{(1)}-2$ ISW Exact answer only	A1	6	Allow 0.718 → M1
				6
3	Volume = $(k)\int_0^{\pi}\sin^2 x\,(dx)$	B1		where $k=\pi, 2\pi$ or 1; limits necessary
	Suitable method for integrating $\sin^2 x$	*M1		eg $\int +/-1+/-\cos 2x\,(dx)$ or single integ by parts & connect to $\int \sin^2 x\,(dx)$
	$\int \sin^2 x\,(dx)=\frac{1}{2}\int 1-\cos 2x\,(dx)$	A1		or $-\sin x\cos x+\int \cos^2 x(dx)$
	$\int \cos 2x\,(dx)=\frac{1}{2}\sin 2x$	A1		or $-\sin x\cos x+\int 1-\sin^2 x\,(dx)$
	Use limits correctly	dep*M1		
	Volume = $\frac{1}{2}\pi^2$ WWW Exact answer	A1	6	**Beware**: wrong working leading to $\frac{1}{2}\pi^2$
				6
4	(i) $\left(1+\frac{x}{2}\right)^{-2}$ $=1+(-2)\left(\frac{x}{2}\right)+\frac{-2.-3}{2}\left(\frac{x}{2}\right)^2+\frac{-2.-3.-4}{3!}\left(\frac{x}{2}\right)^3$	M1		Clear indication of method of ≥ 3 terms
	$=1-x$	B1		First two terms, not dependent on M1
	$+\frac{3}{4}x^2-\frac{1}{2}x^3$	A1		For both third and fourth terms
	$(2+x)^{-2}=\frac{1}{4}\left(\text{their exp of }(1+ax)^{-2}\right)$ mult out	√B1		Correct: $\frac{1}{4}-\frac{1}{4}x+\frac{3}{16}x^2-\frac{1}{8}x^3$
	$\lvert x\rvert<2$ or $-2<x<2$ (but not $\lvert\frac{1}{2}x\rvert<1$)	B1	5	
	(ii) If (i) is $a+bx+cx^2+dx^3$ evaluate $b+d$	M1		
	$-\frac{3}{8}\ (x^3)$	√A1	2	Follow-through from $b+d$
				7

5(i)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1		
	$= \frac{-4\sin 2t}{-\sin t}$	A1		Accept $\frac{4\sin 2t}{\sin t}$ WWW
	$= 8\cos t$	A1		
	≤ 8 **AG**	A1	4	with brief explanation eg $\cos t \le 1$
	(ii) Use $\cos 2t = 2\cos^2 t +/- 1$ or $1 - 2\cos^2 t$	M1		<u>If starting with</u> $y = 4x^2 + 1$, then
	Use correct version $\cos 2t = 2\cos^2 t - 1$	A1		Subst $x = \cos t, y = 3 + 2\cos 2t$ M1
	Produce WWW $y = 4x^2 + 1$ **AG**	A1	3	<u>Either</u> substitute <u>a</u> formula for cos 2*t* M1 Obtain 0=0 or $4\cos^2 t + 1 = 4\cos^2 t + 1$ A1 <u>Or</u> Manip to give formula for cos 2*t* M1 Obtain corr formula & say it's correct A1
	(iii) U-shaped parabola abve *x*-axis, sym abt *y*-axis	B1		Any labelling must be correct
	Portion between $(-1, 5)$ and $(1, 5)$	B1	2	either $x = \pm 1$ or $y = 5$ must be marked
	N.B. If (ii) answered or quoted before (i) attempted,	allow in part		(i) B2 for $\frac{dy}{dx} = 8x$ +B1,B1 if earned. **9**
6	(i) $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1		
	Using $d(uv) = u\,dv + v\,du$ for the $(3)xy$ term	M1		
	$\frac{d}{dx}(x^2 + 3xy + 4y^2) = 2x + 3x\frac{dy}{dx} + 3y + 8y\frac{dy}{dx}$	A1		
	Solve for $\frac{dy}{dx}$ & subst (x, y) = (2,3)	M1		or v.v. Subst now or at normal eqn stage; (M1 dep on either/both B1 M1 earned)
	$\frac{dy}{dx} = -\frac{13}{30}$	A1		Implied if grad normal = $\frac{30}{13}$
	Grad normal = $\frac{30}{13}$ follow-through	√B1		This f.t. mark awarded only if numerical
	Find equ <u>any</u> line thro (2,3) with <u>any</u> num grad	M1		
	$30x - 13y - 21 = 0$ AEF	A1	8	No fractions in final answer **8**
7	(i) Leading term in quotient = $2x$	B1		
	<u>Suff evidence</u> of division or identity process	M1		
	Quotient = $2x + 3$	A1		Stated or in relevant position in division
	Remainder = x	A1	4	Accept $\frac{x}{x^2+4}$ as remainder
	(ii) their quotient + $\frac{\text{their remainder}}{x^2+4}$	√B1	1	$2x + 3 + \frac{x}{x^2+4}$
	(iii) <u>Working with their expression in part (ii)</u>			
	their $Ax + B$ integrated as $\frac{1}{2}Ax^2 + Bx$	√B1		
	their $\frac{Cx}{x^2+4}$ integrated as $k\ln(x^2+4)$	M1		Ignore any integration of $\frac{D}{x^2+4}$
	$k = \frac{1}{2}C$	√A1		
	Limits used correctly throughout	M1		
	$14 + \frac{1}{2}\ln\frac{13}{5}$	A1	5	logs need not be combined.
				10

8	(i) Sep variables eg $\int \frac{1}{6-h}(\mathrm{d}h) = \int \frac{1}{20}(\mathrm{d}t)$	*M1		s.o.i. <u>Or</u> $\frac{\mathrm{d}t}{\mathrm{d}h} = \frac{20}{6-h}$ $\rightarrow$ M1
	LHS = $-\ln(6-h)$	A1		& then $t = -20\ln(6-h)$ (+c) $\rightarrow$ A1+A1
	RHS = $\frac{1}{20}t$ (+c)	A1		
	Subst $t = 0, h = 1$ into equation containing 'c'	dep*M1		
	Correct value of their c = $-(20)\ln 5$ **WWW**	A1		or $(20)\ln 5$ if on LHS
	Produce $t = 20\ln\frac{5}{6-h}$ **WWW AG**	A1	**6**	Must see $\ln 5 - \ln(6-h)$
	(ii) When h = 2, t = $20\ln\frac{5}{4}$ =4.46(2871)	B1	**1**	Accept 4.5, $4\frac{1}{2}$
	(iii) Solve $10 = 20\ln\frac{5}{6-h}$ to $\frac{5}{6-h} = \mathrm{e}^{0.5}$	M1		or $\frac{6-h}{5} = \mathrm{e}^{-0.5}$ or suitable $\frac{1}{2}$-way stage
	h = 2.97(2.9673467…)	A1	**2**	$6 - 5\mathrm{e}^{-0.5}$ or $6 - \mathrm{e}^{1.109}$
	[In (ii),(iii) accept non-decimal (exact) answers but -1 once.**]** Accept truncated values in (ii),(iii).			
	(iv) Any indication of (approximately) 6 (m)	B1	**1**	
				10
9	(i) Use $-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ only	M1		
	Correct method for scalar product	M1		of <u>any</u> two vectors ($-6 + 24 - 4 = 14$)
	Correct method for magnitude	M1		of <u>any</u> vector ($\sqrt{36+64+4} = \sqrt{104}$ or $\sqrt{1+9+4} = \sqrt{14}$)
	68 or 68.5 (68.47546); 1.2(0) (1.1951222) rad	A1	**4**	
	[N.B. 61 (60.562) will probably have been generated by 5**i** – **j** -2**k** and 3**i** – 8**j**]			
	(ii) Indication that relevant vectors are parallel	M1		$-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$ & $3\mathbf{i} + c\mathbf{j} + \mathbf{k}$ with some indic of method of attack eg $-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k} = \lambda(3\mathbf{i} + c\mathbf{j} + \mathbf{k})$
	c **=** -4	A1	**2**	$c = -4$ WW $\rightarrow$ B2
	(iii) Produce 2/3 equations containing t,u (& c)	M1		eg $3 + t = 2 + 3u, -8 + 3t = 1 + cu$ and $2t = 3 + u$
	Solve the 2 equations not containing 'c'	M1		
	t = 2, u = 1	A1		
	Subst their (t,u) into equation containing c	M1		
	c = -3	A1	**5**	
	<u>Alternative method for final 4 marks</u> Solve two equations, one with 'c', for t and u in terms of c, and substitute into third equation	(M2)		
	$c = -3$	(A2)		**11**