OCR RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4722

Core Mathematics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 15 January 2010
Afternoon

Duration: 1 hour 30 minutes

* 4 7 2 2 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(i)** Show that the equation

$$2\sin^2 x = 5\cos x - 1$$

can be expressed in the form

$$2\cos^2 x + 5\cos x - 3 = 0. \quad \textbf{[2]}$$

(ii) Hence solve the equation

$$2\sin^2 x = 5\cos x - 1,$$

giving all values of x between $0°$ and $360°$. **[4]**

2 The gradient of a curve is given by $\frac{dy}{dx} = 6x - 4$. The curve passes through the distinct points (2, 5) and (p, 5).

(i) Find the equation of the curve. **[4]**

(ii) Find the value of p. **[3]**

3 **(i)** Find and simplify the first four terms in the expansion of $(2 - x)^7$ in ascending powers of x. **[4]**

(ii) Hence find the coefficient of w^6 in the expansion of $\left(2 - \frac{1}{4}w^2\right)^7$. **[2]**

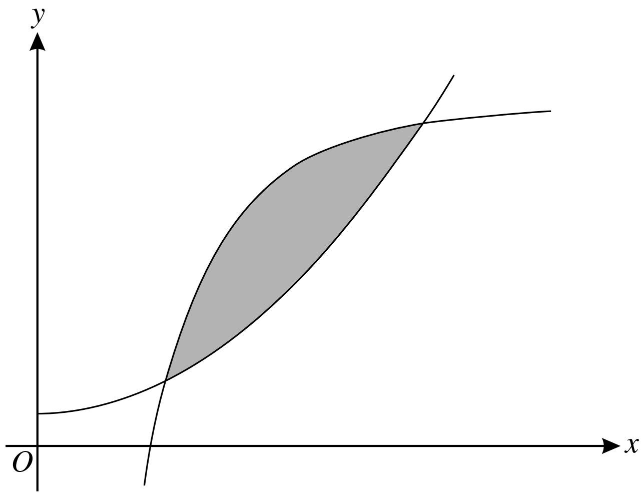
4 **(i)** Use the trapezium rule, with 4 strips each of width 0.5, to find an approximate value for

$$\int_3^5 \log_{10}(2 + x)\, dx,$$

giving your answer correct to 3 significant figures. **[4]**

(ii) Use your answer to part **(i)** to deduce an approximate value for $\int_3^5 \log_{10}\sqrt{2 + x}\, dx$, showing your method clearly. **[2]**

5



The diagram shows parts of the curves $y = x^2 + 1$ and $y = 11 - \frac{9}{x^2}$, which intersect at $(1, 2)$ and $(3, 10)$. Use integration to find the exact area of the shaded region enclosed between the two curves. **[7]**

6 The cubic polynomial $f(x)$ is given by

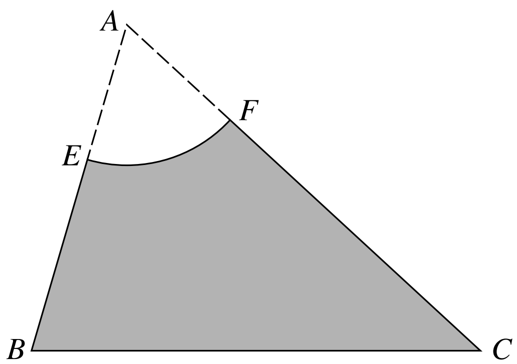
$$f(x) = 2x^3 + ax^2 + bx + 15,$$

where a and b are constants. It is given that $(x + 3)$ is a factor of $f(x)$ and that, when $f(x)$ is divided by $(x - 2)$, the remainder is 35.

(i) Find the values of a and b. **[6]**

(ii) Using these values of a and b, divide $f(x)$ by $(x + 3)$. **[3]**

7



The diagram shows triangle ABC, with $AB = 10$ cm, $BC = 13$ cm and $CA = 14$ cm. E and F are points on AB and AC respectively such that $AE = AF = 4$ cm. The sector AEF of a circle with centre A is removed to leave the shaded region $EBCF$.

(i) Show that angle CAB is 1.10 radians, correct to 3 significant figures. **[2]**

(ii) Find the perimeter of the shaded region $EBCF$. **[3]**

(iii) Find the area of the shaded region $EBCF$. **[5]**

8 A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 8 \qquad \text{and} \qquad u_{n+1} = u_n + 3.$$

(i) Show that $u_5 = 20$. **[2]**

(ii) The nth term of the sequence can be written in the form $u_n = pn + q$. State the values of p and q. **[2]**

(iii) State what type of sequence it is. **[1]**

(iv) Find the value of N such that $\sum_{n=1}^{2N} u_n - \sum_{n=1}^{N} u_n = 1256$. **[5]**

9 **(i)** Sketch the curve $y = 6 \times 5^x$, stating the coordinates of any points of intersection with the axes. **[3]**

(ii) The point P on the curve $y = 9^x$ has y-coordinate equal to 150. Use logarithms to find the x-coordinate of P, correct to 3 significant figures. **[3]**

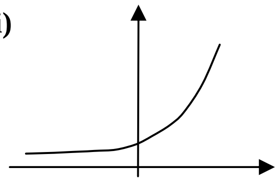
(iii) The curves $y = 6 \times 5^x$ and $y = 9^x$ intersect at the point Q. Show that the x-coordinate of Q can be written as $x = \dfrac{1 + \log_3 2}{2 - \log_3 5}$. **[5]**

4722 Core Mathematics 2

1	**(i)**	$2(1-\cos^2 x) = 5\cos x - 1$	M1		Use $\sin^2 x = 1 - \cos^2 x$
		$2\cos^2 x + 5\cos x - 3 = 0$ **A.G.**	A1	**2**	Show given equation correctly
	(ii)	$(2\cos x - 1)(\cos x + 3) = 0$	M1		Recognise equation as quadratic in cos x and attempt recognisable method to solve
		$\cos x = ½$	M1		Attempt to find x from root(s) of quadratic
		$x = 60°$	A1		Obtain 60° or $^{\pi}/_{3}$ or 1.05 rad
		$x = 300°$	A1√	**4**	Obtain 300° only (or 360° – their x) and no extra in range **SR** answer only is B1 B1
				6	
2	**(i)**	$\int(6x-4)\,dx = 3x^2 - 4x + c$	M1*		Attempt integration (inc. in power for at least one term)
			A1		Obtain $3x^2 - 4x$ (or unsimplified equiv), with or without + c
		$y = 3x^2 - 4x + c \Rightarrow 5 = 12 - 8 + c$ $\Rightarrow c = 1$	M1dep*		Use (2, 5) to find c
		Hence $y = 3x^2 - 4x + 1$	A1	**4**	Obtain $y = 3x^2 - 4x + 1$
	(ii)	$3p^2 - 4p + 1 = 5$	M1*		Equate their y (from integration attempt) to 5
		$3p^2 - 4p - 4 = 0$ $(p-2)(3p+2) = 0$	M1dep*		Attempt to solve three term quadratic
		$p = {}^{-2}/_{3}$	A1	**3**	Obtain $p = {}^{-2}/_{3}$ (allow any variable) from correct working; condone $p = 2$ still present, but A0 if extra incorrect solution
				7	
3	**(i)**	$(2-x)^7 = 128 - 448x + 672x^2 - 560x^3$	M1		Attempt (at least) two relevant terms – product of binomial coeff, 2 and x (or expansion attempt that considers all 7 brackets)
			A1		Obtain $128 - 448x$
			A1		Obtain $672x^2$
			A1	**4**	Obtain $-560x^3$
	(ii)	$-560 \times ({}^{1}/_{4})^3 = {}^{-35}/_{4}$	M1		Attempt to use coeff of x^3 from **(i)**, with clear intention to cube $^{1}/_{4}$
			A1	**2**	Obtain ${}^{-35}/_{4}$ (w^6), (allow ${}^{35}/_{4}$ from $+560x^3$ in **(i)**)
				6	

4	(i)	$\int_3^5 \log_{10}(2+x)\,dx \approx \frac{1}{2}\times\frac{1}{2}\times(\log 5 + 2\log 5.5 +$	M1		Attempt y-coords for at least 4 of the correct 5 x-coords only
		$2\log 6 + 2\log 6.5 + \log 7)$	M1		Use correct trapezium rule, any h, to find area between $x = 3$ and $x = 5$
			M1		Correct h (soi) for their y-values
		≈ 1.55	A1	4	Obtain 1.55
	(ii)	$\int_3^5 \log_{10}(2+x)^{\frac{1}{2}}\,dx = \frac{1}{2}\int_3^5 \log_{10}(2+x)\,dx$	B1√		Divide by 2, or equiv, at any stage to obtain 0.78 or 0.77,
		$\approx \frac{1}{2} \times 1.55$			following their answer to **(i)**
		≈ 0.78	B1	2	Explicitly use $\log\sqrt{a} = \frac{1}{2}\log a$ on a single term
				6	
5		$\int_1^3 \{(11-9x^{-2})-(x^2+1)\}\,dx = \left[9x^{-1} - \frac{1}{3}x^3 + 10x\right]_1^3$	M1		Attempt subtraction (correct order) at any point
		$= (3 - 9 + 30) - (9 - \frac{1}{3} + 10)$	M1		Attempt integration – inc. in power for at least one term
		$= 24 - 18\frac{2}{3}$	A1		Obtain $\pm(-\frac{1}{3}x^3 + 10x)$ or $11x$ and $\frac{1}{3}x^3 + x$
		$= 5\frac{1}{3}$	M1		Obtain remaining term of form kx^{-1}
		OR	A1		Obtain $\pm 9x^{-1}$ or any unsimplified equiv
		$\left[11x + 9x^{-1}\right]_1^3 - \left[\frac{1}{3}x^3 + x\right]_1^3$	M1		Use limits $x = 1, 3$ – correct order & subtraction
		$= [(33 + 3) - (11 + 9)] - [(9 + 3) - (\frac{1}{3} + 1)]$	A1	7	Obtain $5\frac{1}{3}$, or exact equiv
		$= 16 - 10\frac{2}{3}$			
		$= 5\frac{1}{3}$			
				7	
6	(i)	$f(-3) = 0 \Rightarrow -54 + 9a - 3b + 15 = 0$	M1		Attempt f(–3) and equate to 0, or equiv method
		$3a - b = 13$	A1		Obtain $3a - b = 13$, or unsimplified equiv
		$f(2) = 35 \Rightarrow 16 + 4a + 2b + 15 = 35$	M1		Attempt f(2) and equate to 35, or equiv method
		$2a + b = 2$	A1		Obtain $2a + b = 2$, or unsimplified equiv
		Hence $a = 3$, $b = -4$	M1		Attempt to solve simultaneous eqns
			A1	6	Obtain $a = 3$, $b = -4$
	(ii)	$f(x) = (x + 3)(2x^2 - 3x + 5)$	M1		Attempt complete division by $(x + 3)$, or equiv
			A1		Obtain $2x^2 - 3x + c$ or $2x^2 + bx + 5$, from correct $f(x)$
		ie quotient is $(2x^2 - 3x + 5)$	A1	3	Obtain $2x^2 - 3x + 5$ (state or imply as quotient)
				9	

Qn	Part	Answer	Marks		Guidance
7	(i)	$13^2 = 10^2 + 14^2 - 2 \times 10 \times 14 \times \cos\theta$	M1		Attempt to use correct cosine rule in ΔABC
		$\cos\theta = 0.4536$			
		$\theta = 1.10$ **A.G.**	A1	2	Obtain 1.10 radians (allow 1.1 radians) **SR** B1 only for verification of 1.10, unless complete method
	(ii)	arc $EF = 4 \times 1.10 = 4.4$	B1		State or imply $EF = 4.4$cm (allow 4×1.10)
		perimeter $= 4.4 + 10 + 13 + 6$	M1		Attempt perimeter of region - sum of arc and three sides with attempt to subtract 4 from at least one relevant side
		$= 33.4$ cm	A1	3	Obtain 33.4 cm
	(iii)	area $AEF = ½ \text{ x } 4^2 \times 1.1$	M1		Attempt use of (½) $r^2\theta$, with $r = 4$ and $\theta = 1.10$
		$= 8.8$	A1		Obtain 8.8
		area $ABC = ½ \times 10 \times 14 \times \sin 1.1$	M1		Attempt use of (½)$ab\sin\theta$, sides consistent with angle used
		$= 62.4$	A1		Obtain 62.4 or better (allow 62.38 or 62.39)
		hence total area $= 53.6$ cm^2	A1	5	Obtain total area as 53.6 cm^2
				10	
8	(i)	$u_5 = 8 + 4 \times 3$	M1		Attempt $a + (n-1)d$ or equiv inc list of terms
		$= 20$ **A.G.**	A1	2	Obtain 20
	(ii)	$u_n = 3n + 5$ ie $p = 3$, $q = 5$	B1		Obtain correct expression, poss unsimplified, eg $8 + 3(n-1)$
			B1	2	Obtain correct $3n + 5$, or $p = 3$, $q = 5$ stated
	(iii)	arithmetic progression	B1	1	Any mention of arithmetic
	(iv)	$\frac{2N}{2}(16+(2N-1)3)-\frac{N}{2}(16+(N-1)3)=1256$	M1		Attempt S_N, using any correct formula (inc $\sum(3n+5)$)
			M1		Attempt S_{2N}, using any correct formula, with $2N$ consistent (inc $\sum(3n+5)$)
		$26N + 12N^2 - 13N - 3N^2 = 2512$			
		$9N^2 + 13N - 2512 = 0$	M1*		Attempt subtraction (correct order) and equate to 1256
		$(9N + 157)(N - 16) = 0$	M1dep*		Attempt to solve quadratic in N
		$N = 16$	A1	5	Obtain $N = 16$ only, from correct working
					OR: alternative method is to use $^n/_2\,(a + l) = 1256$
			M1		Attempt given difference as single summation with N terms
			M1		Attempt $a = u_{N+1}$
			M1		Attempt $l = u_{2N}$
			M1		Equate to 1256 and attempt to solve quadratic
			A1		Obtain $N = 16$ only, from correct working
				10	

9 (i)		M1		Reasonable graph in both quadrants
		A1		Correct graph in both quadrants
		B1	3	State or imply (0, 6)
(ii)	$9^x = 150$	M1		Introduce logarithms throughout, or equiv with $\log_9$
	$x \log 9 = \log 150$	M1		Use $\log a^b = b \log a$ and attempt correct method to find x
	$x = 2.28$	A1	3	Obtain $x = 2.28$
(iii)	$6 \times 5^x = 9^x$	M1		Form eqn in x and take logs throughout (any base)
	$\log_3 (6 \times 5^x) = \log_3 9^x$	M1		Use $\log a^b = b \log a$ correctly on $\log 5^x$ or $\log 9^x$ or legitimate combination of these two
	$\log_3 6 + x \log_3 5 = x \log_3 9$	M1		Use $\log ab = \log a + \log b$ correctly on $\log (6 \times 5^x)$ or $\log 6$
	$\log_3 3 + \log_3 2 + x \log_3 5 = 2x$	M1		Use $\log_3 9 = 2$ or equiv (need base 3 throughout that line)
	$x(2 - \log_3 5) = 1 + \log_3 2$			
	$x = \dfrac{1+\log_3 2}{2-\log_3 5}$ **A.G.**	A1	5	Obtain $x = \dfrac{1+\log_3 2}{2-\log_3 5}$ convincingly (inc base 3 throughout)
			11	