RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4722

Core Mathematics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 22 May 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 The lengths of the three sides of a triangle are 6.4 cm, 7.0 cm and 11.3 cm.

(i) Find the largest angle in the triangle. **[3]**

(ii) Find the area of the triangle. **[2]**

2 The tenth term of an arithmetic progression is equal to twice the fourth term. The twentieth term of the progression is 44.

(i) Find the first term and the common difference. **[4]**

(ii) Find the sum of the first 50 terms. **[2]**

3 Use logarithms to solve the equation $7^x = 2^{x+1}$, giving the value of x correct to 3 significant figures. **[5]**

4 **(i)** Find the binomial expansion of $(x^2 - 5)^3$, simplifying the terms. **[4]**

(ii) Hence find $\int (x^2 - 5)^3 \, dx$. **[4]**

5 Solve each of the following equations for $0° \leqslant x \leqslant 180°$.

(i) $\sin 2x = 0.5$ **[3]**

(ii) $2\sin^2 x = 2 - \sqrt{3}\cos x$ **[5]**

6 The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 + a$, where a is a constant. The curve passes through the points $(-1, 2)$ and $(2, 17)$. Find the equation of the curve. **[8]**

7 The polynomial $f(x)$ is given by $f(x) = 2x^3 + 9x^2 + 11x - 8$.

(i) Find the remainder when $f(x)$ is divided by $(x + 2)$. **[2]**

(ii) Use the factor theorem to show that $(2x - 1)$ is a factor of $f(x)$. **[2]**

(iii) Express $f(x)$ as a product of a linear factor and a quadratic factor. **[3]**

(iv) State the number of real roots of the equation $f(x) = 0$, giving a reason for your answer. **[2]**

8

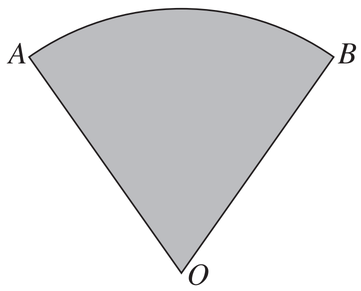

Fig. 1

Fig. 1 shows a sector AOB of a circle, centre O and radius OA. The angle AOB is 1.2 radians and the area of the sector is $60\,\text{cm}^2$.

(i) Find the perimeter of the sector. **[4]**

A pattern on a T-shirt, the start of which is shown in Fig. 2, consists of a sequence of similar sectors. The first sector in the pattern is sector AOB from Fig. 1, and the area of each successive sector is $\frac{3}{5}$ of the area of the previous one.

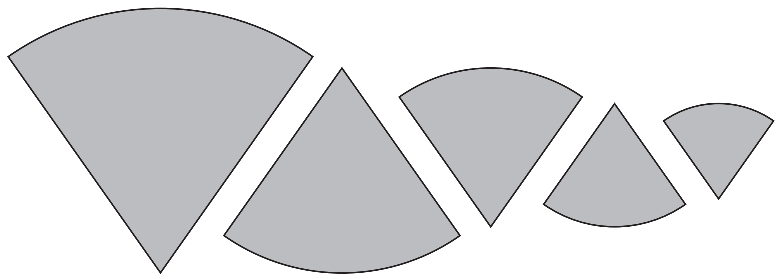

Fig. 2

(ii) (a) Find the area of the fifth sector in the pattern. **[2]**

(b) Find the total area of the first ten sectors in the pattern. **[2]**

(c) Explain why the total area will never exceed a certain limit, no matter how many sectors are used, and state the value of this limit. **[3]**

9 **(i)** Sketch the graph of $y = 4k^x$, where k is a constant such that $k > 1$. State the coordinates of any points of intersection with the axes. **[2]**

(ii) The point P on the curve $y = 4k^x$ has its y-coordinate equal to $20k^2$. Show that the x-coordinate of P may be written as $2 + \log_k 5$. **[4]**

(iii) (a) Use the trapezium rule, with two strips each of width $\frac{1}{2}$, to find an expression for the approximate value of

$$\int_0^1 4k^x \, dx.$$ **[3]**

(b) Given that this approximate value is equal to 16, find the value of k. **[3]**

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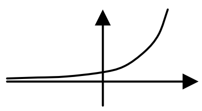
1	**(i)** $\cos\theta = \frac{6.4^2+7.0^2-11.3^2}{2\times6.4\times7.0}$	M1		Attempt use of cosine rule (any angle)
	$= -0.4211$	A1		Obtain one of 115°, 34.2°, 30.9°, 2.01, 0.597, 0.539
	$\theta = 115^\circ$ or 2.01 rads	A1	**3**	Obtain 115° or 2.01 rads, or better
	(ii) area = $\frac{1}{2}\times7\times6.4\times\sin115$	M1		Attempt triangle area using (½)$ab\sin C$, or equiv
	$= 20.3\ \text{cm}^2$	A1	**2**	Obtain 20.3 (cao)
			5	
2	**(i)** $a + 9d = 2(a + 3d)$	M1*		Attempt use of $a + (n - 1)d$ or $a + nd$ at least once for u_4, u_{10} or u_{20}
	$a = 3d$	A1		Obtain $a = 3d$ (or unsimplified equiv) and $a + 19d = 44$
	$a + 19d = 44 \Rightarrow 22d = 44$	M1dep*		Attempt to eliminate one variable from two simultaneous equations in a and d, from u_4, u_{10}, u_{20} and no others
	$d = 2, a = 6$	A1	**4**	Obtain $d = 2$, $a = 6$
	(ii) $S_{50} = {}^{50}/_2$ (2x6 + 49x2)	M1		Attempt S_{50} of AP, using correct formula, with $n = 50$, allow $25(2a + 24d)$
	= 2750	A1	**2**	Obtain 2750
			6	
3	$\log 7^x = \log 2^{x+1}$	M1		Introduce logarithms throughout, or equiv with base 7 or 2
	$x\log 7 = (x+1)\log 2$	M1		Drop power on at least one side
		A1		Obtain correct linear equation (allow with no brackets)
	$x(\log 7 - \log 2) = \log 2$	M1		**Either** expand bracket and attempt to gather x terms, **or** deal correctly with algebraic fraction
	$x = 0.553$	A1	**5**	Obtain $x = 0.55$, or rounding to this, with no errors seen
			5	
4	**(i)** $(x^2-5)^3 = (x^2)^3 + 3(x^2)^2(-5) + 3(x^2)(-5)^2 + (-5)^3$	M1*		Attempt expansion, with product of powers of x^2 and $\pm$ 5, at least 3 terms
	$= x^6 - 15x^4 + 75x^2 - 125$	M1*		Use at least 3 of binomial coeffs of 1, 3, 3, 1
		A1dep*		Obtain at least two correct terms, coeffs simplified
		A1	**4**	Obtain fully correct expansion, coeffs simplified
	OR $(x^2 - 5)^3 = (x^2 - 5)(x^4 - 10x^2 + 25)$	M2		Attempt full expansion of all 3 brackets
	$= x^6 - 15x^4 + 75x^2 - 125$	A1		Obtain at least two correct terms
		A1		Obtain full correct expansion
	(ii) $\int (x^2-5)^3\,dx = \frac{1}{7}x^7 - 3x^5 + 25x^3 - 125x + c$	M1		Attempt integration of terms of form kx^n
		A1√		Obtain at least two correct terms, allow unsimplified coeffs
		A1		Obtain $\frac{1}{7}x^7 - 3x^5 + 25x^3 - 125x$
		B1	**4**	+ c, and no dx or ∫ sign
			8	

5 **(i)**	$2x = 30^o, 150^o$ $x = 15^o, 75^o$	M1 A1 A1	 **3**	Attempt $\sin^{-1}$ 0.5, then divide or multiply by 2 Obtain 15^o (allow $^{\pi}/_{12}$ or 0.262) Obtain 75^o (not radians), and no extra solutions in range
(ii)	$2(1 - \cos^2x) = 2 - \sqrt{3}\cos x$ $2\cos^2x - \sqrt{3}\cos x = 0$ $\cos x\,(2\cos x - \sqrt{3}) = 0$ $\cos x = 0$, $\cos x = ½\sqrt{3}$ $x = 90^o$, $x = 30^o$	M1 A1 M1 A1 B1	 **5**	Use $\sin^2x = 1 - \cos^2x$ Obtain $2\cos^2x - \sqrt{3}\cos x = 0$ or equiv (no constant terms) Attempt to solve quadratic in $\cos x$ Obtain 30^o (allow $^{\pi}/_6$ or 0524), and no extra solns in range Obtain 90^o (allow $^{\pi}/_2$ or 1.57), from correct quadratic only
			SR	answer only B1 one correct solution B1 second correct solution, and no others
			8	
6	$\int(3x^2 + a)\ \mathrm{d}x = x^3 + ax + c$ $(-1, 2) \Rightarrow -1 - a + c = 2$ $(2, 17) \Rightarrow 8 + 2a + c = 17$ $a = 2$, $c = 5$ Hence $y = x^3 + 2x + 5$	M1 A1 A1 M1 A1 M1 A1 A1	 **8**	Attempt to integrate Obtain at least one correct term, allow unsimplified Obtain $x^3 + ax$ Substitute at least one of (–1, 2) or (2, 17) into integration attempt involving a and c Obtain two correct equations, allow unsimplified Attempt to eliminate one variable from two equations in a and c Obtain $a = 2$, $c = 5$, from correct equations State $y = x^3 + 2x + 5$
			8	
7 **(i)**	f(-2) = -16 + 36 – 22 – 8 = -10	M1 A1	 **2**	Attempt f(-2), or equiv Obtain -10
(ii)	f(½) = ¼ + 2¼ + 5½ – 8 = 0 AG	M1 A1	 **2**	Attempt f(½) (no other method allowed) Confirm f(½) = 0, extra line of working required
(iii)	$f(x) = (2x - 1)(x^2 + 5x + 8)$	M1 A1 A1	 **3**	Attempt complete division by $(2x - 1)$ or $(x - ½)$ or equiv Obtain $x^2 + 5x + c$ or $2x^2 + 10x + c$ State $(2x - 1)(x^2 + 5x + 8)$ or $(x - ½)(2x^2 + 10x + 16)$
(iv)	$f(x)$ has one real root ($x = ½$) because $b^2 - 4ac = 25 - 32 = -7$ hence quadratic has no real roots as $-7 < 0$,	B1√ B1√	 **2**	State 1 root, following their quotient, ignore reason Correct calculation, eg discriminant or quadratic formula, following their quotient, or cubic has max at (-2.15, -9.9)
			9	

8 **(i)**	$\frac{1}{2} \times r^2 \times 1.2 = 60$	M1		Attempt $(\frac{1}{2})\, r^2\theta = 60$
	$r = 10$	A1		Obtain $r = 10$
	$r\theta = 10 \times 1.2 = 12$	B1√		State or imply arc length is $1.2r$, following their r
	perimeter = 10 + 10 + 12 = 32 cm	A1	**4**	Obtain 32
(ii)(a)	$u_5 = 60 \times 0.6^4$	M1		Attempt u_5 using ar^4, or list terms
	$= 7.78$	A1	**2**	Obtain 7.78, or better
(b)	$S_{10} = \dfrac{60\left(1 - 0.6^{10}\right)}{1 - 0.6}$	M1		Attempt use of correct sum formula for a GP, or sum terms
	$= 149$	A1	**2**	Obtain 149, or better (allow 149.0 – 149.2 inclusive)
(c)	common ratio is less than 1, so series is convergent and hence sum to infinity exists	B1		series is convergent or $-1 < r < 1$ (allow $r < 1$) or reference to areas getting smaller / adding on less each time
	$S_\infty = \dfrac{60}{1 - 0.6}$	M1		Attempt S_∞ using $\dfrac{a}{1-r}$
	$= 150$	A1	**3**	Obtain $S_\infty = 150$
				SR B1 only for 150 with no method shown
			11	
9 **(i)**		B1		Sketch graph showing exponential growth (both quadrants)
		B1	**2**	State or imply (0, 4)
(ii)	$4k^x = 20k^2$			
	$k^x = 5k^2$	M1		Equate $4k^x$ to $20k^2$ and take logs (any, or no, base)
	$x = \log_k 5k^2$			
	$x = \log_k 5 + \log_k k^2$	M1		Use $\log ab = \log a + \log b$
	$x = 2\log_k k + \log_k 5$	M1		Use $\log a^b = b \log a$
	$x = 2 + \log_k 5$ AG	A1	**4**	Show given answer correctly
OR	$4k^x = 20k^2$			
	$k^x = 5k^2$	M1		Attempt to rewrite as single index
	$k^{x-2} = 5$	A1		Obtain $k^{x-2} = 5$ or equiv eg $4k^{x-2} = 20$
	$x - 2 = \log_k 5$	M1		Take logs (to any base)
	$x = 2 + \log_k 5$ AG	A1		Show given answer correctly
(iii) (a)	area $\approx \frac{1}{2} \times \frac{1}{2} \times \left(4k^0 + 8k^{\frac{1}{2}} + 4k^1\right)$	M1		Attempt y-values at $x = 0$, ½ and 1, and no others
		M1		Attempt to use correct trapezium rule, 3 y-values, $h = \frac{1}{2}$
	$\approx 1 + 2k^{\frac{1}{2}} + k$	A1	**3**	Obtain a correct expression, allow unsimplified
(b)	$1 + 2k^{\frac{1}{2}} + k = 16$	M1		Equate attempt at area to 16
	$\left(k^{\frac{1}{2}} + 1\right)^2 = 16$	M1		Attempt to solve 'disguised' 3 term quadratic
	$k^{\frac{1}{2}} = 3$			
	$k = 9$	A1	**3**	Obtain $k = 9$ only
			12	